LETTERS TO THE EDITOR

PARTICLES IN EINSTEIN UNIVERSE

In recent years Riemannian manifolds $V_4$ with the respective groups of motion $G_n$, $0 \leq n \leq 10$, have been extensively used in particle Physics.\textsuperscript{1-5} One aspect of this study is to investigate the eigen-value problem of the Laplace-Beltrami operator as generalized Klein-Gordon equation with Riemannian background. In this note we report our investigation of this problem with the use of static spherically symmetric Einstein manifold.

Starting with the line element pertaining to Einstein universe\textsuperscript{8}

$$ds^2 = dt^2 - \left(1 - \frac{r^2}{K^2}\right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2),$$

we investigate the eigen-value problem of the following equation:\textsuperscript{5-7}

$$\left(\sqrt{-g} g^{\mu\nu} \psi, \mu, \nu - M^2 \sqrt{-g} \psi = 0. \right) \tag{2}$$

Here $\psi = \psi (x^\mu)$ is a scalar function, and in the spherically symmetric case it will depend on $r$, i.e., $\psi (r)$. We wish to emphasize that (1) is a solution of the Einstein equations\textsuperscript{9}

$$R_{\mu\nu} = 8\pi T_{\mu\nu}, \tag{3}$$

having the energy momentum tensor $T_{\mu\nu} \propto g_{\mu\nu}$, so that (3) describes a space of constant curvature with the Cosmological constant $\Lambda$ being equal to zero. It is known\textsuperscript{8-9} that the Riemannian manifolds $V_4$ described by (1) may be considered as $S^3 \times \mathbb{R}$ which can be embedded isometrically in $R^5$ as a hyperspherical cylinder, $S^3$ being the hypersphere, $Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2 = K^2$ in $R^4 (Z_1, Z_2, Z_3, Z_4)$. The groups of motion of this $V_4$ are therefore $O_4 \times \mathbb{R}$. With this interpretation we take $K$ as the radius of the particle under consideration. Use of (1) in (2) gives following equation:

$$\frac{d}{dr} \left[ r^2 \left(1 - \frac{r^2}{K^2}\right)^{1 \over 2} \frac{d\psi}{dr} \right] + M^2 r^2 \left(1 - \frac{r^2}{K^2}\right)^{-1 \over 2} \psi = 0. \tag{4}$$

If we put $x = \left(1 - \frac{r^2}{K^2}\right)^{1 \over 2}$, (4) becomes

$$\left(1 - x^2\right) \frac{d^2 \psi}{dx^2} - 3x \frac{d\psi}{dx} + M^2 K^2 \psi = 0. \tag{5}$$

Equation (5) is a differential equation of the following type:\textsuperscript{9}

$$\left(1 - x^2\right) \frac{d^2 \psi}{dx^2} - (2\mu - 1) \frac{d\psi}{dx} + \nu (\nu + 2\mu) \psi = 0, \tag{6}$$

with $\mu = 2$, and

$$\nu = -2 \pm \sqrt{4 + M^4 K^2}. \tag{7}$$

For solutions of interest $\nu$ has to be an integer. This is possible if we take

$$M^2 K^2 = n (n + 4), \tag{8}$$

where $n$ is an integer. The solutions of $\psi$ are then the associated Legendre's function $C_n^{\psi} (x)$ as the coefficients of $h^2$ in the expansion of $(1 - 2xh + h^2)^{-1/4}$. First few of them are given below:

$$C_0 (x) = 1, \quad C_1 (x) = 4x,$$

$$C_2 (x) = (12x^2 - 2), \quad C_3 (x) = (32x^3 - 12x), \tag{9}$$

Equation (8) is an interesting relation between $M$ and $K$. We know\textsuperscript{11} that $M = mc/\hbar$, in the Klein-Gordon equation. With the use of natural units where $C = 1$ and $\hbar = 1$, the mass ($M = m$) has the natural dimensionality $a_1^1 L^{-1}$. It is therefore clear that (8) is a correct relationship between mass and the radius of the particle in natural units.

If we take $n = 0$, (8) gives a particle of zero mass which cannot be very clearly identified. For $n = 1$, and with the use of proton mass $M_p = 938 M_V$, it gives $R_p = 0.47 \text{ fm}$ which is in close agreement with the recently determined radius\textsuperscript{12} $R_p = 0.35 \text{ fm}$ of the interior of the proton sphere. For $n > 1$ the interpretation is not clear. It may possibly be given if we consider Baryon and Meson states as the various resonance states of one particle only. The Quarks\textsuperscript{13} that have been taken as the basic constituents of hadrons, have not been discovered so far, and therefore we have no knowledge of the same in terms of $M$ and $K$.

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ORDER OF REACTION FOR THE THERMAL DEGRADATION OF BUTYL RUBBER IN CHLOROBENZENE

It has been observed\(^1\) that the thermal degradation of butyl rubber in solution proceeds through a random chain-scission mechanism. Singh and Wall\(^2\) have shown theoretically that a first order law is incompatible with such a process. It was reported earlier\(^3\) that the thermal degradation of butyl rubber dissolved in \(p\)-xylene, tetralin, decalin and bromobenzene is independent with respect to the polymer concentration in solution. In the present investigation, the degradation of butyl rubber has been carried out in chlorobenzene in the concentration range 0.5-2.0% (wt./vol.) with a view to determining the order of reaction.

The butyl rubber sample (a copolymer of isobutylene and isoprene) was supplied through the courtesy of Dr. A. B. Biswas, Indian Institute of Technology, Bombay, and was used as such without further purification. Chlorobenzene (MB) was purified by standard techniques. Degradation was carried out at 140°C for different intervals of time as described elsewhere.\(^3\) Studies were carried out at four different polymer concentrations. The process of thermal degradation was followed viscometrically. Measurements were made in a modified Cannon-Ubbelhode dilution viscometer suspended in a water thermostat at 37°C. The values of intrinsic viscosity were evaluated from the experimental data using the triple plot method.\(^5\)

Figure 1 shows the plots of \([\eta]/[\eta]_0\) versus time of degradation, where \([\eta]_0\) is the intrinsic viscosity of the undegraded sample and \([\eta]_t\) is the intrinsic viscosity of the sample degraded for time \(t\). The data clearly indicate that the extent of degradation is independent of polymer concentration in the range investigated. Thus, the thermal degradation of butyl rubber in chlorobenzene follows a zero order law with respect to the polymer concentration in agreement with the theoretical results.\(^2\)

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**Reductive Determination of Selenium**

The reduction of Se (IV) to elemental selenium by Fe(II) was found to be very slow in sulphuric acid medium. However, when sufficient concentration of chloride was present in the medium, the reduction occurred readily. This observation led to the development of the following procedure for accurate determination of selenites: To exactly 5-10 ml of solution containing about 0.25 mM of Se(IV), 20-25 ml of 18 M sulphuric acid are added. After cooling to 25°C, 2-5 ml of 10 M hydrochloric acid are added followed by the addition of exactly 20 ml of 0.1 N ferrous sulphate solution. After 5 minutes, the precipitated selenium is filtered and the filtrate diluted to about 250 ml. The excess of Fe(II) in the filtrate is determined by titration with standard vanadate solution. (Each ml of