markable result of mathematical interest is his analysis of the nature of motion when n, the number of discontinuities, is a prime > 1, or is composite. A careful examination of the manner in which this analysis is accomplished shows that he has not used any sophisticated results of prime number theory, but just the definition of a prime number!

Another investigation relating to the vibrations of circular membranes is also of great interest. It is well known that percussion instruments, generally speaking, give rise to inharmonic overtones, and are thus musically imperfect. The Indian Mridanga forms a remarkable exception to this rule as found by Raman experimentally. The character of vibrations of the associated heterogeneous membrane which gives rise to these significant properties was investigated by sand figures, and definite quantitative results obtained. The problem of oscillations of heterogeneous membranes is a complicated one, and it is difficult to solve it completely with all the resources of advanced mathematical physics. It is again a general Sturm-Liouville eigenvalue problem, whose solution demands an extensive knowledge of integral equations theory (Hilbert, Ibid., pp. 255, and 10, p. 273). It is very interesting to note that the result of physical interest mentioned at the end of paragraph 2, p. 279 of the above reference is inherent in Raman's experimental results.

Lastly, a result of rather academic interest is Raman's attempt at a classical derivation of the Compton effect (of which the Raman Effect is the optical analogue) by using the theory of vibrations (*Ind. J. Phys.*, 3, p. 357) based on the result that even the most arbitrary type of wave disturbance can be represented as the superposition of plane trains of waves travelling in all directions in space.

In conclusion, I might mention that Raman always evinced a keen interest in topics of mathematics, even of the purest type. I remember, when I was a student of the M.Sc. Pure Mathematics course of the Calcutta University during 1919-21, and he was the Palit Professor of Physics, his finding time to question me (a young student of about 20 years then) as to what I was learning, and listen to my talking about topics like Algebra of Quantics, Projective and Differential Geometry. He retained this interest in Pure Mathematics, which he often likened to an art like painting or music, throughout his life. Such great men of wide interests, and broad vision are rare indeed.

ELASTIC CONSTANTS AND SUB-HARMONICITY

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1. Introduction

ONE of the subjects which interested C. V. Raman was that of internal couple-stresses produced in crystals subjected to severe deformation. This required the introduction of additional elastic constants. He applied it to the determination of the elastic constants of cubic crystals at the macro-level.

Internal moments are essentially a micro-phenomenon and cannot be of much use in macro-measurements of elastic constants. Recently, unsuccessful attempts have been made to use this concept to explain fatigue. It is also being used in turbulence where it may bear some fruit, as any further work in this subject must now be at the macro-level.

As regards rotation effects, these can be produced by the second order terms in the strain tensor itself, which are neglected in the classical treatment. Actually, the elastic fields are sub- or super-harmonic in character, and, contain rotation or spin effects as particular cases. The classical bi-harmonic or harmonic fields are only degenerate forms, and hence do not exhibit these effects. If this is taken into consideration, the assumption of additional elastic constants for crystals, isotropic at the macro-level, need not be made. We shall now prove the sub- or super-harmonicity of the elastic fields.

2. Equations of Equilibrium

Let the undeformed and the deformed coordinates of the position of a particle be X^r and x^r, $(r = 1, 2, 3)$ respectively, so that the deformation is given by

$$u^r = x^r - X^r, \quad x^r = f^r(X^1, X^2, X^3)$$
$$(x^1, x^2, x^3) = (x, y, z). \tag{2.1}$$

Assuming for simplicity that both systems are rectangular cartesian the Almansi strain tensor is:

$$2e_{ij} = \delta_{ij} - X^a{}_{,i} X_{a,j} = u_{i,j} + u_{j,i} - u^a{}_{,i} u_{a,j}. \tag{2.2}$$

A linear stress-strain tensor law is quite adequate for our purpose, so that

$$\begin{aligned} T_{ij} &= \lambda\delta_{ij}e_{aa} + 2\mu e_{ij} \\ &= \lambda\delta_{ij}e_{aa} + \mu(\delta_{ij} - X^a{}_{,i} X_{a,j}), \end{aligned} \tag{2.3}$$

where

$$2e_{aa} = 3 - X^a{}_{,i} X_{a,i} \tag{2.4}$$

and the comma denotes differentiation with respect to i or j. It should be noted that $e_{\sigma a}$ is the first strain invariant of the Almansi tensor.

Substituting the value of T_{ij} from (2.3) in $J_{ij,j} = 0$ and simplifying, we get

$$e_{aa,i} = \tfrac{1}{2} c J_{ij}(X^r, X^r{}_{,j}). \tag{2.5}$$

In (2.5) J_{ij} is the Jacobian of X^r and $X^r{}_{,j}$ with respect to x^i and x^j and

$$c = \frac{2\mu}{\lambda + 2\mu} = \frac{1 - 2\sigma}{1 - \sigma}. \tag{2.6}$$

A typical form of (2.5) is

$$\begin{aligned} \frac{\partial e_{aa}}{\partial x} = \frac{1}{2} c\,[&J_{xy}(X^1, X^1, y) + J_{xz}(X^1, X^1, z) \\ &+ J_{xy}(X^2, X^2, y) + J_{xz}(X^2, X^2, z) \\ &+ J_{xy}(X^3, X^3, y) + J_{xz}(X^3, X^3, z)]. \end{aligned} \tag{2.7}$$

Eliminating e_{aa} between the three equations taken in pairs, we get

$$J_{ij}(X^r, \nabla^2 X^r) = 0. \tag{2.8}$$

It should be noted that

$$\nabla^2 \equiv \frac{\partial}{\partial x^i}\frac{\partial}{\partial x^i}, \quad \nabla_0{}^2 \equiv \frac{\partial}{\partial X^i}\frac{\partial}{\partial X^i}.$$

A typical form of (2.8) is

$$J_{xy}(X^1, \nabla^2 X^1) + J_{xy}(X^2, \nabla^2 X^2) + J_{xy}(X^3, \nabla^2 X^3) = 0. \tag{2.9}$$

The complete solution of the equation (2.8) is

$$\nabla^2 X^r = \nabla_0 T(X^1, X^2, X^3), \tag{2.10}$$

where

$$\nabla_0 = i\frac{\partial}{\partial X^1} + j\frac{\partial}{\partial X^2} + k\frac{\partial}{\partial X^2},$$

$$J_{ij}(X^i, X^j) \neq 0, \quad i, j = 1, 2, 3$$

and T is an arbitrary function of X^r.
From (2.10) we see that X^r are sub- or superharmonic functions.

MY PROFESSOR

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IT was just a happy and unforgettable event in my life that I met my Professor by sheer accident who had just joined the Indian Institute of Science as its Director and Professor of Physics, and I had gone to the Institute to secure admission in the Electrical Technology Department. When I was leaving the Department after securing admission, I said to myself, why not meet the Nobel Laureate on the plea of becoming his research scholar! I had no hope that I would get admission in the Physics Department as a research scholar as I had graduated in mathematics. After a thorough interview lasting till the evening, he stood up and patted me and said that I could join the Department as his research scholar. I then told him my plight that I had already secured admission in the E.T. Department. He appeared to feel annoyed but then he said that I had to give up the admission in the E.T. Department. I said that I would very gladly do so and join his Department. I was overwhelmed by his kindness and generosity, and academic outlook which appeared to me like a mountain peak and I had chosen my *guru* from that moment.

The new Department of Physics had to be organised. The work that was entrusted to me was to prepare a list of books in Physics and Mathematics for the library. I prepared a long list which contained quite a number of books in the purest of Pure Mathematics which could only be procured from Köhlers Antiquarium in Germany. I had thought that the Professor would make his own selection from my list, but he simply dittoed the list and said that I had to prepare additional lists from time to time.

One day I picked up enough courage to tell him that a teacher of mine who was exceptionally brilliant in Pure Mathematics was at that moment without a job. He said: "Do you really think so?" "Yes," I replied. "All right, ask him to meet me," he said. The teacher met him and he was taken as a research scholar. Professor observed no water-tight com-