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ON THE PROBLEM OF NONTRANSPORT OF ELECTRONS IN DISORDERED SYSTEMS (ANDERSON LOCALIZATION)

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ABSTRACT

The problem of non-transport (Anderson localisation) in cellularly disordered systems is well known to be related to the question of convergence of the Brillouin Wigner perturbation expansion for self-energy involving random energy denominators. In this preliminary communication an explicit criterion for the absolute convergence in the probability sense, of a partial series, has been obtained in terms of the width Δ of the rectangular distribution of the random site-energies and the band width W. The partial series considered involves only terms with non-repeating site indices.

CONSIDER a cellularly disordered crystal described by the Anderson Hamiltonian in the i site representation as

$$H = \sum_{l} \epsilon_l a_l^{+} a_l + \sum_{l \neq m} V_{lm} a_m^{+} a_l \qquad (1)$$

where a_l^+, a_l are the creation, annihilation fermion operators corresponding to the non-degenerate (Wannier) orbital at site l. The ϵ_l's represent the statistically independent random orbital energies having a rectangular distribution of width Δ centred at zero. V_{lm} is the transfer matrix element.

For simplicity we consider a simple-cubic lattice with nearest-neighbour interactions only. Thus the matrix elements $V_{lm} = V\,\delta_{l,\,m+\Delta}$ where Δ spans nearest neighbours. It is well known[1-2], that the problem of localisation is connected with the question of convergence of the Brillouin-Wigner perturbation series for the self-energy[2]:

$$E = \epsilon_0 + \sum_{l_1} V_{0l_1} \frac{1}{E - \epsilon_{l_1}} V_{l_1 0} + \sum_{l_1, l_2} V_{0l_1} \frac{1}{E - \epsilon_{l_1}} V_{l_1 l_2} \frac{1}{E - \epsilon_{l_2}} V_{l_2 0} + \cdots + \sum_{l_1 l_2, \ldots, l_n} \frac{V_{0l_1} V_{l_1 l_2} \cdots V_{l_n 0}}{(E - \epsilon_{l_1})(E - \epsilon_{l_2}) \cdots (E - \epsilon_{l_n})} + \cdots \qquad (2)$$

Anderson[1] has discussed the criterion for the localisation of an electron placed at site '0' in terms of the convergence of an analogous series.

In the present analysis, however, we shall discuss the criterion for the absolute convergence of a partial series consisting of terms with non-repeating indices only. Thus the nth order term of the partial series involves Z^n terms, approximately equal to the number of self-avoiding closed random walks of length n from a given site (Z being the number of nearest neighbours). The absolute value of a typical nth order term in this partial series will be denoted by T_n, where

$$T_n \simeq \prod_{i=1}^{n} |x_i - \epsilon|^{-1}$$

with

$$x_i = \frac{\epsilon_i}{Z|V|}, \quad \epsilon = \frac{E}{Z|V|}.$$

From the hypothesis on $\{\epsilon_i\}$ it follows that the sequence $\{x_i\}$ is a sequence of independently and identically distributed random variables each having a rectangular distribution over the range $(-\Delta/2Z|V|, \Delta/2Z|V|)$. To discuss the convergence of $\sum_n T_n$ we can now use results from modern probability theory.

We write $T_n = \exp S_n$ where $S_n = -\sum_{i=1}^{n} \log|x_i - \epsilon|$. If we set $Y_i = -\log|x_i - \epsilon|$ then it is well known (*see* for instance Feller[3]) that

Prob $\{S_n > 0$ for infinitely many $n\} =$ one or zero

according as the mean $\bar{Y}_1$ of $Y_1 \geqslant O$ or $< O$. Thus if $\bar{Y}_1 \geqslant O$ the $T_n \geqslant 1$ for infinitely many n and hence $\sum_n T_n = \infty$ with probability one. If $\bar{Y}_1 < O$, we use the strong law of large numbers[3] to conclude that $S_n < n\bar{Y}_1/2$ for all large n. Thus if $\bar{Y}_1 < O$, $T_n = \exp S_n < C^n$ where $O < C = \exp(\bar{Y}_1/2) < 1$ and hence $\sum_n T_n < \infty$ with probability one.

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We may summarise the above discussion as follows: The series $\sum_n T_n$ converges with probability one if $\bar{Y}_1 < O$ and diverges with probability one if $Y_1 \geqslant O$.

It may be verified that

$$\bar{Y}_1 = -\frac{1}{4}\left\{(1+x)\log(1+x) + (1-x) \times \log|1-x| + 2\left(\log\frac{\Delta}{2ZV} - 1\right)\right\}$$

$= -f(x)$ say

where $O \leqslant x = (2E/\Delta) < \infty$. Of course, the case $-2E/\Delta = x \leqslant O$ is just the mirror image.

take place within or outside the band according as $O < \Delta/VZ (\equiv 2\Delta/W) < e$ or $e < 2\Delta/W < 2e$. The first case may imply the existence of mobility edges. Thus the mobility edges move inwards as $\Delta/W \to \infty$. For $2\Delta/W > 2e$ all staates are localised. The relationship of the present work to that of Economou and Cohen[4] is not transparent at the moment.

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$2E/\Delta$ \ Δ/VZ	$< e$	$e < \frac{2\Delta}{W} < 2e$	$\frac{2\Delta}{W} > 2e$
$\leqslant 1$	Delocalised	$\exists X_0$ $O \leqslant X_0 \leqslant 1 \ni$ $X \leqslant X_0$ delocalised $X > X_0$ localised This X_0 is the unique zero of f in $[0, 1]$	Localised
> 1	$\exists X_0 > 1 \ni$ $X \leqslant X_0 \Rightarrow$ delocalised $> X_0$ localised This X_0 is the unique zero of f in $(1, \infty)$	Localised	Localised

Using the monoticity of $f(x)$, we get the following criterion for convergence or localisation in the different cases as given above in a tabular form.

In conclusion, we may remark that the transition from the delocalised states in the centre of the band to the localised states near the band edges can

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INDUCED BREEDING OF THE FRESHWATER CATFISH *CLARIAS BATRACHUS* (LINN.) BY USING PITUITARY GLANDS FROM MARINE CATFISH

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INTRODUCTION

THE freshwater catfish, *Clarias batrachus* (Linn.), commonly known as 'magur', has great cultural possibilities. The possession of accessory respiratory organs in this fish renders it suitable for culture in swamps and derelict ponds, where carps cannot be cultured. Because of its hardy nature, the survival rate is quite high, and as high a production as 1,07,500 kg/ha/year has been reported from Thailand by Sidthimunka *et al.* (1966), by stocking at a rate of 20 lakhs of fry per hactare. This compares very favourably with the major carp production of about 4,000 kg/ha/year only in tropical climates (Lakshmanan *et al.*, 1971). Further, the 'magur' is a highly esteemed table fish in the inland areas of our country. In view of these favourable features, investigations on the culture of *Clarias batrachus* have been taken up at the Fisheries College, Mangalore. Since availability of stocking material is a pre-requisite for culture, experiments were conducted successfully to breed the species by hypophysation technique and the results therefrom are reported in this paper. Because