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SOLUTION OF LINEAR IMMISCIBLE DISPLACEMENT PROCESS IN A FINITE POROUS MEDIUM WITH RANDOMLY ORIENTED FRACTURES

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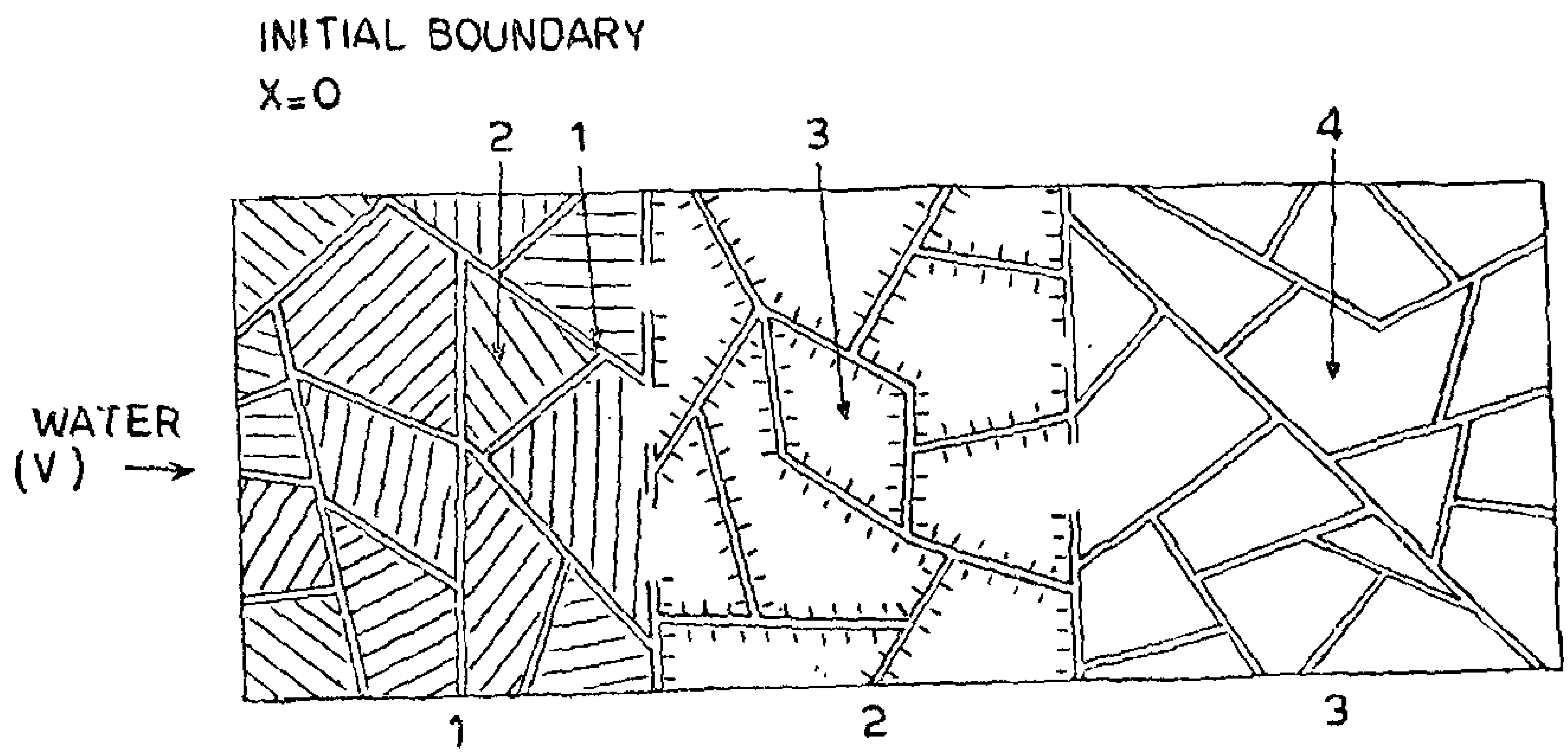
ABSTRACT

In this paper a specific problem of immiscible displacement through a porous medium containing randomly oriented fractures is discussed. An analytical expression for the phase saturation distribution is obtained by adopting a perturbation procedure.

INTRODUCTION

A POROUS medium consisting of extensively developed system of randomly oriented fractures is called a fractured porous medium. In such a medium the entire porous domain is made up of a large number of porous blocks each surrounded by the fractures that may form an interconnected network of narrow passages imbedded in the porous medium (*see* Fig. 1). The physics of oil water flow in fractured media is described by



Impregnation of a one dimensional Fractured Porous medium with water.

1 FRACTURES 2 COMPLETELY IMPREGNATED BLOCKS

3 BLOCKS BEING IMPREGNATED

4 NON-IMPREGNATED BLOCKS

FIG. 1

Bokserman *et al.*[1]. Recently Verma[2,3] and Venkateswarlu[4-6] have discussed specific oil-water displacement problems in porous media with additional physical phenomena, and offered their analytical solutions. In the present investigation, we consider extensively developed fractured system of the type described by Bokserman *et al.* Further we assume oil and water as the two immiscible phases involved in the displacement process.

Water at a constant velocity (V) is injected into a finite fractured medium containing oil (Fig. 1). It is assumed that the entire oil on the initial boundary, $X = 0$ (X, being measured in the direction of displacement) is displaced through a small distance due to the impact of the injecting water. It follows from this that at $X = 0$, $K_0 = 0$ for all time becomes the boundary condition of the problem.

Flow in Fractured Porous Medium

In a fractured porous medium the volume of water entering the blocks (under the action of capillary suction) in an elementary volume of the medium is called the 'Impregnation function' $\phi(t)$ which is defined by Mattax and Kyte[7] as below:

$$\phi(t) = D(\epsilon t)^{-\frac{1}{2}}$$

where D and ϵ are the constants which depend on the nature of the fractured medium and 't' is the time. Ryzhik has pointed out that in a fractured medium the equations of continuity for the flowing phases include an additional term the capillary suction function $\phi[T - \tau(\xi)]$ which is defined as below:

$$\phi[T - \tau(\xi)] = D[T - \tau(\xi)]\,; \quad T = \epsilon t \qquad (1)$$

$\tau(\xi) = \bar{a}\xi^2$ ($\bar{a}$ is a constant).

$\xi = \dfrac{x}{l}$ (l is the mean block size).

Equation of Motion for Saturation

From Darcy's law the seepage velocity for water (v_w) and oil (v_0) and the continuity equations for the flowing phases (after Ryzhik as in ref. 1) can be written as below:

$$v_w = -K\frac{K_w}{\mu_w}\frac{\partial p}{\partial x} \qquad (2)$$

$$v_0 = -K\frac{K_0}{\mu_0}\frac{\partial p}{\partial x} \qquad (3)$$

$$m\frac{\partial S_w}{\partial t} + \frac{dv_w}{\partial x} + \phi[T - \tau(\zeta)] = 0 \qquad (4)$$

$$m\frac{\partial S_0}{\partial t} + \frac{\partial v_0}{\partial x} - \phi[T - \tau(\xi)] = 0 \qquad (5)$$

where 'm' and 'K' are the porosity and permeability of medium; K_w and K_0 are the relative permeability to water and oil, which are functions of water saturation (S_w) and oil saturation (S_0) respectively; μ_w and μ_0 are the viscosity of water and oil. The densities of the flowing phases are considered to be constant in the present analysis.

Adding equations (4) and (5) and since the sum of the two flowing phases is unity. we may write:

$$\frac{\partial v_w}{\partial x} + \frac{\partial v_0}{\partial x} = 0. \qquad (6)$$

Using equations (2), (3) and (6) and integrating under the boundary condition that at $X = 0$, $K_0 = 0$ for all time, we obtain:

$$\frac{\partial p}{\partial x} = -\frac{V}{K\left[\frac{K_w}{\mu_w} + \frac{K_0}{\mu_0}\right]}. \qquad (7)$$

Putting the value of v_w and $\delta p/\delta x$ in equation (4), changing t to T with the help of equation (1), we have the equation of motion for phase saturation as follows:

$$m\epsilon\frac{\partial S_w}{\partial t} + V\frac{\partial}{\partial x}\left[\frac{K_w\mu_0}{K_w\mu_0 + K_0\mu_w}\right] + \phi[T - \tau(\xi)] = 0. \qquad (8)$$

For definiteness, the experimental results of Jones [as in ref. 9] for the relation between the relative permeability and the phase saturation are taken in the following form:

$$K_w = S_w^3\,;\ K_0 = 1 - aS_w\,;\ (a = 1\cdot 11). \qquad (9)$$

Using equation (9), the equation of motion can be rewritten as follows:

$$m\epsilon\frac{\partial S_w}{\partial t} + V\left[\frac{3PS_w^2 - 2PaS_w^3}{(1 - aS_w + PS_w^3)^2}\right]\frac{\partial S_w}{\partial x} + \phi[T - \tau(\xi)] = 0. \qquad (10)$$

Perturbation Solution

Since the fractured medium considered is of Bokserman *et al.* type therefore we assume ϵm ($\epsilon \sim 10^{-4}$ to 10^{-6} and $m \sim 10^{-2}$ to 10^{-3}) as a perturbation parameter and write equation (10) as follows:

$$\left[\frac{3PS_w^2 - 2PaS_w^3}{(1 - aS_w + PS_w^3)^2}\right]\frac{\partial S_w}{\partial x} = -\frac{D}{\sqrt{T - Rx^2}}. \qquad (11)$$

Evgen'ev[8] has recently shown that 'P' the ratio of viscosity of oil to water is large in most of the cases, and so we assume 1/P as a small quantity. Accordingly we simplify equation (11) and integrating it under the boundary condition that at $X = 0$, $K_0 = 0$ for all times, we have :

$$\left\{\frac{1-\alpha S_w}{PS_w^3}\right\} = \frac{D}{V\sqrt{R}} \sin^{-1}\left\{x\sqrt{\frac{R}{T}}\right\} \qquad (12)$$

$$x = \sqrt{\frac{T}{R}} \sin\left[\frac{V\sqrt{R}}{D}\left\{\frac{1-\alpha S_w}{PS_w^3}\right\}\right].$$

The solution by perturbation method consists in substituting the value of $\delta S_w/\delta T$ [as obtained from equation (12)] in equation (10) which can be written as :

$$V\left[\frac{3PS_w^2 - 2P\alpha S_w^3}{(1-\alpha S_w + PS_w^3)^2}\right]\frac{\partial S_w}{\partial x} + \frac{D}{\sqrt{T - Rx^2}} + f(S_w, x, t) = 0 \qquad (13)$$

where

$$f(S_w, x, t) = \epsilon m \frac{\partial S_w}{\partial T}.$$

In the light of our previous remark that $1/P$ is a small quantity we simplify $f(S_w, x, t)$ and write equation (13) as :

$$V\left[\frac{3PS_w^2 - 2P\alpha S_w^3}{(1-\alpha S_w + PS_w^3)^2}\right]\frac{\partial S_w}{\partial x} + \frac{D}{\sqrt{T - Rx^2}} + \frac{\epsilon}{\sqrt{T - Rx^2}}\left[\frac{1}{2\alpha\sqrt{T}} - \frac{S_w}{2\sqrt{T}}\right] = 0. \quad (14)$$

Rearrangement and further simplification yields :

$$\frac{V}{PZ}\left\{\frac{1}{S_w^3} + \frac{n}{2}\frac{1}{S_w^2} - \alpha\frac{1}{S_w^2} - \alpha\frac{n}{2}\frac{1}{S_w}\right\} = -\frac{1}{\sqrt{T - Rx^2}} \qquad (15)$$

where

$$n = \frac{\epsilon}{\sqrt{T}Z} \quad \text{and} \quad Z = D + \frac{\epsilon}{2\alpha\sqrt{T}}.$$

This is a first order differential equation which on integration gives :

$$\frac{V}{2PZ}\left[\frac{1}{S_w^2} + \frac{1}{S_w}(n - 2\alpha) + \alpha n \log S_w\right] + F = \frac{1}{\sqrt{R}}\sin^{-1} x\sqrt{\frac{R}{T}} \qquad (16)$$

where F is a constant of integration which is evaluated as follows :

at $x = 0$, $K_0 = 0$

and

$1 - \alpha S_w = 0$,

hence

$S_w = 1/\alpha$

$$F = -\frac{V}{2PZ}\left[\alpha^2 + (n - 2\alpha)\alpha + n\alpha \log\left(\frac{1}{\alpha}\right)\right].$$

Putting the value of F in equation (16), we have :

$$x = \sqrt{\frac{T}{R}} \sin\left[\frac{V\sqrt{R}}{2PZ}\left\{\frac{1}{S_w^2} + (n - 2\alpha)\frac{1}{S_w} + n\alpha \log S_w\right\} + F\right]. \qquad (17)$$

This is the desired analytical expression for we can determine the phase saturation distribution for any assigned value of x and t.

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