RELATION BETWEEN THE n^{th} POWER OF THE ADJACENCY MATRIX AND TH NUMBER OF POINTS OF A COMPLETE GRAPH

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ABSTRACT

In this note a theorem is proved by induction, which expresses the n^{th} power of the adjacency matrix of a complete graph in the form of sum of a series of $n \times n$ matrices. The elements of these matrices being functions of p, the number of vertices of the complete graph K_p. This relation is useful in finding the number of walks of length n from any point of the graph to the other.

DEFINITIONS: A graph G consists of a finite non-empty set V of P points, together with a set X of unordered pairs of distinct points of V. The points of V are called vertices and the elements of X are called edges. Each pair $x = u, v$ of points in X is a line of the graph G and x is said to join u and v. We write $x = uv$ and say that u is adjacent to v.

A graph G is said to be complete, if the edge set X consists of all possible pairs of distinct points of V, and is denoted by K_p.

A graph is said to be labelled, if each of its vertices are designated by a name, *viz.*, $V_1 V_2 . V_p$. A walk in a graph is an alternating sequence of vertices and edges, which are incident with each other. The length of a walk is the number of edges that it contains.

The Adjacency matrix of a graph with P vertices is defined as a Square matrix of order P^2, denoted by A whose elements are such that

$$a_{ij} = \begin{cases} 1 \text{ if } v_i \text{ is adjacent to } v_j \\ 0 \text{ otherwise} \end{cases}$$

The purpose of this note is to establish an identity relating the Adjacency matrix and the number of vertices P of a complete graph.

Theorem.—The adjacency matrix of a complete graph K_p with P ($\geqslant 3$) vertices satisfies the following Identity for all positive integral values of n.

When n is odd we have,

$$[A]^n_{pxp} = I_{pxp} + \sum_{r=0}^{(n-2)/2} [(p-1)^{2r+1}(p-2)]_{pxp}$$

When n is even, we have

$$[A]^n_{pxp} = [A]_{pxp} + \sum_{n=0}^{(n-2)/2} [(p-1)^{2r}(p-2)]_{pxp}$$

Here I_{pxp} is the Identity matrix and $[(\quad)]_{pxp}$ represents a square matrix of order p^2, the elements of which are in the brackets.

Proof: The proof is by Induction on n.

for $n = 1$ $\quad A = A$

for $n = 2$ $\quad [A]^2 = I_{pxp} + [(p-2)]$

for $n = 3$ $\quad [A]^3 = [A] + [(p-1)(p-2)]$

for $n = 4$ $\quad [A]^4 = I_{pxp} + [(p-2)] + [(p-2)(p-1)^2]$.

These four cases can be verified by actual multiplication of the matrix (A) we have,

$$A = a_{ij};$$

$$a_{ij} = \begin{cases} 0 & i = j \\ 1 & i \neq j \end{cases}$$

$$A^2 = a_{ij}^{(2)}$$

$$a_{ij}^{(2)} = \begin{cases} (p-1) & i = j \\ (p-2) & i \neq j \end{cases}$$

$$A^3 = a_{ij}^{(3)};$$

$$a_{ij}^{(3)} = \begin{cases} (p-1)(p-2) & i = j \\ (p-1) + (p-2)^2 & i \neq j \end{cases}$$

$$A^4 = a_{ij}^{(4)};$$

$$a_{ij}^{(4)} = \begin{cases} (p-1)(p-1) + (p-2)^2 & i = j \\ (p-2)2(p-1) + (p-2)^2 & i \neq j. \end{cases}$$

Case 1.—The identity was seen to be true for 2 and 4. Let us assume that the Identity is true for some even number K then for K + 1, an odd number we must obtain the other form of the Identity. Therefore, for $n + k$ even we have,

$$[A]^k = I_{pxp} + \sum_{r=0}^{(k-2)/2} [(p-1)^{2r}(p-2)]_{pxp}.$$

Now

$$[A]^{k+1} = [A][A]^k$$

$$[A]^{k+1} = [A]I + \sum_{r=0}^{(k-2)/2} [A][(p-1)^{2r}(p-2)]$$

We have

$$[A][(p-1)^{2r}(p-2)] = [(p-1)^{2r+1}(p-2)]$$

$$[A]^{k+1} = [A] + \sum_{r=0}^{(k-2)/2} [(p-1)^{2r+1}(p-2)]$$

Put $k+1 = m$ some odd integer, then $k = m-1$, Hence we have

$$[A]^m = [A] + \sum_{r=0}^{(m-?)/2} [(p-1)^{2r+1}(p-2)],$$

Case 2.—The Identity was seen to be true for 1 and 3. Let the Identity be true for some odd integer K^1, then K^1+1 will be an even integer. now we must obtain the other form of Identity which is true for even integer.

We have for $n = K^1$ an odd number

$$[A]^{k^1} = [A] + \sum_{r=0}^{(?-?)\,2} [(p-1)^{2r+1}(p-2)]_{rep}.$$

Now

$$[A]^{k^1+1} = [A]\,[A]^{k^1}$$

$$[A]^{k^1+1} = [A]\,[A] + \sum_{r=0}^{(n-3)/2} [A]\,[(p-1)^{2r+1} \times (p-2)]_{pxp}$$

$$[A]^2 = [I] + [(p-1)^0(p-2)]$$

$$[A]\,[(p-1)^{2r+1}(p-2)] = [(p-1)^{2r+2}(p-2)]$$

$$[A]^{k^1+1} = I_{pxp} + [(p-1)^0(p-2)] + \sum_{r=0}^{(k-3)/2} [(p-1)^{2r+2}(p-2)]_{pxp}$$

K^1+1 is an even number say n.

then

$$[A]^n = I_{pxp} + \sum_{r=0}^{((K^1-?)\,2+1)} [(p-1)^{2r}(p-2)]$$

$$[A]^n = I_{pxp} + \sum_{r=0}^{(n-2)/2} [(p-1)^{2r}(p-2)]_{pxp}$$

Hence the proof:

The Identity can also be proved for even number and odd numbers separately by considering the respective cases for numbers like n and $n+2$. It may be noted that the above proved Identity helps one to calculate the (ij) element $a_{ij}^{(n)}$ of A^n, which represents the number of walks of length n from i to j.

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THE PHYSIOLOGICAL SIGNIFICANCE OF THE INTERACTION OF BILIRUBIN WITH RECONSTITUTED COLLAGEN FIBRILS*

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THE extensive yellowing of the body surface in hyperbilirubinaemic new borns and the restoration to the normal colour of the skin after recovery from jaundice suggest that skin takes active part in the homeostasis of bilirubin at least during the diseased condition. Evidence has been adduced earlier to show that skin epithelium and skin strips of the mouse, rat, guinea pig and man possess a mechanism to accumulate and release bilirubin[1,2]. When skin strips saturated with bilirubin are exposed to light (80 watt) in Krebs Ringer buffer, there is a rapid bleaching of the skin accompanied by the release of water soluble and non-diazotizable degradation products of bilirubin[3].

The uptake of bilirubin by skin is sensitive to temperature indicating that binding of bilirubin to skin strips involves participation of collagen. Results of studies on the binding of bilirubin to collagen fibrils are reported now which suggest that the interaction between collagen and bilirubin may be involved in the uptake of bilirubin by skin.

Collagen was prepared from rat tail tendons according to Glimcher and Krane[4] and purified by dialysis against 0·02 M Na_2HPO_4. The precipitate obtained was redissolved in 1% acetic acid and stored as a lyophilized material. It contained 13·5% hydroxyproline[5]. When required it was dissolved in 100 mM acetic acid and converted to the reconstituted fibrillar form by dialysing at 2° against 200 mM Tris HCl buffer pH 8·6 or 7·5 as required following essentially the procedure of Gross and Krick[6]. The interaction of bilirubin with collagen fibrillar aggregate was followed in 5 ml 100 mM Tris HCl buffer pH 8·6 or 7·5 at 37° C in a two phase system where collagen was present as an opaque rigid gel composed of striated fibrils and bilirubin was in aqueous solution. Different amounts of serum albumin or competing anions could be added to this system as desired. The collagen fibrils were recovered by centrifugation and washed repeatedly with 100 mM Tris HCl buffer and the washed fibrils extracted

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