STABILITY OF A HETEROGENEOUS CONDUCTING FLUID WITH THE COMBINED EFFECT OF CORIOLIS AND RADIAL GRAVITATIONAL FORCE

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ONE interesting application of magneto-hydrodynamic stability theory is in the study of oscillations in the atmosphere in the presence of solar magnetic field. In particular, considerable interest is attached to the flow of a fluid with a vertical gradient of density in meteorological problems. Recent space science research investigations show that there is a considerable effect of earth's or the solar magnetic field on a stratified conducting fluid in the atmosphere. Still more important, there is the coriolis force to be considered. The stability of such fluids, is, therefore, of importance in meteorological problems. Recently, Rudraiah (1970) has investigated the stability of heterogeneous incompressible, non-viscous perfectly conducting fluid between fixed coaxial cylinders with an applied magnetic field in the azimuthal direction and a radial gravitational force. The present purpose of this article is to examine the combined effect of coriolis and radial gravitational forces with magnetic field on the stability of such a system.

We consider the flow of a heterogeneous incompressible inviscid and perfectly conducting fluid between two co-axial cylinders of radii a and b, and with the flow and the magnetic field in the azimuthal direction. The stability equation, against axisymmetric disturbances of the type exp i $(\alpha z - \beta t)$ (where β is a complex quantity and α is a real positive number) and linearizing the basic equations is

$$D\left(\frac{\rho_0}{r} D r u_r\right) + \frac{1}{C^2} \chi(r)\, u_r - \alpha^2 \rho_0 u_r = 0, \quad (1)$$

where

ρ_0 is the density, u_r is the radial velocity

$$C = \frac{\beta}{\alpha}, \quad D = \frac{d}{dr}$$

$$\chi(r) = \left\{\frac{1}{r^3} D(\rho_0 V_0^2 r^2) - S r D\left(\frac{H_0}{r}\right)^2 - \left(\frac{V_0^2}{r} + G\right) D\rho_0 - 2 T r C D\left(\frac{\rho_0}{r}\right)\right\}$$

$$S = \frac{\mu H_0^2}{\rho_0 U^2}, \text{ cowling number}$$

$$G = \frac{g}{\rho U^2}.$$

$$T = \frac{\Omega}{\alpha U},$$

Ω angular velocity

$$C = C_r + i C_i,$$

The boundary conditions are

$$u_r = 0 \quad \text{at} \quad r = a, \quad r = b \quad (2)$$

From the general definition of stability theory, we know that if C is real, the motion is stable and if C is complex, the motion is unstable. We find the nature of C from the stability equation (1) by using the boundary conditions (2). For this, we multiply the equation (1) by $r\bar{u}_r$ where $\bar{u}_r$ is the complex conjugate of u_r, integrating between a and b and using the boundary condition (2) and equating the imaginary part of the resulting equation to zero, we obtain

$$\frac{2C_i}{(C_r^2 + C_i^2)} \int_a^b \{K_1 + (C_r^2 + C_i^2) K_2\}\, r\,|u_r|^2\, dr = 0, \quad (3)$$

where

$$K_1 = \frac{1}{r^3} D(\rho_0 V_0^2 r^2) - S r D\left(\frac{H_0}{r}\right)^2 - \left(\frac{V_0^2}{r} + G\right) D\rho_0$$

$$K_2 = T\rho_0\left(\beta + \frac{1}{r}\right),$$

$$\beta = -\frac{1}{\rho_0} D\rho_0.$$

Hence, if k_2 does not change its sign in (a, b) and if

$$T > \frac{K_1}{\rho_0\left(\beta + \frac{1}{r}\right)}, \quad (4)$$

then the motion is stable. We notice that the factor $\rho_0(\beta + 1/r)$ does not change its sign in (4), thus whether the motion is stable or unstable will depend on the sign of K_1.

(*i*) If K_1 does not change the sign, the integrand will never be zero, and hence $C = 0$. This means that the motion is stable.

(*ii*) Even if K_1 changes its sign, but if the magnitude of the coriolis force is such that (4) is satisfied, the motion is stable.

If, the coriolis force, magnetic field and gravitational forces are absent, then as can be seen from (3), the motion will be stable if $\rho_0/r^3 D(V_0^2 r^2)$ does not change its sign in (a, b).

Thus we can conclude that the combined effect of magnetic field and coriolis force is to add to the stability of the system.

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1. Rudraiah, N., *Publ. Astro. Soc., Japan*, 1970, **22**, 41.