LETTERS TO THE EDITOR

PHOTOELASTICITY OF AN EPOXY ADHESIVE

THE exceptional properties of epoxy resins have led to the development of several new techniques in photoelasticity and generally in improving the accuracy and versatility of this method of stress analysis. D' Agostino *et al.*[1] have recommended CN501 for work at room temperature after studying several epoxy resins. Amba Rao[2] reported a rise in figure of merit of Araldites by lowering the content of the hardener while casting. The effect of proportion of phthalic anhydride hardener on the resins, Araldite 6020 and Bakelite ERL 2774, in imparting stress optical properties was investigated by Leven and Wahl[3]. Reduction in hardener proportion has led to ease of casting, least residual stresses, and reduced figure of merit. Leven and Sampson[4] recommended a mixed hardener to reduce exothermal stress in the material during curing. The casting technique, mechanical and photoelastic properties of some epoxy resins of CIBA (INDIA) Ltd. at room temperature were reported by Jaintiprasad[5]. Patil[6] worked on the mechanical and photoelastic properties of some indigenous epoxy resins. The casting technique of CY230 with HY951 and freezing stress in it at 60° C was reported by Prem Saran Nigam[7]. Chandrasekhara *et al.*[8] reported the photoelastic properties of this resin at room and upper critical temperatures.

Prompted by the above interesting reports on epoxy resins, an attempt is made to investigate the photoelastic behaviour of an epoxy resin of CIBA (India) Ltd. to judge its suitability for utilization in the photoelastic method of stress analysis. Araldite AY 103 with its corresponding hardener HY956 is chosen for this purpose after a study[9] of such materials. The resin has been studied at three different proportions of the hardener, by initially casting at 55° C and post-curing. A measured quantity of Araldite is heated gradually to 80° C to expel all the air bubbles and cooled to 50° C. Then the required quantity of hardener is added to it. The mixture is throroughly stirred, heated to 55° C, and transferred into moulds constructed with perspex plates, to set. The clear and transparent castings so formed are removed easily from the moulds after 24 hours. While preparing bigger castings heat generated due to polymerization can be reduced by reducing the hardener proportion or by two stage curing[10]. For the sake of uniformity, a single casting is used in each composition to prepare various test specimens. The specimens are annealed at 55° C for about 12 hrs and cooled to room temperature in 6 hrs. The post-curing is done in 4 cycles. In each cycle the castings are heated to 100° C in 1 hr, and maintained there for 5 hrs. Then they are cooled to room temperature in 10 hrs. At room temperature, the material stress fringe values (f) are determined in tension using the conventional calibration members and the Young's moduli (E) are determined by the ultrasonic rotating plate method[11]. At elevated temperatures, these constants are determined by enclosing the specimens in a specially constructed thermostat within the polariscope set up. The 'f' values are determined as above. E values are determined by selecting the samples as beams and measuring their deflection under three point loading. The results are presented in Table I and Fig. 1. Further, the accuracy with which the three dimensional work can be carried out with this resin is shown by determining the stress concentration factor due to an axial hole in a flat bar subjected to tension by freezing a system of stress in it. The loaded bar is heated to 70° C in 30 minutes in the thermostat and maintained there for 1 hr. Then it is cooled to room temperature in $4\frac{1}{2}$ hrs. The details of the bar and the system of frozen stress are given in Fig. 2. Repeated check measurements were made in all the cases and the reported values are the averages of such observations.

Results in Table I clearly indicate that the figure of merit of an Araldite hardener combination can be varied in two ways. They are: (1) variation of the hardener proportion in a fixed quantity of Araldite while casting and (2) post-curing a cast specimen containing fixed quantities of Araldite and hardener. From the present results, a fall in the figure of merit is observed by post-curing the specimen of composition 18 phr. This is due to a higher rate of rise in f than in E. Though there is a possibility of a continuous rise in E, showing stabilization after a number of post-curing cycles, it is to be remembered that f also acquires its maximum and stabilized value. Hendry[12] has shown the effect of prolonged curing on Catalin 800, where the material fringe value doubled itself. At this stage the figure of merit may be large but the increased value of f affects the accuracy of stress analysis by way of producing less number of fringes for an applied load. Moreover, the process of post-curing is tedious. Hence the first method, namely, variation of hardener proportion is quite effective

TABLE I

Photoelastic properties of Araldite AY103 and Hardener HY956

Composition (parts by weight of hardener per 100 gm of the resin, phr.	Material Stress Fringe value f. lb/in/order at λ = 5461 Å		Young's Modulus E × 10^{-5} psi		Figure of Merit Q		Critical temp. for stres freezing T_c °C	No. of fringes per inch thickness at the stress-fringe proportional limit	Proportional limit psi at 30° C
	at 30° C	at T_c	at 30° C	psi at T_c	at 30° C	at T_c			
18 Cast at 55° C	70·17	1·893	4·915	3398	7004	1795	70	69·04	4843
18 Cast at 55° C and post-cured	70·96	2·248	4·962	3654	6992	1626	76	68·85	4904
17 Cast at 55° C and post-cured	69·77	2·169	4·633	3362	6642	1550	75	67·02	4671
16 Cast at 55° C and post-cured	68·76	2·093	4·330	3085	6296	1474	73	63·33	4355

and easy to obtain a high figure of merit in the Araldite-hardener combinations.

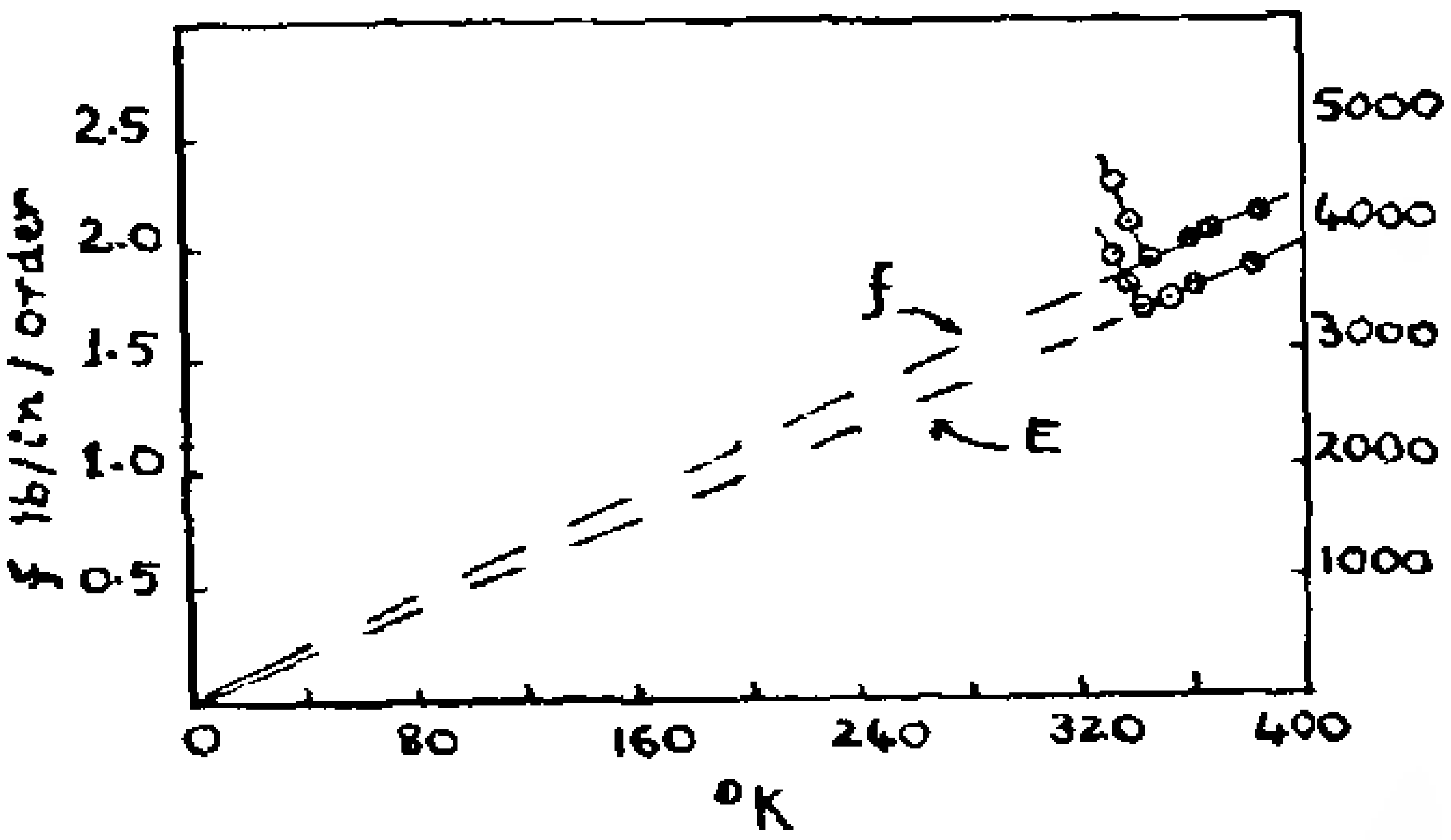

FIG. 1. Variation of material fringe value (*f*) and Young's modulus (E) with absolute temperature, around upper critical temperature.

Out of the materials studied, the composition 18 phr may be considered as a highly sensitive photoelastic material for two-dimensional studies, since its properties like the figure of merit and the number of fringes at the proportional limit are comparable with that of a standard photoelastic material CN501 and better than CY230. For three-dimensional studies, it appears to be better than the generally used formulation CY230 and HY951, as per the literature cited above. Since it is a plasticized resin, any complicated model can be cast to any size without cracks. Its low critical temperature is an advantage, since the process of stress freezing can be done conveniently at moderate temperatures. However care should be taken to avoid the rise in temperature while cutting and machining the samples to overcome the danger of relieving the frozen stress. Fig. 1 shows the behaviour of the

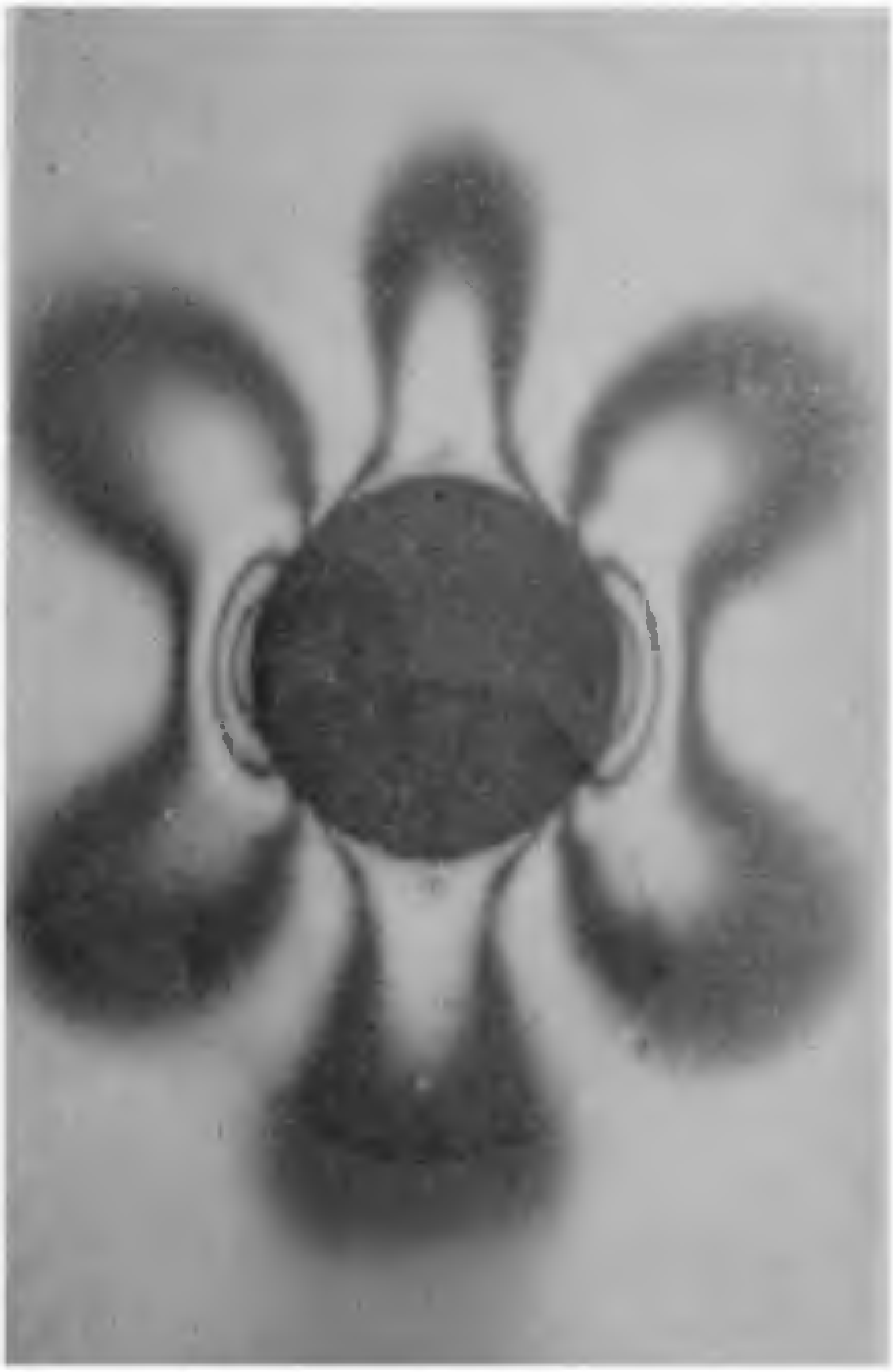

FIG. 2. Frozen stress fringe pattern of a flat bar with an axial hole, subjected to tension.

Tensile load = 2·977 lb, width of the bar D = 1·045" and thickness = 0·1456". Radius of the hole r = 0·1495" and $2r/D$ = 0·2990.

material beyond its upper critical temperature, supporting the theory of entropy elasticity and the results of Tuzi *et al.*[13] on moderately crosslinked polymers.

The effective fringe order at the hole on the transverse section of symmetry is determined as 4·944, using the analyser as compensator. Hence, the maximum stress at the hole $\sigma_{\max} = 4 \cdot 944 \times 1 \cdot 893/0 \cdot 1456 = 64 \cdot 28$ lb/sq. in. The nominal stress $\sigma_{\text{nom}} = 2 \cdot 977/1 \cdot 045 \times 0 \cdot 1456 = 19 \cdot 56$ lb/sq. in. So, the stress concentration factor, $\sigma_{\max}/\sigma_{\text{nom}} = 3 \cdot 286$. This value is very close to the Howland's[14] theoretical value 3·33, when compared with a similar result 3·15 obtained by Leven[15] on Fosterite. This ascribes reliability to the material for more accurate stress analysis by frozen stress technique.

Considering the high photoelastic nature of this resin, besides the ease of casting to any shape or size, negligible heat edge effects, availability in bulk, etc., priority may be given to this resin for photoelastic studies.

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January 3, 1978.

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A SHORT NOTE ON SECOND ORDER ROTATABLE DESIGNS

In statistical designs of experiments in the field of industry, numerous problems involve the fitting of a response surface in which the response depends upon one or more controllable variables or factors. Box and Hunter[1] introduced a new class of designs called rotatable designs in which the variance of the estimated response at a point is a function of the square of the distance of the points from a suitable origin, so that the variances of all estimated responses at points equidistant from the origin are the same. A design involving *m* factors is called a rotatable design of second order (SORD) if the response function can be expressed as a quadratic in the given variables. The valuable contributions on this subject are due to Box and Behnken[3], Box and Draper[2], Das[4], Das and Narshimham[5], Saha and Das[8], Pal[6], etc. In this paper, necessary and sufficient conditions have been obtained for second order rotatable designs constructed through PB arrays to have minimum number of design points and the result provides the parameters of the array.

For a definition of partially balanced (PB) array, Rafter *et al.*[7] may be consulted. Method of construction of a SORD using PB array is given by Das[4].

We prove the following Theorems.

Theorem 1

The necessary and sufficient condition for a SORD constructed from a PB array $(m, \mathrm{N}, 2, 2)$ with $\lambda^2_{\alpha\beta} > \lambda_{\alpha\alpha}\lambda_{\beta\beta}$ to have minimum number of design points is $\mathrm{N} = m$.

Proof

Since in this method of construction the number of design points is directly proportional to the number of assemblies N the minimum value of N will provide minimum number of design points. For the existence of a SORD from PB array we should have (Theorem 1, Saha and Das[8]).

$$\lambda_{\alpha\beta} > 2\lambda_{\alpha\alpha} \text{ or } \lambda_{\alpha\beta} > 2\lambda_{\beta\beta}. \tag{2.1}$$

Necessity. From the linear relation

$$\mathrm{N} = \lambda_{\alpha\alpha} + 2\lambda_{\alpha\beta} + \lambda_{\beta\beta},$$

it is evident that for minimum N each of the parameters should be minimum. With this view, let us choose $\lambda_{\alpha\alpha} = 0$ and $\lambda_{\alpha\beta} = 1$ so that (2.1) is satisfied. Now, we have

$$\mathrm{N} = \lambda_{\beta\beta} + 2.$$

Under the given condition of the Theorem, we have the following inequality (Theorem 3.2, Rafter and Seiden[7]),