

FILTRATION OF GEOPHYSICAL FIELDS USING EIGEN VECTORS

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ABSTRACT

A digital filter based on the eigen vectors of the correlation matrix of the geophysical field is presented. The advantage of the method is indicated with the help of two field examples.

INTRODUCTION

THE processing of geophysical data involves two basic problems, namely, detection of anomalies and their integrated interpretation. Digital filters are increasingly designed to meet this requirement. Energy filter¹, based on the criterion of the maximisation of the ratio of the energy of the anomaly and the energy of the interferences, enables in particular to resolve problems of detection of oil and gas layers on the basis of the anomalies of the structures in gravity and magnetic prospecting. In the present study anomaly detection procedure is presented by decomposing the geophysical field using orthogonal system of eigen vectors of the correlation matrix.

EIGEN VECTORS CORRELATION MATRIX

Suppose the set of functions $h_i(X_k)$ $i = 1, 2, \dots, m$; $k = 1, 2, \dots, n$ from an orthogonal system: (i.e.)

$$\sum_{k=1}^n h_i(X_k) h_j(X_k) = \delta_{ij} \quad (1)$$

where δ_{ij} is the Kronecker's δ and X_k is the station of observation.

Any function of the geophysical field $f(X_k)$ can be expressed as a linear combination of this system of weight functions²:

$$f(X_k) = \sum_{i=1}^m C_i h_i(X_k) \quad (2)$$

where the coefficients C_i are determined by using eq. (1)

$$C_i = \sum_{k=1}^n f(X_k) h_i(X_k). \quad (3)$$

Weight functions h_i can be found from the minimisation of the mean-square error, given by

$$E_m^2 = \sum_{k=1}^n [f(X_k) - \sum_{i=1}^m C_i h_i(X_k)]^2. \quad (4)$$

Minimisation of the expression on the right hand side of eq. (4) leads to the problem of finding the solution of the system of the following equations:

$$\sum_{k=1}^n R_{l-k} h_i(X_k) = \lambda_l h_l(X_l) \quad (5)$$

$l = 1, 2, \dots, n; \quad i = 1, 2, \dots, m$

where $[R_{l-k}]$ is the correlation matrix of the initial values of the field. For stationary random function $f(X_k)$ correlation matrix depends only on the shift $(l-k)$. Hence the correlation matrix here can be determined by the autocorrelation functions of the

initial values of the geophysical field. However care should be taken to exclude the regional component before taking the initial data for consideration.

Noting that the autocorrelation function is even, we can write the correlation matrix as:

$$[R_{l-k}] = \begin{bmatrix} R(0) & R(1) & \dots & R(p-1) \\ R(1) & R(0) & \dots & R(p-2) \\ \vdots & \vdots & \ddots & \vdots \\ R(p-1) & R(p-2) & \dots & R(0) \end{bmatrix} \quad (6)$$

where $R(p)$ is given by $R(p) = R'(p)/R'(0)$

$$R'(p) = \frac{1}{n-p} \sum_{i=1}^{n-p} f(X_i) f(X_{i+p})$$

and

$$R_{l-k} = R(|l-k|).$$

It is evident that, from eq. (5), the λ_i are eigen values of the matrix (R_{l-k}) and h_i are the corresponding eigen vectors.

ENERGY FILTER

Application of the above theoretical considerations for the detection of the anomaly, on the background of noises, involves only the calculation of the weight functions of the filter based on the criterion of the maximisation of the ratio of the energy of the signal to the power of the interferences^{1,3}. Apart from the enhancement of the ratio of signal to noise this criterion takes into account all the useful content of the signal.

The output of any filter is expressed as:

$$Y_k = \sum_{i=1}^m h_i f(X_{k-i}) \quad (7)$$

where h_i denote the weight function.

Let us suppose $f(X_k) = a_k + n_k$ where $a_k = a(X_k)$ is the anomaly and $n_k = n(X_k)$ the noise.

The energy of the anomaly of the output is defined by:

$$E = \sum_{k=1}^{n+m} C_k^2 \quad (8)$$

where

$$C_k = \sum_{i=1}^m h_i a_{k-i}. \quad (9)$$

The power of the interferences of the output of the filter is given by

$$P = \sum_{k=1}^{n+m} V_k^2 \quad (10)$$

where

$$V_k = \sum_{i=1}^m h_i n_{k-i} \quad (11)$$

The ratio of the energy of the anomaly to the power of interferences is:

$$\lambda = \frac{E}{P} \quad (12)$$

Substituting from eqs. (8), (9), (10), (11) in and eq. (12) simplifying we obtain,

$$\lambda = \frac{\sum_{i=1}^m \sum_{j=1}^m h_i R_{i-j} h_j}{\sum_{i=1}^m \sum_{j=1}^m h_i Q_{i-j} h_j} \quad (13)$$

where $[R_{i-j}]$ and $[Q_{i-j}]$ are the correlated matrices of the anomaly and interferences respectively.

For maximising λ we differentiate the right hand side of (13) with respect to h_i and equate the derivatives to zero⁴. And thus obtain a system of linear equations as:

$$\sum_{j=1}^m [R_{i-j} - \lambda Q_{i-j}] h_j = 0 \quad (14)$$

$i = 1, 2, \dots, m$

System (14) constitutes the 'generalised eigen value problem'⁵ where the signal to noise ratio λ is the eigen value and the filter $[h_i]$ is the associated eigen vector.

If the interferences are uncorrelated and their dispersion is equal to unity,

(i.e.)

$$Q(m) = 0, \quad m \neq 0; \quad Q(0) = 1$$

$[Q_{i-j}]$ becomes unit matrix I.

as a result system (14) reduces to:

$$\sum_{j=1}^m [R_{i-j} - \lambda I] h_j = 0 \quad (15)$$

$i = 1, 2, \dots, m$

Since we want the filter corresponding to maximum value of the signal to noise ratio, we select $[h_i]$ corresponding to λ_{\max} . Thus from eq. (15),

$$\sum_{j=1}^m R_{i-j} h_j = \lambda_{\max} h_i \quad (16)$$

$i = 1, 2, \dots, m$

System (16) is analogous to system (5). However in case of (16) it is necessary only to find one eigen value of the correlation matrix. Since noise is assumed to be uncorrelated this λ_{\max} is calculated from the correlation matrix of the field data, by using iterative techniques which require not much computer time. We then construct the vector $[h_i]$ using eq. (16) and filter the geophysical field using eq. (7). Since the correlation matrix is of controlling importance the dimen-

sion of the autocorrelation function is usually limited by the first minimum.

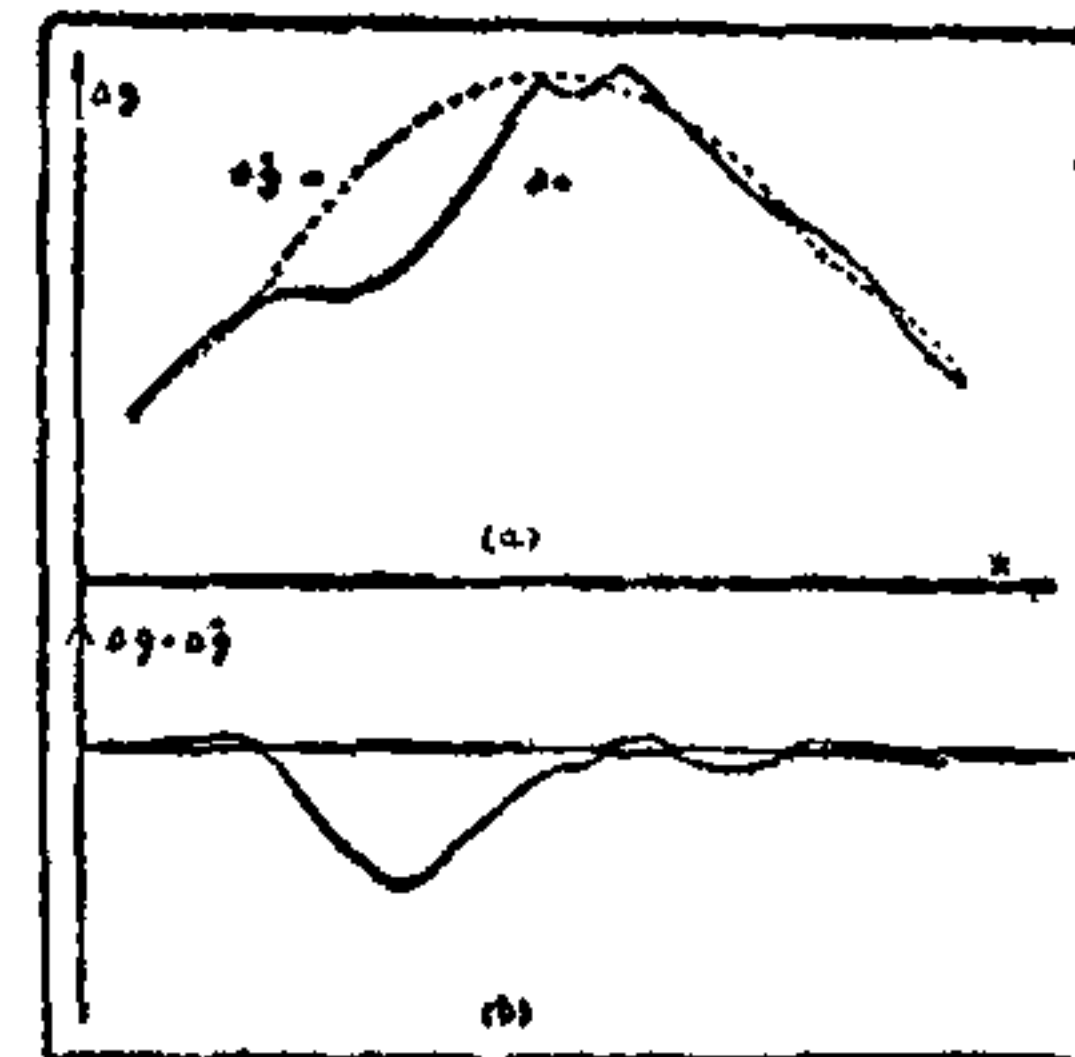


FIG. 1. Filtration of gravity field by latent vectors.

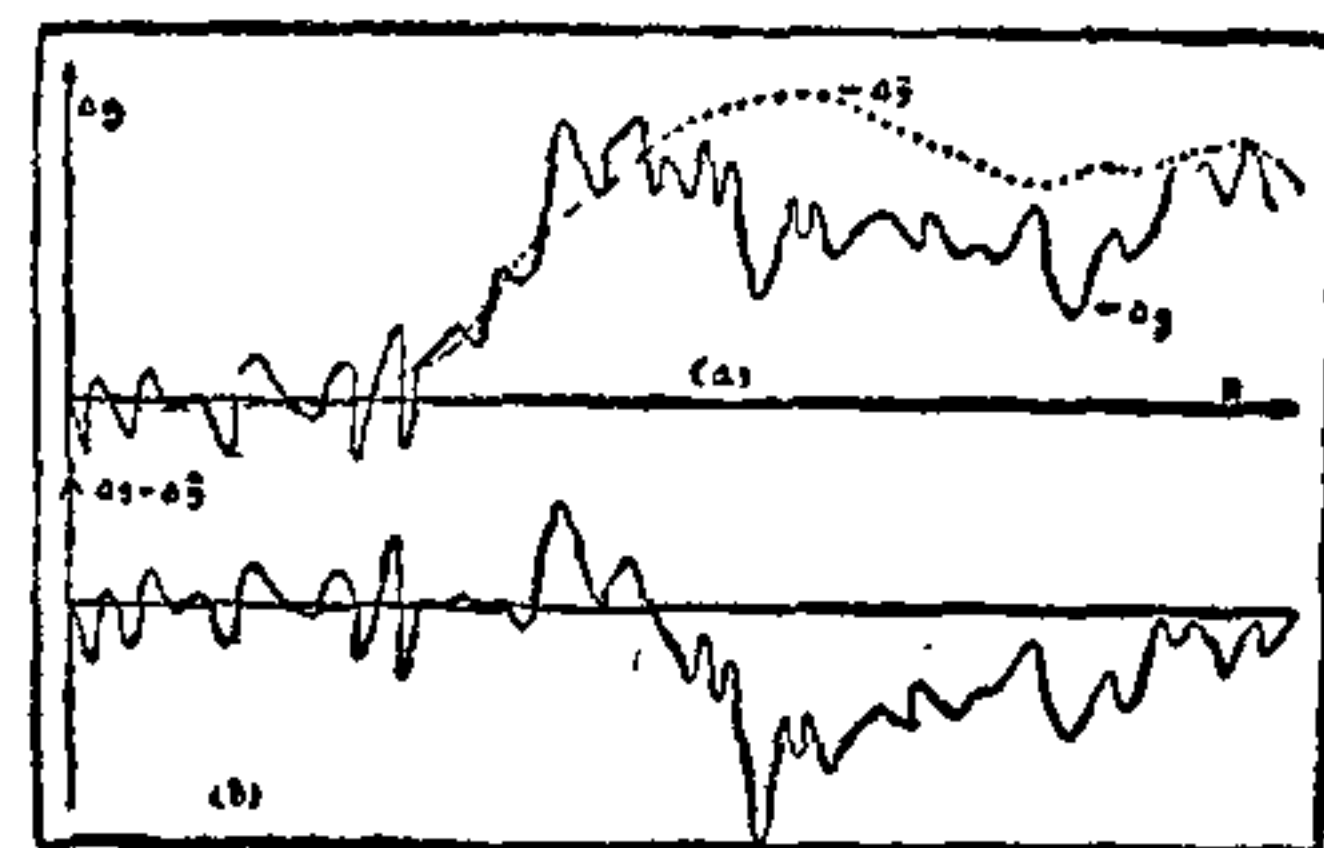


FIG. 2. Filtration of gravity field by latent vectors.

Figures 1 and 2 illustrate the advantage of the processing of geophysical data based on the above method. The initial gravity data (Δg) from the field surveys for the detection of oil and gas deposits is shown in Figs. 1(a) and 2(a) by a thick line. The filtered gravity field ($\Delta \bar{g}$) is shown by a broken line. The difference curves of the two fields ($\Delta g - \Delta \bar{g}$) are shown in Figs. 1(b) and 2(b) respectively. The anomaly is more pronounced, evidently, in Figs. 1(b) and 2(b).

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