

GRAVITATIONAL FIELD OF A TACHYON

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ABSTRACT

A static axially symmetric exact solution of Einstein's equation $R_{ik} = 0$ is interpreted as describing the gravitational field of a tachyon. It is found that under gravitation a tachyon will repel a material particle.

INTRODUCTION

TACHYON is the name given to a 'particle' which moves faster than light. It is known that the existence of free Tachyons is consistent with the consequences of the basic principles of relativity¹, though the problem of causality in relation to Tachyons is still an open question^{2,3}. However, it would be interesting to see how one can describe the gravitational effect produced by such 'Faster than Light particles'.

Mathematically one expects the gravitational field produced by a Tachyon to have all the geometrical characteristics of Schwarzschild's field for a point particle; i.e., it will be represented by a static solution of Einstein's equations $R_{ik} = 0$ containing a single parameter m and admitting a shear-free, hypersurface-orthogonal null congruence. However, since it will not be possible to get an observer who would observe the Tachyon at rest, the field will have to be axially symmetric (and not spherically symmetric) and must contain an additional characteristic constant a such that $|a| > 1$, the velocity of light being put equal to unity. Further, it should not be possible to transform away this parameter a by a co-ordinate transformation in the case when $|a| > 1$ and in the case $|a| < 1$, it should be possible to choose co-ordinates in such a way that the solution reduces to the spherically symmetric Schwarzschild's solution for a material particle.

THE SOLUTION

The desired gravitational field is described by the line-element

$$ds^2 = - \left(1 - a^2 + \frac{m}{\rho}\right)^{-1} d\rho^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) + \left(1 - a^2 + \frac{m}{\rho}\right) dt^2, \quad (2.1)$$

$$r^2 = \rho^2 (1 - a \cos \theta)^{-2}. \quad (2.2)$$

This metric is static, axially symmetric and satisfies Einstein's equations $R_{ik} = 0$. Again it admits a shear-free geodesic null congruence which is everywhere orthogonal to the hypersurface

$$u \equiv t + \int \left(1 - a^2 + \frac{m}{\rho}\right) d\rho = \text{constant}. \quad (2.3)$$

The function r can now be interpreted as the radial co-ordinate and (r, θ, ϕ) as the usual spherical polar co-ordinates in the 3-space Π defined by $u - r = \text{constant}$. Thus this axially symmetric solution possesses all the geometrical characteristics of the Schwarzschild's solution for a material particle.

Again if $|a| < 1$, one can transform (2.1) to the form

$$ds^2 = (1 - 2\sqrt{(1-a^2)} M/\rho) du'^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) - 2(1-a^2)^{-1/2} du' d\rho,$$

where

$$M = -\frac{m}{2} (1-a^2)^{-3/2}, \quad u' = u \sqrt{(1-a^2)}.$$

This, in its turn, can be transformed to Schwarzschild's solution

$$ds^2 = \left(1 - \frac{2M}{r'}\right) du'^2 - r'^2 (d\theta'^2 + \sin^2 \theta' d\phi'^2) - 2du' dr'$$

by the substitution

$$\cos \theta' = \frac{\cos \theta - a}{1 - a \cos \theta}, \quad (2.4)$$

$$r = \frac{r'(1 - a \cos \theta')}{\sqrt{(1-a^2)}}. \quad (2.5)$$

Transformations (2.4) and (2.5) show that the origin O' moves relative to O with a uniform velocity a in the flat background along the common axis $\theta = 0 = \theta'$.

Thus the metric (2.1) represents the field of a Schwarzschild-particle moving with a velocity a along the axis $\theta = 0$ when $|a| < 1$. And it has all the geometrical characteristics of Schwarzschild's solution even when $|a| > 1$. Therefore in the case when $|a| > 1$, we take this metric as giving us the gravitational field of a Tachyon.

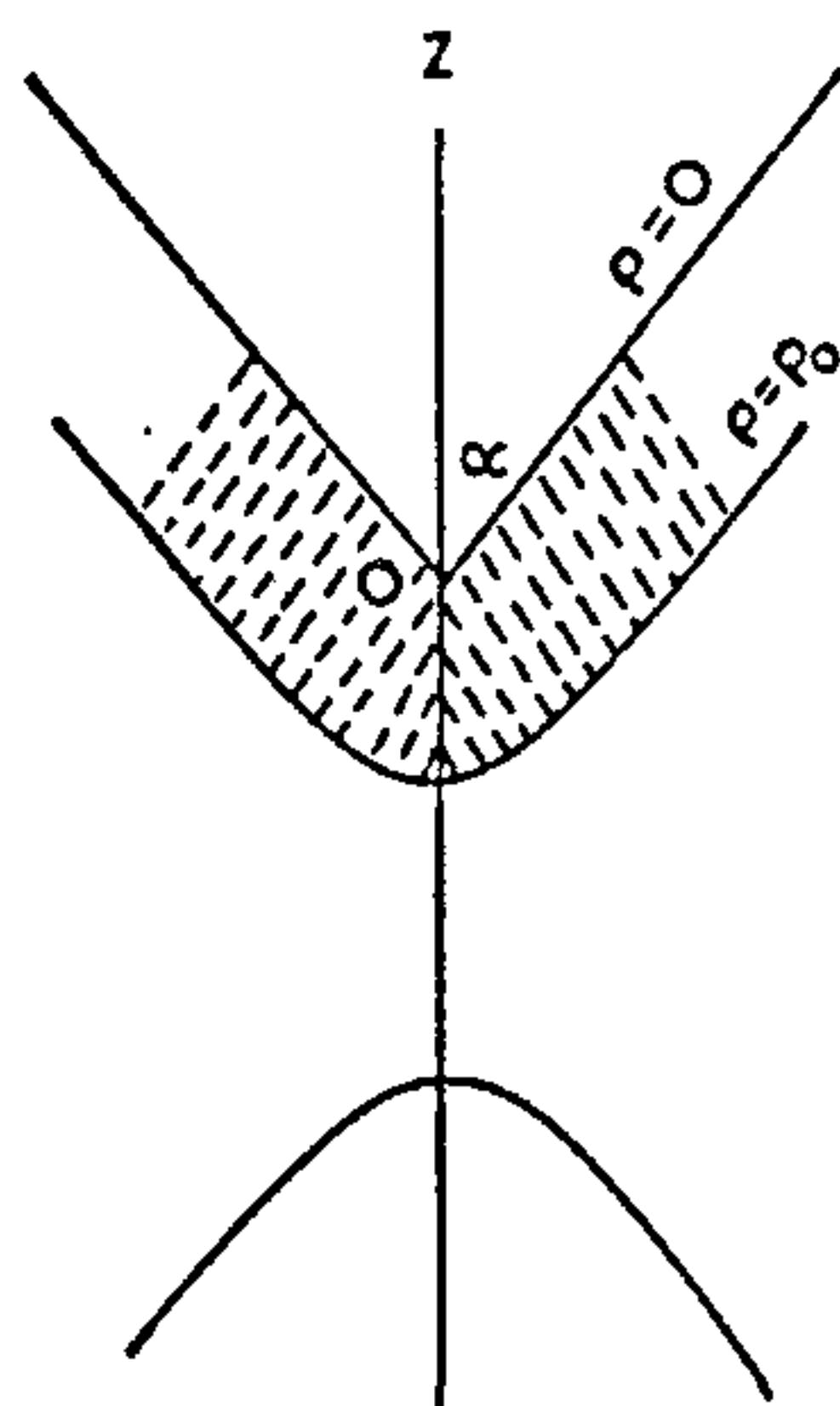
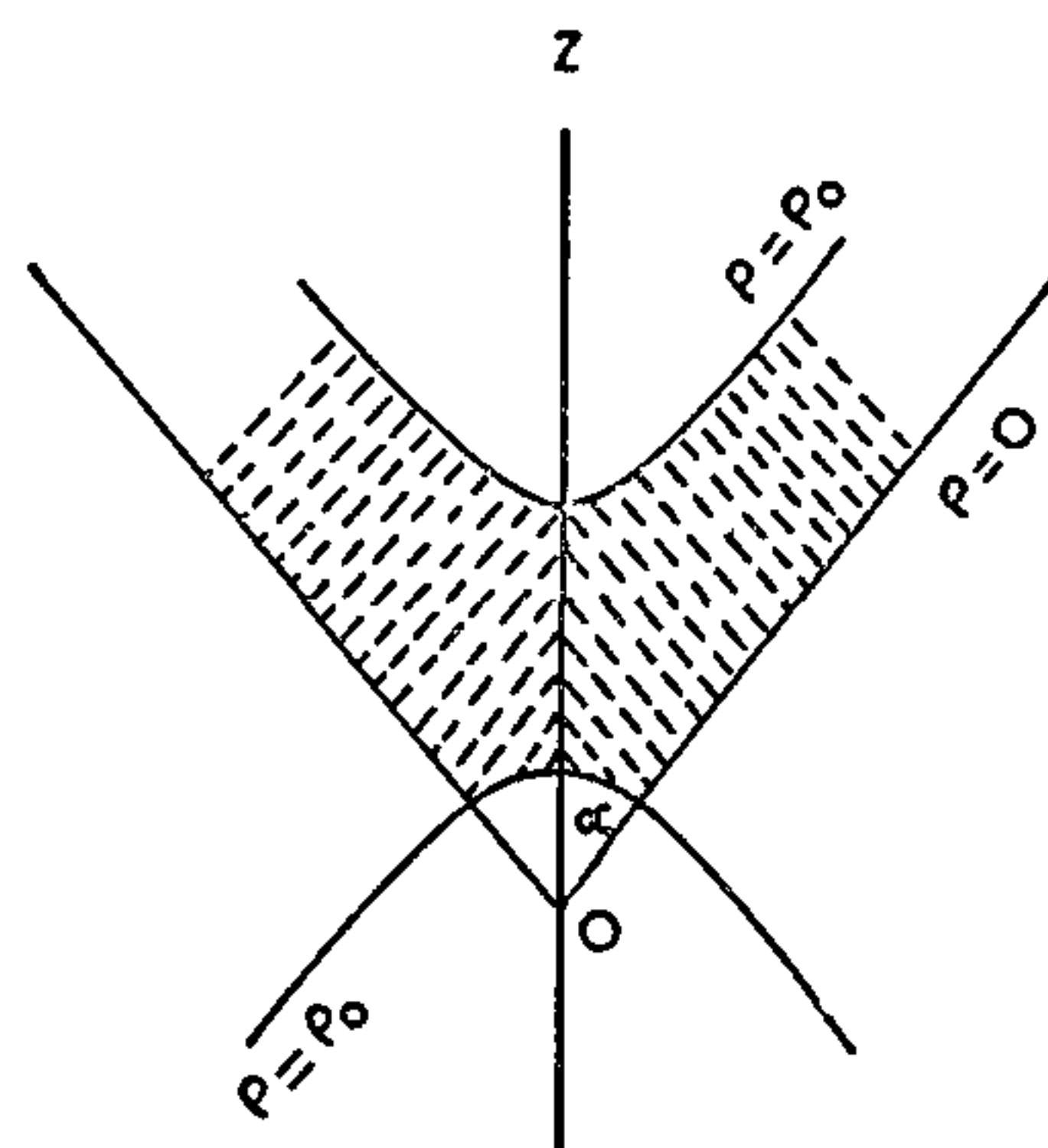
DISCUSSION OF THE GRAVITATIONAL FIELD

The line-element (2.1) has a singularity at $\rho = 0$, i.e., at $r = 0$ and $\sec \theta = a$ when $|a| > 1$, which gives a right circular cone of semivertical angle α ($\sec \alpha = a$) as the singularity surface. Again the surface $\rho = m(a^2 - 1)^{-1} = m \cot^2 \alpha = \rho_0$ is a null surface which acts as the 'Schwarzschild's surface' for this gravitational field. This surface is a hyperboloid of revolution and of two sheets in the 3-space Π , its equation being

$$\tan^2 \alpha (\xi + \rho_0 \operatorname{cosec} \alpha \cot \alpha)^2 = R^2 = \rho_0^2 \cot^2 \alpha$$

with $Z = r \cos \theta$, $R = r \sin \theta$.

The metric (2.1) will define a static gravitational field within that region of the 3-space II in which $1 - a^2 + m/\rho > 0$, i.e., in which $\rho > \rho_0$. This region depends on the sign of the parameter m . Figures 1 and 2 indicate the portion of the (R, ξ) plane (the meridian plane) which generate the regions of validity by rotation round the ξ -axis in the 3-space II, corresponding to the cases $m > 0$ and $m < 0$.

FIG. 1 $m > 0$ FIG. 2 $m < 0$

FIGS. 1-2. The region of validity is obtained by rotating the shaded portion about OZ.

It will be seen from the figures that in both cases the 'black holes' are open and spatial infinity is included in black holes. For the case $m < 0$, the black regions are separated and the singularity $r = 0$ is enclosed in a black hole.

assuming the existence of 'Tachyons' implies assuming the existence of particles which repel material particles under gravity.

1. Camenzind, M., *GRG Journal*, 1970, 1, 41.
2. Glueck, M., *Phys. Rev.*, 1969, 183, 1514.
3. Rolnick, W., *Ibid.*, 1969, 183, 1105.

SURVEY FOR A HIGH METHIONINE VARIETY IN THE WORLD COLLECTION OF CHICKPEA

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RESearch towards quantitative and qualitative improvement of pulse crops is essential because of the vital role that these crops play in providing a reasonably balanced protein component in the diets of primarily cereal eating people. Though, in comparison to cereals, pulses have a higher protein and lysine content, they lack in the sulphur containing essential amino-acids, methionine and cystine, which results in their reduced net protein utilization (NPU).

Indian Agricultural Research Institute maintains the germplasm stock of various pulses in which more than 6,000 cultivated and non-cultivated strains of chickpea are available¹. The same are being rapidly screened for the crude protein content and methionine content. Already, out of some 1,300 strains screened so far, four high methionine genotypes have been isolated. The results of the survey concluded so far are given.