LETTERS TO THE EDITOR

A SUSPECTED CONVERSE OF A THEOREM REGARDING SPHERICALLY SYMMETRIC SPACE-TIMES

PRASANNA[1] has shown that, for the spherically symmetric metric,

$$ds^2 = -A\,dr^2 - B\,d\Omega^2 + C\,dt^2, \tag{1}$$

where

$$d\Omega^2 = d\theta^2 + \sin^2\theta\,d\phi^2$$

and A, B, C are functions of r and t, the quantity

$$\frac{(f_1 f_4 - f_5^2)}{f_2 f_3} = I \quad (f_2 \neq 0,\ f_3 \neq 0) \tag{2}$$

is an invariant under arbitrary non-singular Gaussian transformations preserving spherical symmetry.

The following problem has arisen from the same note and the notations of the same note are followed.

It is well-known that a necessary and sufficient condition for the metric (1) to be of class one is that

$$I = 1, \quad i.e.,\ f_1 f_4 - f_2 f_3 = f_5^2. \tag{3}$$

If (1) describes a perfect fluid distribution, the components of the curvature tensor R_{hij} satisfy the condition

$$\left(\frac{f_1}{A} + \frac{f_4}{C}\right)^2 - \left(\frac{f_2 B}{AC} + \frac{f_3}{B}\right)^2 = \frac{4 f_5^2}{AC}. \tag{4}$$

If (1) is of class one and it also describes a perfect fluid distribution, (3) and (4) imply

either,

$$\frac{f_1}{A} - \frac{f_4}{C} = -\frac{f_2 B}{AC} + \frac{f_3}{B}, \tag{5}$$

or

$$\frac{f_1}{A} - \frac{f_4}{C} = \frac{f_2 B}{AC} - \frac{f_3}{B}. \tag{6}$$

The condition (5) is necessary and sufficient for the metric (1) to be conformally flat. Condition (6) has to be interpreted.

A spherically symmetric, conformally flat metric, describing a perfect fluid distribution, is necessarily of class one. This can be verified by imposing (5) on (4).

I understand from Prof. V. V. Narlikar that Dr. P. C. Vaidya had raised the question whether the converse of the above result holds good: that is, is a spherically symmetric metric of class one, giving a perfect fluid distribution necessarily conformally flat?

We give here a metric which shows that the converse does not hold good. The metric is,

$$ds^2 = -\frac{2r^2 + P}{r^2 + P}\,dr^2 - r^2\,d\Omega^2 + \frac{r^2 + P}{4K}\,dt^2, \tag{7}$$

where P and K are arbitrary positive constants.

For this metric the surviving components of the curvature tensor are,

$$R_{1212} \equiv R_{1313}/\sin^2\theta \equiv f_1 = -\frac{r^2 P}{(2r^2 + P)(r^2 + P)},$$

$$R_{1414} \equiv f_2 = -\frac{P}{4K(2r^2 + P)},$$

$$R_{2323}/\sin^2\theta \equiv f_3 = -\frac{r^4}{(2r^2 + P)},$$

$$R_{2424} \equiv R_{3434}/\sin^2\theta \equiv f_4 = -\frac{r^2}{4K}\frac{(r^2 + P)}{(2r^2 + P)},$$

$$R_{1224} \equiv R_{1334}/\sin^2\theta \equiv f_5 = 0. \tag{8}$$

The expressions for the pressure and density are,

$$8\pi p = \frac{1}{(2r^2 + P)},$$

$$8\pi\rho = \frac{2r^2 + 3P}{(2r^2 + P)^2}. \tag{9}$$

It can be verified that (7) satisfies (3) and (4). It does not satisfy (5) but it satisfies (6).

Hence it can be safely said that, a spherically symmeric metric of class one, describing a perfect fluid distribution, is either conformally flat or it satisfies (6), and so the converse does not hold good in general. (7) is a new metric with interesting features. It satisfies the condition (6) which arises as an alternative to (5). Our interest here is mainly in demonstrating that a suspected converse does not hold good.

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investigation and for his valuable guidance in preparing this note. I also wish to thank the U.G.C. for the award of a Research Scholarship which has made this work possible.

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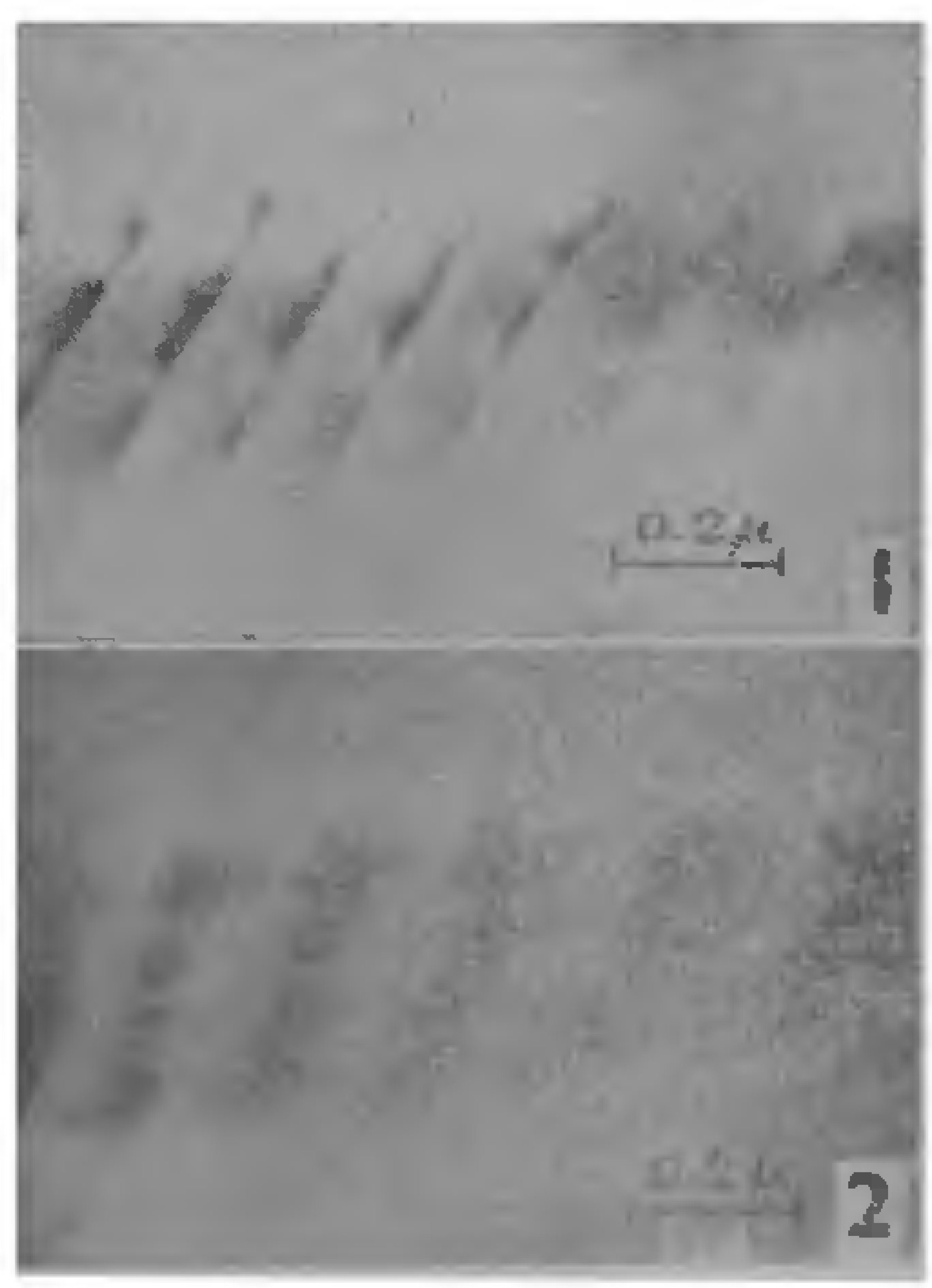
1. Prasanna, A. R., *Curr. Sci.*, 1969, 38, 455.

ON THE FORMATION OF DISLOCATION LOOPS IN GADOLINIUM

Electron-microscope observations of the formation of dislocation loops in basal oriented thin single crystal film of h.c.p. metal gadolinium have been made. Evidence and arguments are advanced to show that the loops are formed as a result of the interaction between vacancies and dislocations.

Basal oriented Gd films with thickness of about 200 Å were prepared by a method described previously.[1] Many basal grains contained a family of parallel dislocations which originated during deformation. These dislocations by their appearance seemed to be extending from the top surface to the bottom surface of the (0001) oriented thin film. Diffraction contrast experiments relevant to h.c.p. crystals[2] showed that the dislocations were of nearly screw-character with Burgers Vector, $\vec{b} = \vec{c} + \vec{a}$, and were lying on $(11\bar{2}2)$ pyramidal planes. These dislocations, when subjected to several successive temperature gradient anneals employing electron-beam pulse annealing in the microscope, decomposed into rows of dislocation loops. A typical transmission electron micrograph exhibiting $\vec{c} + \vec{a}$ dislocations and the corresponding rows of dislocation loops is shown in Figs. 1 and 2.

Screw dislocations are known to decompose to helical dislocations and then to loops by interaction[3,4] with vacancies through climb-process. It is thought that the generation of dislocation loops in the present case occurs as a result of the formation and interaction of helixes. The inclined $\vec{c} + \vec{a}$ dislocations are intersecting the (0001) foil at opposite ends and will thus be subjected to strong image-forces.[5] The image-forces will pin up the dislocation at opposite ends where these intersect the film surfaces, since the dislocations are lying on $(11\bar{2}2)$ planes and thus cannot become oriented perpendicular to (0001) foil plane. The film is subjected to temperature gradient anneal employing pulse annealing technique, there will thus be a vacancy current,[1] the dislocations will interact with these vacancies and produce helical dislocations in the usual way. Evidence for the presence of helical dislocations was obtained during intermediate stages of pulse anneal and this shows that helix formation precedes the loop formation. It has been conjectured that the temperature gradient-induced vacancy current will be very suitable for climb in opposite senses of the nearby nearly screw dislocations.[3] No experimental observations have been however reported regarding this. In the present case, since the vacancy current is temperature gradient-induced, the vacancies will migrate away from one place and towards a nearby place in the crystal foil. This flow will induce climb in opposite senses in the two nearby $\vec{c} + \vec{a}$ dislocations, leading to formation of two helixes of opposite senses. These helixes will then interact and produce row of dislocation loops.[3,4] It is evident that the equilibrium distances between rows of loops will not be the same as that between initial dislocations, this feature can be easily noticed in Figs. 1 and 2.



Figs. 1–2

For temperature gradient-induced vacancy flow the non-equilibrium concentration c of vacancies in terms of the local equilibrium concentration c_0 can be expressed by the expression[1,6]

$$\ln \frac{c}{c_0} = \frac{Q^* - H}{RT}\left(\frac{T}{T_0} - 1\right)$$