LETTERS TO THE EDITOR

AN EXACT SOLUTION OF EINSTEIN-MAXWELL EQUATIONS

CONSIDER a space-time whose metric is given by

$$ds^2 = -(dx^1)^2 - e^{\beta}(dx^2)^2 - e^{\gamma}(dx^3)^2 + e^{\delta}(dx^4)^2 \quad (1)$$

where β, γ, δ are functions of x^1 alone. The purpose of this note is to present a solution of Einstein-Maxwell equations which for empty, charge and current free space are

$$\left.\begin{aligned} &R^i_j - \tfrac{1}{2} R g^i_j = -8\pi E^i_j \\ &F_{ij;k} + F_{jk;i} + F_{ki;j} = 0 \\ &F^{ij}_{\ \ ;j} = 0 \end{aligned}\right\} \quad (2)$$

where

$$E^i_j = \frac{1}{4\pi}\left(-F_{jk}F^{ik} + \frac{1}{4}g^i_j F_{kl}F^{kl}\right) \quad (3)$$

for the metric (1). Since the field is static and function of x^1 alone, F_{14} is the only non-vanishing independent component of the electromagnetic field tensor. We have obtained the solution of equations (2) for the metric (1) as

$$\begin{aligned} ds^2 = &-(dx^1)^2 - (4/y^2)(e/4c)^{2k/(k+l)} \\ &[(-1+\sqrt{1+q^2y^2/kl})(qy/\sqrt{kl})^{-1}]^{\mp 2\sqrt{k/l}}(dx^2)^2 \\ &-(4/y^2)(e/4c)^{2l/(k+l)} \\ &\times[(-1+\sqrt{1+q^2y^2/kl})(qy/\sqrt{kl})^{-1}]^{\mp 2\sqrt{l/k}} \\ &+(y^2/4)(dx^4)^2 \end{aligned} \quad (4a)$$

$$\begin{aligned} F_{14} = qe^{(\delta-\beta-\gamma)/2} - e/2cqy^3 \\ \times[(-1+\sqrt{1+q^2y^2/kl})(qy/\sqrt{kl})^{-1}]^{\pm(k+l)/\sqrt{kl}} \end{aligned} \quad (4b)$$

where y is given by

$$\begin{aligned} &[(-1+\sqrt{1+q^2y^2/kl})(qy/\sqrt{kl})^{-1}]^{\mp(k+l)/\sqrt{kl}} \\ &[\mp(k+l)/\sqrt{kl} + \sqrt{1+q^2y^2/kl}]y^{-1} \\ &= [(k^2+l^2+kl)/kl)](\pm c\sqrt{kl}\,x^1 + d). \end{aligned} \quad (5)$$

Here k, l, q, c and d are arbitrary constants. Clearly k and l must be non-zero and of the same sign. The letter e which stands for ± 1 may be so chosen that the signature of the metric (1) is maintained. We have to take either upper or lower signs throughout in (4) and (5) to get the solution. Similar solutions can be written for other variables also.

The empty space solution for the metric (1) may be written as

$$\begin{aligned} ds^2 = &-(dx^1)^2 - (ax^1+b)^{p_1}(dx^2)^2 - (ax^1+b)^{p_2} \\ &\times(dx^3)^2 + (ax^1+b)^{p_3}(dx^4)^2 \end{aligned} \quad (6)$$

analogous to the Narlikar-Karmarkar curious solution[1], where a and b are arbitrary constants and the constants p_1, p_2, p_3, satisfy the conditions

$$\left.\begin{aligned} &p_1 + p_2 + p_3 = 2 \\ &p_1p_2 + p_2p_3 + p_3p_1 = 0 \end{aligned}\right\}. \quad (7)$$

The particular case

$$p_1 = p_2 = 4/3,\ p_3 = -2/3 \quad (8)$$

of (6) is the transform of the Taub solution[2].

If we choose

$$\left.\begin{aligned} C &= \pm a[\mp(k+l) - \sqrt{kl}]^{-1} \\ &\times(q/2\sqrt{kl})^{\mp(k+l)/\sqrt{kl}} \\ d &= b\sqrt{kl}[\mp(k+l) - \sqrt{kl}]^{-1} \\ &\times(q/2\sqrt{kl})^{\mp(k+l)/\sqrt{kl}} \end{aligned}\right\} \quad (9)$$

and take the limit of the solution (4) as $q \to 0$, it reduces to the solution (6) after a suitable change in the scale of co-ordinates x^2, x^3, x^4 with

$$\left.\begin{aligned} p_1 &= 2(\mp k - \sqrt{kl})[\mp(k+l) - \sqrt{kl}]^{-1} \\ p_2 &= 2(\mp l - \sqrt{kl})[\mp(k+l) - \sqrt{kl}]^{-1} \\ p_3 &= 2\sqrt{kl}[\mp(k+l) - \sqrt{kl}]^{-1}. \end{aligned}\right\} \quad (10)$$

The solution (4) may, therefore, be interpreted as the field of a charged source of metric (6). The interpretation of q which is related to the charge on the source, depends on the nature of the source.

If $k = l$ the solution (4) becomes plane symmetric and for corresponding choice of c and d given by (9) it can be reduced to Taub solution and flat space-time as $q \to 0$ for the choice of upper and lower signs respectively. The charged Taub solution is found to be

$$\begin{aligned} ds^2 = &-(dx^1)^2 - \frac{eu^2}{4c}[(dx^2)^2 + (dx^3)^2] \\ &+(1+ku/q)u^{-2}(dx^4)^2 \end{aligned} \quad (11a)$$

$$F_{14} = 4ecqu^{-3}\sqrt{1+ku/q} \quad (11b)$$

where u is related to x^1 through η as

$$\begin{aligned} u &= \eta^{1/3} + (q^2/k^2)\eta^{-1/3} + q/k \\ k\eta &= q[3(ckx^1+d) + \sqrt{9(ckx^1+d)^2 - q^2/k^2}] \end{aligned} \quad (11c)$$

and

$$c = -4ak/3q^2, \quad d = -4bk^2/3q^2. \qquad (11d)$$

This solution can be written in the form of Patnaik metric[3],

$$ds^2 = (1 + ku/q)\,u^{-2}\,[(dx^4)^2 - (d\bar{x}^1)^2] - \frac{eu^2}{4c}\,[(dx^2)^2 + (dx^3)^2] \qquad (12)$$

if

$$\bar{x}^1 = \frac{eq}{4ck^2}\,[-u + ku^2/2q + (q/k)\log(1 + ku/q)] + A \qquad (13)$$

where A is a constant. The case of lower signs yields the solution

$$ds^2 = -(dx^1)^2 - \frac{eu^2}{4c}\,[(dx^2)^2 + (dx^3)^2] + (1 - ku/q)\,u^{-2}\,(dx^4)^2 \qquad (14a)$$

$$F_{14} = 4ecqu^{-3}\sqrt{1 - ku/q} \qquad (14b)$$

where

$$\left.\begin{aligned} ku &= q\,(1 - 4\cos^2\alpha), \\ 3\alpha &= \cos^{-1}[3k\,(cdx^1 - d)/q], \\ c &= -aq^2/4k^3, \quad d = bq^2/4k^2. \end{aligned}\right\} \qquad (14c)$$

This solution can also be reduced to the Patnaik form

$$ds^2 = (1 - ku/q)\,u^{-2}\,[(dx^4)^2 - (d\bar{x}^1)^2] - \frac{eu^2}{4c}\,[(dx^2)^2 + (dx^3)^2] \qquad (15)$$

if

$$\bar{x}^1 = -\frac{eq}{4ck^2}\,[u + ku^2/2q + (q/k)\log(1 - ku/q)] + B \qquad (16)$$

where B is a constant. The letter e, in the solution (11) and (14), is $+1$ or -1 according as $c >$ or < 0.

The study of geodesic motion, nature of the source of the gravitational field, the pseudo-energy-momentum tensor, etc., etc., for the new metric (4), will be published elsewhere.

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1. Narlikar, V. V. and Karmarkar, K. R., *Curr. Sci.*, 1946, **15**, 69.
2. Taub, A. H., *Ann. Maths.*, 1951, **53**, 472.
3. Patnaik, S., *Proc. Camb. Phil. Soc.*, 1970, **67**, 127.

FINE STRUCTURE OF B – X SYSTEM OF SbO MOLECULE

THE spectrum of SbO molecules has been investigated by several authors[1-8]. The spectrum consists of seven band systems, *viz.*, A–X, B–X, C–X, D–X, E–X, F–X and G–X extending from infra-red to ultraviolet region. It is established that ground state of SbO molecule is a regular $^2\Pi$ state with doublet separation of 2276 cm^{-1}. Analyses of D–X and C–X systems of SbO molecule by S.B. Rai *et al.*[7] and Rai *et al.*[6] reveal an appreciable Λ-type doubling in the ground state. Hence we expect similar results in the case of B–X system. However, analysis of B–X system by previous workers[5] does not reveal such Λ-type doubling. Hence a re-investigation of B–X system of SbO molecule has been undertaken at high dispersion and resolution. Results obtained are reported in the present note.

The spectrum of SbO molecule has been excited in a high frequency discharge (15 MHz). A pure sample of Sb_2O_3 was kept in a conventional type of discharge tube. 0, 0 and 1, 0 bands of sub-systems of B–X level of SbO molecule were photographed in the 9th order at a dispersion of ·35 Å/mm on two metre plane grating spectrograph. Measurements were made on an Abbe comparator against iron arc lines as standards. The sharp lines could be measured within an error of ·03 cm^{-1}.

The bands in B–X system are degraded to longer wavelength side and are double headed. 0, 0 and 1, 0 bands of B–X sub-system showed well resolved R_{12}, R_2, Q_2 and P_2 branches of which R_{12} and R_2 are head forming, P_2 branch lines are weak in intensity. The doubling of rotational lines in some branches of two component systems has been attributed to the isotopic effect of antimony. The J numbering of rotational lines and their analysis has been carried out by the method suggested by Youngner and Winans[8]. The analysis revealed a good amount of Λ-type doubling in the lower state $^2\Pi_{1/2}$ and $^2\Pi_{3/2}$ (0·06667 cm^{-1} and 0·02337 cm^{-1}) and spin splitting in the upper state $^2\Sigma$ (0·0133 cm^{-1}). The combination differences of 0, 0 and 1, 0 bands of sub-system of B–X of SbO molecule for the common lower state were compared and found to agree within experimental errors.

Λ-type doubling constant q and spin splitting constant γ have been calculated by the method of combination defect. Results obtained are given in table I. It is observed that $B_v^i/B_v = \rho^2\,(0{\cdot}9974)$ agrees with calculated value of $\rho^2 = (0{\cdot}9980)$. The ground state of SbO molecule is $^2\Pi_r$ state arising from the electronic configuration $(z\sigma^2 y\sigma^2 \omega\pi^4 x\sigma^2 v\pi)$. The excitation of an electron from $v\pi$ orbitial to $u\sigma$ oribital gives rise to $B^2\Sigma$ state. Thus B–X system can be attributed to $B^2\Sigma - X^2\pi_r$ transition.