LETTERS TO THE EDITOR

UPPER BOUND ON THE NUMBER OF CONSTRAINTS IN BALANCED ARRAYS

We generalise two of the results of Rafter and Seiden[1] regarding a balanced array BA $(m, \mathrm{N}, 2, 2)$ with index set

$$\Lambda_{2,2} = \{\mu_0{}^{(2)}, \mu_1{}^{(2)}, \mu_2{}^{(2)}\}$$

to a balanced array BA $(m, \mathrm{N}, 2, t)$ with index set

$$\Lambda_{2,t} = \{\mu_0{}^{(t)}, \mu_1{}^{(t)}, \ldots, \mu_t{}^{(t)}\}.$$

THEOREM. Let A be a BA $(m, \mathrm{N}, 2, t)$ with index set $\Lambda_{2,t}$ and let

$$\mathrm{D} = \left\{\sum_{j=1}^{t-1} \binom{t-2}{j-1} \mu_j{}^{(t)}\right\}^2 - \left\{\sum_{j=0}^{t-2} \binom{t-2}{j} \mu_j{}^{(t)}\right\} \times \left\{\sum_{j=2}^{t} \binom{t-2}{j-2} \mu_j{}^{(t)}\right\},$$

(i) If $\mathrm{D} > 0$, then

$$m \leqslant \frac{\mathrm{N}}{\mathrm{D}} \sum_{j=1}^{t-1} \binom{t-2}{j-1} \mu_j{}^{(t)},$$

and (ii) if $\mathrm{D} = 0$, then $m \leq \mathrm{N} - 1$.

Proof. Let n_j be the number of columns of A which contain j ones and let

$$\bar{j} = \frac{1}{\mathrm{N}} \sum_{j=1}^{m} j n_j .$$

Since $n_j \geqslant 0$, for all j, it follows that

$$0 \leqslant \sum_{j=0}^{m} (j - \bar{j})^2 n_j = \sum_{j=1}^{m} j^2 n_j - \mathrm{N}(\bar{j})^2.$$

Using lemma 2.1 of Rafter and Seiden, we see that

$$\bar{j} = \frac{m}{\mathrm{N}} \sum_{i=0}^{t-1} \binom{t-1}{i} \mu_{i+1}^{(t)},$$

and

$$\sum_{j=1}^{m} j^2 n_j = m \sum_{i=0}^{t-2} \binom{t-2}{i} \mu_{i+1}^{(t)} + m^2 \sum_{i=0}^{t-2} \binom{t-2}{i} \mu_{i+2}^{(t)}$$

Thus substituting these values in the above inequality, we have

$$0 \leqslant m \sum_{i=0}^{t-2} \binom{t-2}{i} \mu_{i+1}^{(t)} + m^2 \sum_{i=0}^{t-2} \binom{t-2}{i} \mu_{i+2}^{(t)} - \frac{m^2}{\mathrm{N}} \left\{ \sum_{i=0}^{t-1} \binom{t-1}{i} \mu_{i+1}^{(t)} \right\}^2$$

Since $m > 0$, so

$$0 \leqslant \sum_{i=0}^{t-2} \binom{t-2}{i} \mu_{i+1}^{(t)} + \frac{m}{\mathrm{N}} \left[\mathrm{N} \sum_{i=0}^{t-2} \binom{t-2}{i} \mu_{i+2}^{(t)} - \left\{ \sum_{i=0}^{t-1} \binom{t-1}{i} \mu_{i+1}^{(t)} \right\}^2 \right]$$

Since

$$\mathrm{N} = \sum_{i=0}^{t} \binom{t}{i} \mu_i^{(t)}, \text{ we have}$$

$$0 \leqslant \sum_{i=0}^{t-2} \binom{t-2}{i} \mu_{i+1}^{(t)}$$

$$\frac{m}{\mathrm{N}} \left[\left\{ \sum_{i=0}^{t} \binom{t}{i} \mu_i^{(t)} \right\} \right.$$

$$\times\left\{\sum_{i=0}^{t-2}\binom{t-2}{i}\mu_{i+2}^{(t)}\right\}$$

$$-\left\{\sum_{i=0}^{t-1}\binom{t-1}{i}\mu_{i+1}^{(t)}\right\}^2\Bigg].$$

On simplification, we get

$$m\Bigg[\left\{\sum_{j=1}^{t-1}\binom{t-2}{j-1}\mu_j^{(t)}\right\}$$

$$-\left\{\sum_{j=0}^{t-2}\binom{t-2}{j}\mu_j^{(t)}\right\}$$

$$\times\left\{\sum_{j=2}^{t}\binom{t-2}{j-2}\mu_j^{(t)}\right\}\Bigg]$$

$$\leqslant N\sum_{j=1}^{t-1}\binom{t-2}{j-1}\mu_j^{(t)}.$$

Hence the first part of the theorem.

For the second part, $D = 0$ reduces to,

$$(\mu_1^{(2)})^2 - \mu_0^{(2)}\,\mu_2^{(2)} = 0,$$

by making repeated use of the recurrence relation

$$\mu_i^{(t-1)} = \mu_i^{(t)} + \mu_{i+1}^{(t)},\quad i = 0,\ 1,\ 2, \ldots,\quad t-1,$$

between the elements of BA $(m, N, 2, t)$ and BA $(m, N, 2, t-1)$.

This combined with the fact that BA $(m, N, 2, t)$ is also a BA $(m, N, 2, 2)$ we get $m \leqslant N - 1$, by Theorem 3.4 of Rafter and Seiden, for the given BA $(m, N, 2, t)$. This is to be noticed here that in this case (*i.e.*, when $D = 0$) the upper bound is not a function of t.

This completes the proof.

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1. Rafter, J. A. and Seiden, E., *Annals of Statistics*, 1974, 2 (6), 1256.

ULTRASONIC PARAMETERS OF DICARBOXYLIC ACIDS

In the present investigation, an attempt has been made to apply both Schaaffs and Kittels' theories to the ultrasonic behaviour of dicarboxylic acids.

Using the composite ultrasonic interferometer[1] designed in this laboratory, the ultrasonic velocity and temperature coefficient of velocity of six dicarboxylic fatty acids in their liquid state at 170° C have been determined and reported earlier[2]. The variation of these two parameters with molecular weight of the acid is plotted in Fig. 1. It may be observed that ultrasonic velocity decreases, while the temperature coefficient of velocity increases with molecular weight of the acid.

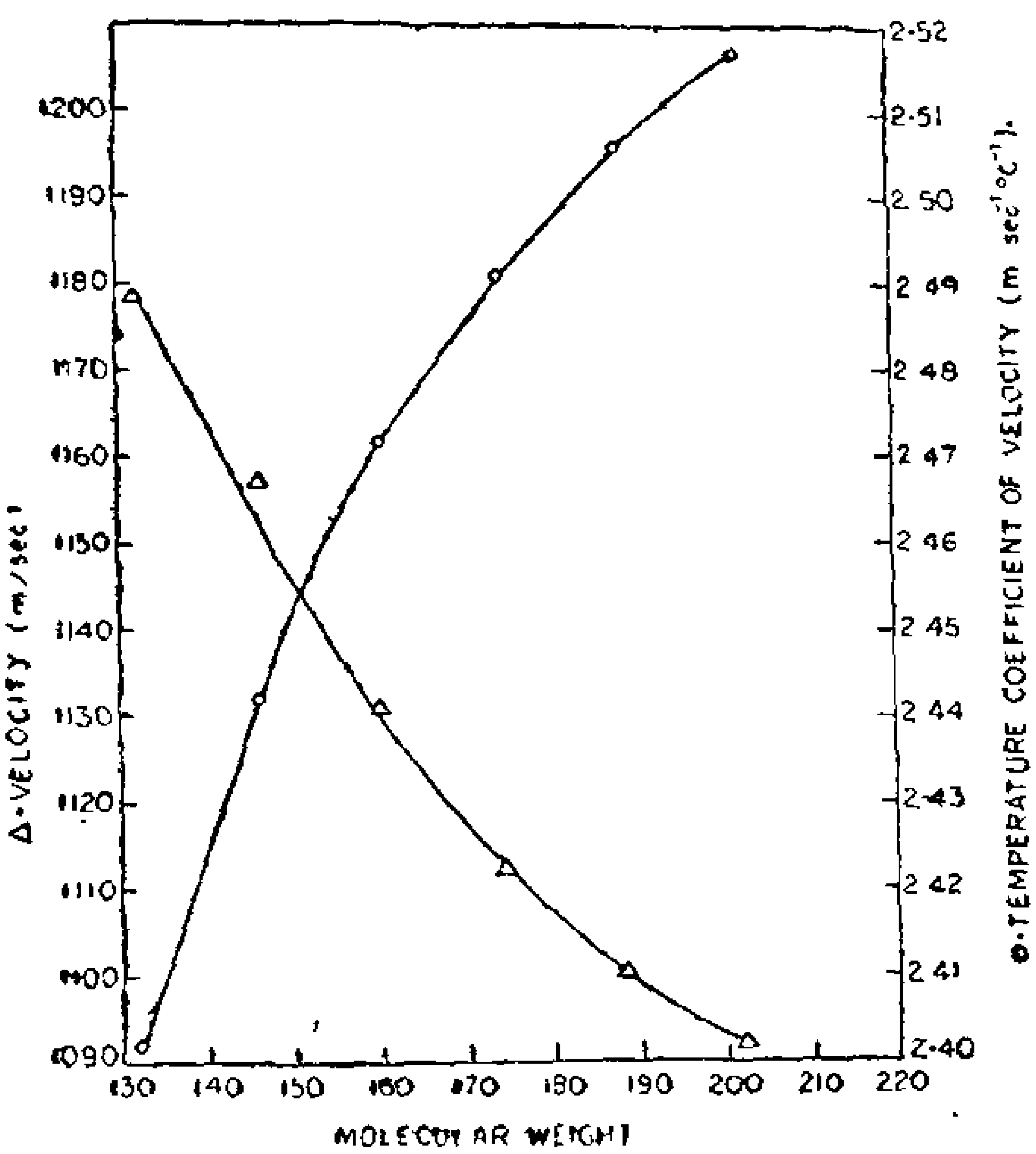

Fig. 1. Dependence of ultrasonic velocity (at 170° C) and temperature coefficient on molecular weight.

With a view to verifying whether these substances obey Schaaffs[3] theory or not, Schaaffs' parameters have been evaluated for the first time for six dicarboxylic acids using the previously published velocity and density data[2]. The van der Waals' b, the effective molecular volume B, the space filling factor 'r_f' and collision factor 's' have been calculated using the relevant formulae[4] given below and shown in columns 4, 6, 8 and 9 of Table I.

$$b = \frac{M}{\rho}\left\{1 - \frac{RT}{MC^2}\left(\sqrt{1 + \frac{MC^2}{3RT}} - 1\right)\right\} \tag{1}$$

$$B = \frac{4}{3}\cdot\pi r_m^3 N \tag{2}$$