# A STATIC UNIVERSE FILLED WITH SPINNING MATTER AND MAGNETIC FIELD

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RECENTLY, the Einstein-Cartan theory has become a fashionable alternative to Einstein's original theory of gravitation[1-3]. Actually, is constitutes an extension of the latter theory, with the spin being incorporated from the beginning as a dynamical quantity. The spin angular momentum is related algebraically to the torsion tensor of the underlying manifold which no longer has a simple Riemannian structure. But, at the present observational level, the consequences of the ECT (Einstein-Cartan theory) are practically indistinguishable from those of general relativity, especially since the equations in empty space are identical in the two theories. The torsion of space-time does not vanish in regions filled with matter. A characteristic spin-spin repulsive interaction arises in the ECT which dominates the behaviour of matter at extremely high densities, say, above $10^{54}$ g $cm^{-3}$, and is able to prevent the occurrence of singularities in cosmology. Non-singular models of the Universe have been constructed explicitly[4-7], which behave quite analogously to Friedmann-like models with a minimum non-zero radius of the Universe[8]. Trautman's original value of this minimum radius, ca. 1 cm, may be raised by ca. 13 orders of magnitude if the *f*-gravity is taken into account[9]. Then the maximum density of the Universe is only two or three orders of magnitude higher than the nuclear density, which sounds quite reasonable. Another approach to include the short range of the *f*-gravity (mediated by massive spin 2-mesons) through a reinterpretation of the cosmological constant[10-12] yields practically the same results : the possible prevention of the singularity. Instead of the spin-spin repulsive interaction characteristic for the ECT, there appears in the theory of Sivaram *et al.*[12] a repulsive scalar component of the massive *f*-gravity. The role of the cosmological constant in preventing the singularity is essential in the latter theory[10-12]. On the other hand, Einstein's static universe exists only with a nonvanishing cosmological constant. These two facts, together with the common consequence of the two approaches, makes it meaningful to ask for the possibility of a static cosmological model in the ECT. Such a model is only a very remote analogy of Einstein's static universe, because the presence of spins makes it anisotropic ; numerical estimates yield values different from those of Einstein's model.

Our model is possible for the metric corresponding to Case 1 of Kantowski and Sachs[13] (a semi-closed model) :

$$ds^2 = -X^2(t)\,dx^2 - Y^2(t)\,[d\theta^2 + \sin^2\theta\, d\phi^2] + dt^2 c^2 \tag{1}$$

when we assume that the metric functions X and Y do not depend on time. The cosmological substratum is a perfect fluid of energy density $\rho$ and pressure $p$, with a density S of angular momentum, aligned along the $x$-axis. Also the magnetic field H points along this axis. The detailed equations are derived elsewhere[14], and we give here only the final results for the two metric functions and the average radius R of the Universe :

$$X = \sqrt{\frac{(2-\gamma)\,4\pi G}{\gamma}}\,\frac{8\pi G H S_0}{c^5},\quad Y = \frac{c^2}{\sqrt{8\pi G}\,H},$$

$$R^3 = \sqrt{\frac{(2-\gamma)\,4\pi G}{\gamma}}\,\frac{S_0}{Hc} \tag{2}$$

Here $c$ is the light velocity, G — Newtonian gravitational constant, $S_0$ — total spin of aligned matter. $\gamma$ is the coefficient ($1 \leqslant \gamma < 2$) in the linear equation of state : $p = (\gamma - 1)\rho$. Energy density of the fluid and spin density may be expressed in terms of the magnetic field :

$$\rho = \frac{1}{2-\gamma}H^2,\quad S = \sqrt{\frac{\gamma}{(2-\gamma)\,4\pi G}}\,cH \tag{3}$$

Let us assume that matter is composed of particles of mass M (identified with the nucleon mass) and spin $\frac{1}{2}\hbar$, and let $n$ denote the numerical particle density while N is the total number of particles in a universe of radius R. Then we have : $\rho = nMc^2$, $S = \frac{1}{2}\hbar n$, $S_0 = \frac{1}{2}\hbar N$. Particle number density $n$ and the magnetic field are well determined by the elementary constants :

$$n = \frac{\gamma}{\pi}\frac{Mc^4}{\hbar^2 G} \approx 0{\cdot}6\cdot 10^{70}\,\gamma \text{ particles per cm}^3,$$

$$H = \sqrt{\frac{(2-\gamma)\,\gamma}{4\pi G}}\,\frac{2Mc^3}{\hbar} \approx \sqrt{(2-\gamma)\,\gamma}\cdot 10^{38}\ \text{Oe}.$$

The transverse radius Y does not depend on the amount of particles present, and is equal to

$$Y = \hbar/2Mc\,\sqrt{2\gamma(2-\gamma)} \approx 0{\cdot}7\cdot 10^{-14}\text{ cm}\cdot\frac{1}{\sqrt{\gamma(2-\gamma)}},$$

while for a total of $N \approx 10^{80}$ baryons we get the "longitudinal" dimension $X \approx (2-\gamma)\cdot 0{\cdot}3\cdot 10^{40}$ cm, and the average radius $R = 2 \cdot 5/\sqrt[3]{\gamma}$ cm. Both the longitudinal and transversal dimension may come close to each other for a very "stiff" equation of state, with $\gamma$ approaching the value of 2. At the same time, magnetic field H goes to zero for $\gamma \rightarrow 2$. It is interesting to find that the "longitudinal" dimension of our universe goes linear with the total particle number N, while the "average radius" rises with $N^{1/3}$.

It is not possible to argue that the static model we present here has something to do with the actual universe in which we live. But in fact it may be related to the unstable initial stage of evolution of an expanding non-singular universe; such a stage might have lasted an indefinite amount of time before the instability resulted in expansion. Our model is classical, and it does not take into account the effects of pair creation, etc., which should occur at such high values of the magnetic field. In spite of all possible counterarguments against the reality of models of such kind, this may be an interesting result in itself, proving the possibility of deriving an anisotropic, static model with aligned spins and magnetic field in the framework of the ECT. No analogy to this in the general theory of relativity can be found, apart from the remote case of the Einstein static universe, where the term with the cosmological constant plays a similar stabilizing role like our spin (or torsion)-induced term in the ECT.


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