IN the beginnings of general relativistic cosmology, the concept of an oscillating universe, with alternating cycles of expansion and contraction, has been popular. The Hawking–Penrose singularity theorems\(^1\), have pointed to the inadequacy of the concept of a pulsating universe within the standard framework of general relativity (with zero cosmological constant). Already for a single expansion-contraction cycle, there are two barriers (one at a finite time in the past, another at a finite time in the future) beyond which the space-time manifold cannot be extended. The well-known trouble with the cosmological singularity is doubled for closed models.

Of course, the singularity does not mean any breakdown of physics. Only classical physics stops here, and it is often conjectured that with some quantum theory of gravity, replacing the classical formulation of general relativity at some sufficiently contracted state of matter evolution, it should be possible to pass in a smooth manner from a stage of contraction to a subsequent of expansion. The singularity should be conceived thus only as in unrealistic consequence of the extrapolation of the classical general relativity theory to situations where it evidently cannot apply.

We have, so far, no consistent quantum formulation of the general theory of relativity. But when a characteristic physical quantity of the quantum theories—the spin, has been incorporated as a dynamical quantity into the general relativistic theory of space-time, this proved to be a sufficient step toward the expulsion of the unphysical, singularities from cosmology. This “general relativity with spin” is called the Einstein–Cartan theory of gravitation, briefly ECT\(^2\). The geometric structure of the space-time manifold is generalized in it from a purely Riemannian one into a general affine one, with non-vanishing torsion which is algebraically related to the density of the spin angular momentum of matter. Torsion is identically zero where matter density is zero, thus equations in general relativity and in the ECT are the same for empty space; at the present observational level, these two theories are practically indistinguishable.

The characteristic spin-spin interaction of matter in the ECT which dominates the behaviour of matter of extremely high densities, above ca. 10\(^{10}\) g cm\(^{-3}\), is able to prevent singularities. With the simple model of a Weyssenhoff fluid\(^4\) of energy density \(\rho\), pressure \(p\), and spin density vector \(\mathbf{S}\) (orthogonal to the four-velocity \(u\), in a comoving frame of reference), one of the two alternative approaches may be adopted: Either (I) the spin vector \(\mathbf{S}\) conforms to the symmetries of the space-time under study, i.e., the spins of matter are aligned along some characteristic direction of anisotropy, or (II) the spins are randomly distributed, and in averaging over them only some spin-square terms remain.

Cosmological models with aligned spins have to be at least partially anisotropic; extensions of the isotropic Robertson–Walker models to the ECT are thus impossible. The study of aligned spin models started two years ago with the Bianchi type I\(^7\). A lot of exact solutions and various results derived in the meantime are recently summarized\(^4\). More than two years ago, Trautman\(^7\) thought it interesting to know whether a closed model would also bounce because of spin and torsion. The possibility of such a bounce, for aligned spin models, has not been proved till now for the general Bianchi type IX models which are a generalization of the closed Robertson–Walker cosmologies. But it has been proved very recently\(^8\), that pulsating models with 4-parameter Lie group of isometry which does not permit any simply transitive group of motions are possible. These models constitute non-singular ECT analogues of the three-cylinder semiclosed cosmological models of Kantowski and Sachs\(^9\). In contrast to the closed Robertson–Walker models, they are always shearing. In ECT, the main scale-factor \(R\) for them is always \(R \geq R_{\text{min}} > 0\) provided a certain condition holds\(^8\) which expresses the fact that the effect of the spin term should overwhelm the effect of the shear when \(R \rightarrow R_{\text{min}}\). This condition for the prevention of the singularity looks much like analogous conditions for Bianchi type I models\(^9\).
Two other questions raised by Trautman, have been solved in the meantime: the effect of the pressure, and of the magnetic field $H$ which should be responsible for the alignment of spins. When energy density $\rho$ and pressure $p$ are related by a linear equation of state: $p = \gamma \rho$, with $0 < \gamma < 1$, then non-singular ECT solutions of several Bianchi types already exist. Also the pulsating s-closed models of Kantowski-Sachs type allow for non-zero pressure, while being non-singular in the ECT. Finally, exact non-singular cosmological models of axial symmetry, with a non-vanishing magnetic field pointing in the direction of spin alignment, have been constructed. The ECT allows even for static models containing a uniform magnetic field and spinning matter which were discussed earlier in this journal.

With random spin models, the situation is even simpler than for models with aligned spins. Since spins may locally fluctuate in direction, and only averages are meaningful, there are now no troubles with adapting the spinning Weyssenhoff fluid to the high symmetries of the Robertson-Walker models. Closed, flat and open Robertson-Walker models with random spin distribution are possible. Below we give the simple expression for the radius function $R$ of a pulsating Robertson-Walker model, filled with spinning dust of an averaged spin square density $<S^2 > = S_0^2/R^4$. This is: $2R - D = (D^2 - S_0^2) \sin D/2 \left(1 + \sqrt{DR - R^2 - \frac{4}{3} S_0^2}\right)$, where $D$ is some integration constant $(D^2 > S_0^2)$, and $t$ is the cosmic time. This model pulsates between the minimum radius $\frac{1}{2} (D - \sqrt{D^2 - S_0^2})$ and the maximum radius $\frac{1}{2} (D + \sqrt{D^2 - S_0^2})$. A characteristic difference with respect to the situation in general relativity (where $S_0 = 0$) occurs: The spin-spin interaction slightly raises the minimum radius from zero and at the same time it decreases also slightly the maximum radius from its general relativistic value.

We gave here this exact solution for a non-singular pulsating universe with randomized spins merely as an example; other related solutions are being derived and will be published together elsewhere. Our aim is to underline here that the possibilities of random spin models in the ECT are even higher than those of aligned spin models, both with respect to variety of types and to the possibility of incorporating the spinning matter in an already given Bianchi (or another) type. Thus Bianchi type II non-tilted models do not allow for a generalization into ECT models even into singular ones, as the spin distribution is aligned, i.e., it conforms to the symmetries of this type. But these, as well as all known models of general relativity may be incorporated into the ECT as models with randomly oriented spins, and in most cases (till now I could find no counterexample) they may be rendered free from singularity by the spin-spin interaction. Since the time when Trautman asked whether a bounce of closed models because of torsion is possible, there accumulated thus much positive evidence in favour of this bounce. The only dubious case is that of Bianchi type IX with aligned spins.