LETTERS TO THE EDITOR

STEADY STATE FILM PROFILES OF VISCOUS DRAINAGE OF A CONDUCTING FLUID IN THE PRESENCE OF A TRANSVERSE MAGNETIC FIELD OVER A NATURALLY PERMEABLE WALL

JEFFREYS[1] studied the classical problem of drainage with a stationary wall and found the thickness of the liquid film formed on the wall. Recently Padma, Ramacharyulu and Ramana Rao[2] have extended this problem to include the effect of porosity by considering the viscous drainage over a naturally permeable wall. Annapurna and Ramanaiah[3] considered the problems of drainage and lifting of a conducting fluid by introducing both the inertia and transverse magnetic effects on the liquid film-thickness. In this note we extend our work[2] to include the effects of a transverse magnetic field. neglecting the induced magnetic field.

Following the analysis of Annapurna and Ramanaiah[3], the Navier-Stokes equation for a steady state, then takes the form

$$O = \nu \frac{\partial^2 v}{\partial x^2} - g - \frac{\sigma B_0^2}{\rho} v. \tag{1}$$

We solve eq. (1) subject to the slip boundary conditions as proposed by Beavers and Joseph[4], namely

$$\left.\frac{\partial v}{\partial x}\right|_{x=0} = \frac{\alpha}{\sqrt{k}}(v_B - v_D), \left.\frac{\partial v}{\partial x}\right|_{x=h} = 0, \tag{2}$$

where the various quantities have already been explained in our work[2].

The Darcy velocity in this case reduces to

$$v_D = -\frac{gk}{\nu + mk}, \tag{3}$$

where

$$m = \frac{\sigma B_0^2}{\rho}.$$

Solving eq. (1) subject to the boundary conditions of eq. (2) and making use of eq. (3), we obtain

$$v = \frac{g}{m}\left[\frac{\alpha}{\sqrt{k}} \cdot \frac{\nu \cosh\left\{(x-h)\sqrt{\frac{m}{\nu}}\right\}}{(\nu + mk)\left\{\frac{\alpha}{\sqrt{k}}\cosh h\sqrt{\frac{m}{\nu}} + \sqrt{\frac{m}{\nu}}\sinh h\sqrt{\frac{m}{\nu}}\right\}} - 1\right] \tag{4}$$

Putting $\sigma = h/\sqrt{k}$ as in our work[2], M (Hartmann number) $= h\sqrt{m/\nu}$ and in terms of the non-dimensional quantities defined by eq. (8) of Annapurna and Ramanaiah[5], we obtain

$$Y = -H^2T\left[\frac{(\sigma^2 + M^2)(\alpha\sigma\cosh M + M\sinh M)^2 - \alpha^2\sigma^4}{M^2(\sigma^2 + M^2)(\alpha\sigma\cosh M + M\sinh M)^2}\right]. \tag{5}$$

By letting M → 0, we realize eq. (4) of our work[2]. Also by allowing $\alpha, \sigma \to \infty$ independently, we recover from eq. (5),

$$Y = -H^2T\frac{\tanh^2 M}{M^2} \tag{6}$$

When we put $v_0 = 0$, *i.e.*, in the absence of viscous litsing and allowing T → ∞ (for steady state) and replacing MH by M in eq. (15) of their work[3], we recover the above equation.

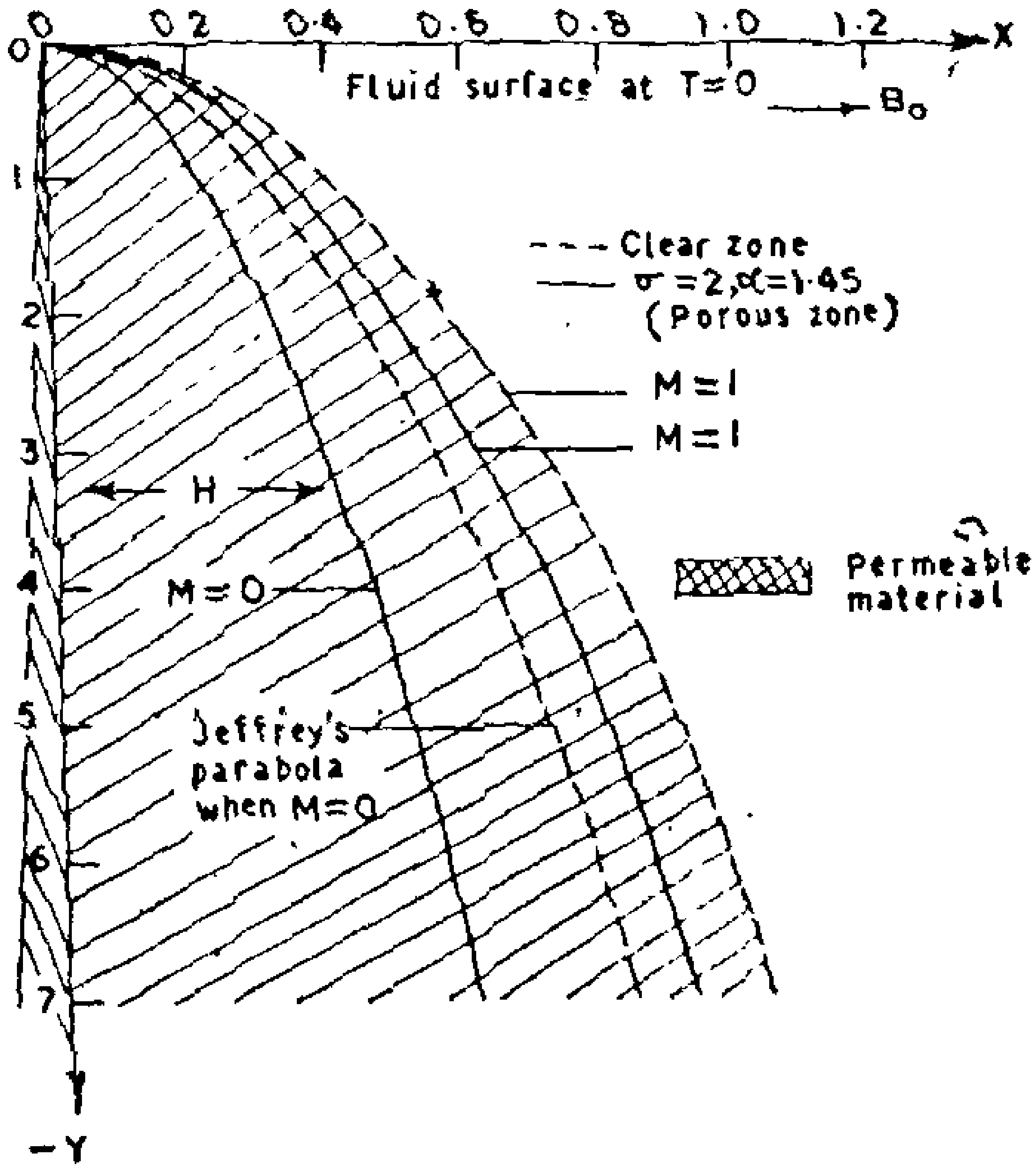

FIG. 1. Steady state film profiles at T = 10.

In Fig. 1, we have shown the porous zone indicated by thick line for M = 0 and 1 by taking $\alpha = 1{\cdot}45$, $\sigma = 2$. Also the clear zone, indicated by a dotted line, has been shown for the same values of the

Hartmann number. For either zone, it is concluded that the effect of increasing the Hartmann number is always to increase the film thickness. This can be reasonably expected as the applied magnetic field B_0, transverse to the wall, *i.e.*, in the positive *x*-direction as showin in Fig. 1 (see also Fig. 1 of Annapurna and Ramanaiah[3]) will always have a tendency to pull the fluid in its own direction.

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LUNAR TIDES IN EQUATORIAL IONOSPHERIC ABSORPTION*

Introduction

LUNAR tides in ionospheric absorption of radiowaves have been studied by several workers.[1–6] In these studies both diurnal and semi-diurnal tides have been observed, the latter being predominant. However, such studies of lunar tides in ionospheric absorption at equatorial latitudes have been inconclusive. This communication deals with the study of the lunar tidal effect on absorption measured at Waltair (17° 43′ N) and the results are compared with those available for the other latitudes.

Data and Method of Analysis

The noontime values of absorption (L) obtained from A_1 technique on two frequencies 2·4 MHz and 5·6 MHz during the period 1971–76 and 1975–76 respectively have been utilised in the present analysis. The 'L' values affected by solar flares, sudden commencement etc. are excluded from the data. Following the method of Chapman and Bartles[7], the data have been subjected to harmonic analysis to obtain the amplitude and phase.

Results and Discussion

The results of the present analysis (Table I) show that the amplitude of the semi-diurnal tide is greater than that of the diurnal tide at both the frequencies. Moreover, the amplitude of the diurnal and semi-diurnal tides decreases with increase of the operating frequency as seen from Table I. Ganguly[6], while studying absorption of radiowaves covering a large number of frequencies, has observed maximum tidal effect for frequencies near foE and a decrease in amplitude of the tide for frequencies greater than the critical frequency of the E-layer.

TABLE I

Diurnal and semi-diurnal periodicities in the Lunar tide of ionospheric absorption at Waltair

Period of study	Frequency MHz	$L_1(A)$ a_1	φ_1	$L_2(A)$ a_2	φ_2	Time of max. of L_2 (Lunar hours)
1971–76	2·4	0·35	17	0·52	59	1·0
1975–76	5·6	0·10	358	0·28	231	7·3

Another important feature evident from Table I is the phase reversal in the E-region and F-region tides. Chakravarthy and Rastogi[8] observed a phase reversal in D-region and F-region lunar tides over Singapore. The results of multifrequency studies of Rao[9] and Ganguly[6] indicated a phase reversal between the D-region and F-region lunar tides, though they did not mention explicitly reversals in their studies.

TABLE II

Seasonal variation of lunar semi-diurnal component for 2·4 MHz

	Amplitude (*in db*) a_2	Phase (*in degrees*) φ_2	Time of max. a_2 after transit (in lunar hrs.)
Winter	0·88	114	11·2
Equinoxes	0·65	19	2·4
Summer	0·64	60	1·0

The seasonal variation of the semi-diurnal component for the frequency 2·4 MHz for which the data coverage was maximum is shown in Table II. The magnitude of the lunar tidal variation of ionospheric absorption is larger in winter than the corresponding values for summer and equinox. This is in good agreement with the seasonal studies made at Colombo[10], Singapore[8] Calcutta[6] and Freiburg[4]. Further, the lunar tide of maximum absorption varies slightly with season, during the sunspot minimum period. It may be worth while mentioning a similar conclusion