SCHWARZSCHILD INTERIOR SOLUTION IN CONFORMALLY FLAT CO-ORDINATE SYSTEM

Spherically symmetric perfect fluid distributions with uniform density \( \rho = \rho(t) \) in a suitably chosen co-ordinate system may be divided into two categories: (i) solutions obeying an equation of state and (ii) solutions where no equation of state exists (non-uniform pressure). Where-as Robertson-Walker cosmological solutions are well-known examples of the first category, the Schwarzschild interior solution and some of its generalizations given by Thompson and Whitrow\(^3\), and Vaidya\(^4\) belong to the second category. But the geometry of both these categories of solutions is conformally flat. The conformally flat form of the Robertson-Walker solutions was given by Infeld and Schild\(^1\). In the present note we obtain the conformally flat form of the Schwarzschild interior solution.

The Schwarzschild interior metric may be expressed as
\[
ds^2 = -(1 - a^2 r^{-2})^{-1} \, \frac{\partial r^2}{\partial r} - r^2 \, d\Omega^2 + (A - B \sqrt{1 - a^2 r^{-2}})^2 \, dt^2
\]
where
\[
d\Omega^2 = d\theta^2 + \sin^2 \theta \, d\phi^2, \quad a^2 = 8\pi\rho/3,
\]
\(\rho\) being a constant. We wish to transform the metric given by (1) into the conformally flat form
\[
ds^2 = a^{-2} (r, t) \left( - \frac{dr^2}{dr} - r^2 \, d\Omega^2 + dt^2 \right).
\]
To obtain the conformal function \(a(r, t)\) we have to solve the first order partial differential equations
\[
g_{ij} = g'_{ij} \frac{\partial x'^i}{\partial x^a} \frac{\partial x'^j}{\partial x^b}.
\]
Making use of (1) and (2) we write (3) explicitly as
\[
a^{-2} = (1 - a^2 r^{-2})^{-1} \left( \frac{\partial r'}{\partial r} \right)^2 - (A - B \sqrt{1 - a^2 r^{-2}})^2 \left( \frac{\partial t'}{\partial r} \right)^2
\]
\[
a^{-2} = -(1 - a^2 r^{-2})^{-1} \left( \frac{\partial r'}{\partial t} \right)^2 + (A - B \sqrt{1 - a^2 r^{-2}})^2 \left( \frac{\partial t'}{\partial t} \right)^2.
\]
\[
0 = -(1 - a^2 r^{-2})^{-1} \frac{\partial r'}{\partial r} \frac{\partial t'}{\partial r} + (A - B \sqrt{1 - a^2 r^{-2}})^2 \frac{\partial t'}{\partial t} \quad (6)
\]
\[
a^{-1} r = r'.
\]
Adding (4) and (5) and dividing the resulting expression by (6) we get on simplification the relationship
\[
\frac{\partial t'}{\partial r} \frac{\partial r'}{\partial t} = \frac{\partial t'}{\partial t} \frac{\partial r'}{\partial r}.
\]
Substituting from (8) in (6) we readily obtain
\[
(A - B \sqrt{1 - a^2 r^{-2}}) \frac{\partial r'}{\partial r} \pm (1 - a^2 r^{-2})^{-1/2} \frac{\partial r'}{\partial t} = 0.
\]
Eliminating \(\partial t'/\partial r\) from (4) and (9) we get
\[
\left( \frac{\partial}{\partial t} (ra^{-1}) \right)^2 \left( \frac{\partial}{\partial r} (ra^{-1}) \right)^2 = a^2 (1 - a^2 r^{-2} a^{-2}).
\]
We have made use of (7) in writing (10) which will now be used for the determination of the conformal function \(a(r, t)\). It may be remarked here that eq.(10) is nothing but the expression for density in conformally flat perfect fluid solutions, which in the present case is a constant (see Krishna Rao\(^2\)).

Treating \((r/a)\) as the dependent variable and \(r, t\) as independent variables we can solve eq.(10) using Charpit's method and the solution is given by
\[
(2a/\sigma) = P + P^{-1}
\]
where
\[
P = \frac{1}{r} \sqrt{1 + r^2 - 1} \, e^{\sqrt{1 + r^2} \sigma} \pm t.
\]
(12)
It can also be verified that the velocity of the fluid particles in the conformally flat system is given by
\[
\frac{dr}{dt} \equiv v = \mp \frac{r}{(1 + r^2)^{1/2}}.
\]
(13)
The surfaces \(r = \) constant are now given by \(P = \) constant. In the \((r, t)\)-plane, \(P = \) constant are hyperbolas since \(1/r \sqrt{1 + r^2 - 1}\) is very nearly equal to unity. Thus the conformal co-ordinate system bears a strong resemblance
to the Kruskal co-ordinate system of the exterior Schwarzschild space-time.

Department of Mathematics, J. Krishna Rao.
South Gujarat University,
Surat-1.

and

Department of Mathematics, R. B. Patel.
Sardar Patel University,
Vallabh Vidyanagar,


ELECTRONIC ABSORPTION SPECTRUM
OF 2,3-DIFLUOROANILINE IN
VAPOUR PHASE

The ultraviolet absorption spectrum of aniline vapour was first studied by Ginsburg and Matsen\(^1\). High resolution vapour absorption spectra of aniline and its isotopic derivatives have been studied by Brand et al.\(^2\). As regards earlier investigations on derivatives of aniline Singh and Singh\(^3\), Sashidhar and Rao\(^4\) and Murty and Samanthama\(^5\) have studied the ultraviolet spectra of isomeric fluoroanilines. High resolution studies on para-fluoroaniline have also been made by Tiwari\(^6\) and Thakur\(^7\). Singh and Singh\(^8\) have also studied the ultraviolet spectra of 2,5-difluoro- and 2,5-dichloroanilines. The present study was undertaken to record and study the ultraviolet absorption spectrum of 2,3-difluoroaniline in vapour phase which has not been done so far primarily due to the difficulty in synthesizing the substance. The compound has been recently synthesized by Dr. A. M. Roe\(^8\).

The near ultraviolet absorption spectrum of 2,3-difluoroaniline was recorded with cells of length 25 and 50 cm and the vapour pressure of substance was varied by keeping the bulb containing the liquid at different temperatures from 0\(^\circ\) C to 25\(^\circ\) C. The cells were made of pyrex glass tubes of 2.5 cm diameter. Plane quartz discs were cemented to the two ends of the absorption cell. A small bulb attached in the middle of the cell served as container for the substance. A side tube with a constriction made at a suitable place was attached to the absorption cell for evacuation and subsequent sealing off. A 1500 W high pressure Xenon Arc Lamp was used as the source of continuous radiation. The light from the lamp was rendered approximately parallel by means of a quartz lens so that after passing through the absorption cell it converged at the slit of the spectrograph. The spectra were photographed on a Zeiss Q-24 Medium Quartz and a two Prism Quartz Steinhell Spectrograph with a slit width of 20\(\mu\). Exposure times ranging from \(\frac{1}{4}\) to 1 hour were needed to record the spectra on Iford N-40 plates with sufficient intensity. Iron arc lines were superposed centrally to provide standard lines for measurement.

The absorption bands of 2,3-difluoroaniline are best developed in a cell of 50 cm with the sample bulb kept at 10\(^\circ\) C. On raising the temperature of the sample bulb and consequently the vapour pressure in the absorption cell, the bands on the shorter wavelength end become diffuse and finally a continuous absorption sets in. The absorption spectrum lies in the region 3000-2700 Å. Over 100 bands have been measured some of which are sharp while others are broad and diffuse. The conspicuous feature of the spectrum is the appearance of the bands as close doublets of average separation 10 cm\(^{-1}\). About fifty doublets could be easily seen. The average separation between all the doublets, however, is not constant.

The molecule 2,3-difluoroaniline belongs to \(C_s\) point group. The electronic transition corresponding to 2600 Å system of benzene is \(B_{2u}^0 \rightarrow A_1^0\). When \(D_{4h}\) symmetry is reduced to \(C_s\) on substitution, both the species \(A_1\) and \(B_{2u}\) of the point group \(D_{4h}\) go over to \(A\). Thus the transition involved in the present case is \(A' \rightarrow A\) and is an allowed one with the transition moment lying in the molecular plane.

In the present case, on the longer wavelength side of the absorption spectrum a band has been observed at 34447 cm\(^{-1}\) which appears with strong intensity even at the smallest cell length and lowest vapour pressure. This is the most intense band in the entire absorption spectrum. We have taken this as the (0–0) band of the system. This is the first component of the doublet. This choice is in accordance with the group theory and Franck-Condon principle. This band is shifted by 415 cm\(^{-1}\) to the violet side compared to the (0–0) band of aniline at 34032 cm\(^{-1}\).

The correspondence between the excited state frequencies observed in the absorption