ON THE MASS SPECTRUM OF ELEMENTARY PARTICLES

K. P. SINHA AND C. SIVARAM*

Indian Institute of Science, Bangalore-560012, India

ABSTRACT

The physical origin of some of the scaling factors introduced in the mass formula given in a prevous paper is explained in terms of interactions involving magnetic charges. The role of gavitation in determining the mass is also clarified in greater detail.

INTRODUCTION

IN a recent paper a formula for calculating the masses of various elementary particles was developed on the basis of a dynamical theory[1]. The central idea of this paper was that the various elementary particles are excited states of a primordial object which is executing harmonic oscillations about some equilibrium configuration determined by the interplay of gravitational and other (*e.g.*, electromagnetic) interactions. Accordingly the restoring forces and hence the oscillation frequencies are controlled by these factors. That the gravitational interaction (general relativity) might play some role in the problem of the mass quantization of elementary particles has also been emphasized recently by some other authors[2,3]. The purpose of the present paper is to amplify and clarify certain details that were not given but promised in the earlier paper.

THE QUANTUM GRAVITATIONAL MASS CONSTANT

As shown by Motz[3], by equating the Gaussian curvature of the space occupied by a particle having rest mass M_P to the inverse Compton length, one gets the relation:

$$16\pi G M_P^2 = \hbar c \tag{1}$$

where G is the Newtonian gravitational constant. We have introduced a factor 16π in the present paper so as to be consistent with a covariant quantum gravitational mass constant as given by Rosen[4].

Thus

$$M_P = \left(\frac{\hbar c}{16\pi G}\right)^{\frac{1}{2}} = 3 \times 10^{-6}\, g \tag{2}$$

which was referred to by us as the Planck mass in our previous paper[1]. Motz has named the particle having the mass

$$\left(\frac{\hbar c}{G}\right)^{\frac{1}{2}} = M_U,$$

the 'uniton'.

Now the solution of the Einstein field equations

$$E_{\alpha\beta} = -\frac{8\pi G}{c^4} T_{\alpha\beta},$$

for a spherically symmetric mass M with charge e, that relates the electromagnetic interaction to the metric of the surrounding space is given by the well-known line element found by Reissner[5] and by Nordström[6], as:

$$ds^2 = \left(1 - \frac{2m}{r} + \frac{e^2 G}{r^2}\right) c^2 dt^2 - \left(1 - \frac{2m}{r} + \frac{e^2 G}{r^2}\right)^{-1} dr^2 - r^2 d\Omega^2$$

where

$$m = \frac{GM}{c^2}, \quad d\Omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2$$

Ross[2] has postulated that a particle assumes an equilibrium radius such that the effective gravitational potential is a minimum. With this postulate, it can be shown that the above metric exhibits an equilibrium point at r_0, where r_0 is given as:

$$r_0 = \frac{e^2}{M_P c^2} = 9 \times 10^{-35}\,\text{cm} \tag{3}$$

[for the Planck mass given by Eq. (2)]. We shall now show that at this value of the radius there is no force acting on an extensible object having a charge e. Consider a displacement of the equilibrium radius r_0 by an infinitesimal amount Δr. Making use of the geodesic equation we have (using the co-ordinates, $X^0 = ict$, $X^1 = r$, $X^2 = \theta$, $X^3 = \phi$),

$$-c^2 \Gamma'_{00} = \text{acceleration}, \tag{4}$$

where the Γ's are the Christoffel symbols of the second kind. For the above line element, we have:

$$-c^2 \Gamma'_{00} \approx \left(\frac{GM}{r^2} - \frac{e^2 G}{r^2 c^2}\right).$$

Thus the force acting on the particle of mass m turns out to be:

$$\begin{aligned} F &\approx -\frac{GMm}{r_0^2}\left(1 + \frac{\Delta r}{r_0}\right)^{-2} + \frac{Ge^2 m}{r_0^3 c^2}\left(1 + \frac{\Delta r}{r_0}\right)^{3} \\ &\approx -\frac{GMm}{r_0^2} + \frac{2GMm}{r_0^2}\left(\frac{\Delta r}{r_0}\right) + \frac{Ge^2 m}{r_0^3 c^2} - \frac{3Ge^2 m}{r_0^3 c^2}\left(\frac{\Delta r}{r_0}\right) \end{aligned} \tag{5}$$

neglecting the higher order terms.

* Research Associate, Raman Research Institute, Bangalore.

At the equilibrium radius r_0,

$$\Delta r = 0, \text{ and } \frac{GMm}{r_0^2} = \frac{Ge^2m}{r_0^3c^2}. \quad (6)$$

Accordingly the force F on displacement can be written as:

$$F \approx \left(\frac{2GMm}{r_0^3} - \frac{3Ge^2m}{c^2r_0^4}\right) \cdot \Delta r \quad (7)$$

which is zero at the radius r_0, when $\Delta r = 0$.

As no forces act on the object when it is at radius r_0, it is indeed the equilibrium radius. Equation (7) suggests that F is a restoring force, the quantity within brackets being related to a stiffness constant. The corresponding frequency of oscillation is then given by:

$$\omega_0 = \sqrt{\left(\frac{3Ge^2}{c^2r_0^4} - \frac{2GM}{r_0^3}\right)}. \quad (8)$$

If we were to take a covariant formulation of the equations of motion in general relativity then there will be a few additional terms in equation (8); however, these turn out to be much smaller in magnitude. This point will be elaborated upon and discussed elsewhere.

The foregoing shows that on disturbing the object from its equilibrium configuration, elastic restoring forces of a general relativistic origin are set up as a result of which the object oscillates around the equilibrium point. On using the value of the Planck mass (quantum gravitational mass) as given by equation (2) and the corresponding equilibrium radius r_0 [given by equation (3)], the frequency ω_0 [as given by equation (8)], turns out to be $\sim 5 \times 10^{44}$ C/sec.

As remarked in our earlier paper[1] and as also noted by Motz[3], the uncertainty principle forces the final configuration to expand to the correct size of the particle. Let us consider the case of the electron. The frequency ω_0 of the primordial object having a bare mass equal to the Planck mass corresponds to a Compton length $\lambda\!\!\!^{-}_0 \approx c/\omega_0 \sim 6 \times 10^{-33}$ cm. In other words the object is observable at this distance with energy $\hbar\omega_0$. As the dynamically stable configuration expands, the frequency decreases inversely as the distance. This can be interpreted as energy lost in increasing the radius of curvature. Applying the uncertainty principle the frequency (ω_e) corresponding to the electron rest mass can be written as:

$$\omega_e = \frac{\lambda\!\!\!^{-}_0}{\lambda\!\!\!^{-}_e}\,\omega_0 = 7 \cdot 77 \times 10^{20}\ \mathrm{C/S} \quad (9)$$

where $\lambda\!\!\!^{-}_e = 3 \cdot 86 \times 10^{-11}$ cm $\simeq$ electron Compton wavelength. As shown in Ref. 1, the ground state of an oscillator having the frequency ω_e gives the electron rest mass. W can rewrite equation (9) as:

$$\omega_e = \frac{\lambda\!\!\!^{-}_0}{\lambda\!\!\!^{-}_e}\,\omega_0 \cong \exp\left(\frac{-x}{\alpha}\right)\omega_0 = 7 \cdot 77 \times 10^{20}\ \mathrm{C/S};$$

where

$$\alpha \equiv \frac{e^2}{\hbar c},\ x = 0 \cdot 4.$$

The scaling factor $\exp(-x/\alpha)$ is similar to that used in the formula for the electron mass given by Rosen[2]. The advantage in putting the factor in this form is seen by letting $\alpha \to 0$, then the mass also tends to zero. This might explain why the massless lepton counterpart of the electron, *i.e.*, the neutrino has no charge.

THE RENORMALISATION OF FREQUENCIES

In the earlier paper[1] we saw that for mesons and baryons the frequencies were respectively scaled up by

$$\frac{1}{2\alpha}\,\omega_e \quad \text{and} \quad \frac{1}{\alpha^{3/2}}\,\omega_e,$$

α being the fine structure constant. To arrive a this scaling, we invoke the concept of magnetic charges (monopoles) introduced by Dirac[7] and recently revived by Schwinger[8].

It may be remarked that the introduction of magnetic charges brings in symmetry in Maxwell's equations and explains the universality of the electric charge. Following Dirac and Schwinger we introduce the magnetic charge 'f'. Now there is no observational evidence for the existence of free magnetic charges in nature, so far (for example, *see* Amaldi[9]). Each magnetic charge f must have a countercharge $-f$, as all particles are observed to be magnetically neutral. If we were to consider the three charges e, f and $-f$, the system will have an angular momentum[8]. On quantizing this, we get the relation

$$\frac{ef}{c} = n\hbar \quad (10)$$

For the fundamental magnetic charge we choose $n = 1$, (Dirac allows half-integral values also). Thus we have:

$$f_0 = \left(\frac{\hbar c}{e^2}\right) e = \frac{e}{\alpha} = 137e. \quad (11)$$

This shows that the fundamental magnetic charge is $1/\alpha$ (≈ 137) times larger than electric charge. According to the currently accepted view there are two kinds of hadrons, namely, mesons (bosons) and baryons (fermions). These are now believed to be composites, *i.e.*, the baryons (*e.g.*, proton) are supposed to consist of a hard core (parton) surrounded by meson clouds. Following Schwinger[8] we take two different values of magnetic charge, *i.e.*, $2f_0$

and f_0 and their oppositely charged counterparts. As the elementary particles are known to be magnetically neutral, we envisage the following neutral composites: mesons comprise $(f_0, -f_0)$ and the baryon hard core comprises $(2f_0, -f_0, -f_0)$, with the reverse configuration for the antiparticles. Let us then consider the renormalisation effects owing to the presence of these magnetic charges. The coupling constant g_m involving magnetic charges in analogy with the case of electric charges, can be written as (using Eq. 11):

$$g_m = \sqrt{\frac{f_0^2}{\hbar c}} = \sqrt{\frac{e^2}{\alpha^2 \hbar c}} = \frac{1}{\alpha^{1/2}}. \qquad (12)$$

Since the interaction is super-strong it is adequate to consider only lowest order terms[10]. For mesons with two magnetic charges we will have two vertices and for the baryon core three vertices, *i.e.*, from the three magnetic charges. Accordingly, the renormalization factor for mesons is

$$\frac{1}{2}\left(\frac{1}{\alpha^{1/2}}\right)^2 = \frac{1}{2\alpha};$$

the number 2 being a weight factor. Similarly for baryon core the three vertices give a factor

$$\left(\frac{1}{\alpha^{1/2}}\right)^3 = \frac{1}{\alpha^{3/2}}:$$

Thus for mesons and the parton (baryon hard core) the renormalised frequencies will be given by

$$\omega_M = \frac{1}{2\alpha}\,\omega_e \qquad (13)$$

$$\omega_P = \frac{1}{\alpha^{3/2}}\,\omega_e. \qquad (14)$$

As explained before[1], a relativistic oscillator model is needed for mesons and baryons. Thus for mesons the eigen masses are given by:

$$m(n_M) = \frac{\hbar\omega_e}{2\alpha}\,[n_M + 1]^2. \qquad (15)$$

The baryons are composed of the parton hard core and the meson clouds, each having independent oscillations. Thus the eigen masses are given by

$$m(n_M, n_P) = \hbar\omega_e\left[\frac{(n_M+1)^2}{2\alpha} + \frac{(n_P+1)^2}{\alpha^{3/2}}\right]. \qquad (16)$$

The parton core being in the unexcited state we can put $n_P = 0$ and rewrite Eq. (16) as:

$$m(n_M, n_B) = \hbar\omega_e\left[\frac{(n_M+1)^2}{2\alpha} + \frac{n_B}{\alpha^{3/2}}\right] \qquad (17)$$

with $n_B = 1$, for baryons and $n_B = 0$ for mesons.

The present model of hadrons is in conformity with the Vigier model of elementary particles[11]. In this model the particles are assumed to be relativistic droplets having six degrees of freedom. In the present case we can think of quantized rotation for the parton hard core and the meson cloud each having a different centre of mass. This will give rise to six degrees of freedom. Work in this direction is in progress. It may be remarked that the meson cloud is held to the parton by strong interaction forces which are much weaker than the super-strong forces holding the parton.

In the foregoing we have discussed the physical basis of the formula developed earlier. The previous paper gave the results of the computations.

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SOME ASSORTED LEPTONIC PUZZLES*

SANDIP PAKVASA

Department of Physics and Astronomy, University of Hawaii, Honolulu, Hawaii-96822
(Visiting Scientist, July 1972, Centre for Theoretical Studies, Indian Institute of Science, Bangalore, India)

I WOULD like to talk about what seem to me to be some outstanding puzzles which I call leptonic because they all involve leptons in one way or another. According to current conventional wisdom, not all of them have to do with leptons, as we shall see. I will first indicate these puzzles, and then discuss briefly some recent speculations which bear on them[1-4].

The oldest, most venerable of these is the muon puzzle, sometimes phrased as why a muon?

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