LETTERS TO THE EDITOR

AN INTERESTING GEOMETRICAL PROPERTY OF VAIDYA'S ANALYTICAL SOLUTION FOR GRAVITATIONAL COLLAPSE WITH RADIATION

In a recent paper Vaidya[3] has given an analytical solution for gravitational collapse with radiation. The metric may be expressed in spherical polar co-ordinates r, θ, ϕ and time t as

$$ds^2 = -e^A dr^2 - e^B (d\theta^2 + \sin^2\theta\, d\phi^2) + e^C dt^2 \quad (1)$$

where

$$e^A = \frac{S^2(t)\, e^{-2R(r,t)}}{(ar^2 + br + 1)^2}, \quad e^B = \frac{S^2(t)\, r^2\, e^{-2R(r,t)}}{(ar^2 + br + 1)},$$

$$e^C = e^{2R(r,t)}, \quad (2)$$

a, b being arbitrary functions of $u(r,t)$ which is conserved along the world-lines of the flow of radiation. We have

$$u' = f(u)\,(ar^2 + br + 1)^{-1}$$
$$\dot{u} = -f(u)\, e^{2R(r,t)}\, S^{-1}, \quad (3)$$

f being arbitrary. Here and in what follows a prime and overhead dot represent respectively a differentiation with respect to r and t. The relation between u and R is given by

$$2R'(ar^2 + br + 1) = -u'\left[\left(\frac{da}{du}\right) r^2 + \left(\frac{db}{du}\right) r\right]. \quad (4)$$

The purpose of this note is to show that the space-time described by the metric (1) with (2)–(4) is conformally flat.

It was shown by Krishna Rao[1] that the most general spherically symmetric non-empty space-time [given by (1) with A, B, C as functions of r and t only] is of type D in Petrov-Pirani classification. The only non-vanishing physical component of Weyl's conformal tensor is given by

$$\epsilon = (1/12)\,[e^{-A}(2B'' - 2C'' - C'^2 - A'B' + B'C' + C'A') + e^{-C}(2\ddot{A} - 2\ddot{B} + \dot{A}^2 - \dot{A}\dot{B} + \dot{B}\dot{C} - \dot{C}\dot{A})] + (1/3)\, e^{-B}.$$

When $\epsilon = 0$, the space-time is conformally flat. Although it is long and tedious one can now compute in a straightforward manner and show that $\epsilon = 0$ for the metric (1) with (2)-(4) and hence we conclude that the space-time geometry of the radiating star under gravitational collapse is conformally flat. It is worth pointing out that Vaidya's solution is a particular case of a general result of Krishna Rao[2] which may be stated as follows:

Given a perfect fluid solution of Einstein's field equations having a conformally flat metric one can always include the presence of a pure radiation stress-energy tensor $\sigma k_a k_b$, $k_a k^a = 0$, without disturbing the conformally flat nature of the space-time.

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1. Krishna Rao, J., *Curr. Sci.*, 1966, **35**, 589.
2. —, "Propagation of pure radiation in material distributions," To appear in *Prof. V. V. Narlikar's 60th Birthday Commemoration Volume.*
3. Vaidya, P. C., *Ap. J.*, 1966, **144**, 943.

DUAL CORE MODEL FOR He^3

Following the author's previous work[1] an attempt has been made to estimate the ground state energy of He^3 nucleus based on the suggested dual core model. Earlier investigators[2] have made estimates of this ground state energy on three nucleon variational calculations whereas, in the present work, He^3 nucleus is treated as a resonating structure of two deuterons with single neutron forming a deuteron with either of the protons, existing as an entity. The high instability of the free diproton and dineutron nuclei and the nuclear stability of H^3 and He^3 nuclear structures justify the stabilizing role of the third nucleon of the He^3 nucleus. The presence of the diproton as an entity inside the He^3 is also rendered plausible by the experimental evidence of the pronounced p-p final state interaction.[3]

In the present model, the neutron moves essentially in an average field formed by itself and the diproton. The two protons give rise to two distinct symmetrical cores separated by a distance R which is of the order of the He^3 nuclear radius, as required by the saturation property of nuclear binding forces. When a neutron is near one of these two cores it moves in a potential field consisting of the overall potential $-U_0$ and the individual harmonic

potential field arising due to that core. The total Hamiltonian of the system due to these potential fields and also the diproton Coulomb fields has been set up and a variational calculation is used to evaluate the ground state energy.

To arrive at the experimental value of the binding energy, the overall potential U_0 is expressed as $U_0 = 2\,V_0\,\lambda\,e^{-\lambda}$ with $\lambda = (R - b)/b$ and the effective range of nuclear forces $b = 1{\cdot}5$ fm. The variable parameter V_0 thus represents the individual nucleon contribution to the overall potential of He^3 nucleus. To furnish He^3 binding energy of 7·72 Mev, a value of 20 Mev is required for V_0.

However, both ψ'_A and ψ'_S wave functions yield bound states and this difficulty is resolved, when the overall potential is modified as:

$$U_0 = 3\,V_0\,\lambda\,e^{-\lambda}\left[\frac{(1 + P_M)}{2}\right]$$

where P_M is the Majorana exchange force operator between the two protons. The Majorana operator allows only the ψ_S as the stable state with the protons existing as a singlet spin system.

Using the experimental binding energy 7·72 Mev, the values $V_0 = 14{\cdot}4$, $11{\cdot}4$ and $9{\cdot}2$ Mev are obtained respectively with individual harmonic oscillator frequency of the symmetrical cores as $\nu = 0{\cdot}32$, $0{\cdot}24$ and $0{\cdot}17\,fm^{-2}$. A decrease in the frequency ν leads to an increase of the overlap of neutron binding orbitals and hence to a lesser value of the individual nucleon contribution to the overall potential.

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1. Narayana, K. L., *Proc. Nucl. and Solid State Physics Symposium*, Govt. of India, 1968: *Curr. Sci.*, 1969, **38** (11), 261.
2. Jaffe, A. I. and Reiner, A. S., *Physics Letters*, 1969, **26 B**, 719; Brink, D. M. and Peierls R. E., *Comments on Nucl. and Particle Physics* (U.S.A), 1968, **11**, 83; West, G. B., *Phys. Rev.*, 1965, **139 B**, 1246 and references therein.
3. Private discussions with Prof. O. M. P. Bilaniuk, *See also* Bilaniuk, O. M. P. and Slobodrian, R. J., *Physics Letters*, 1963, **7**, 77; Artyomov, K. P. *et al.*, *Ibid.*, 1964, **12**, 53; Conzette, H. E. *et al.*, *Phy. Rev. Letters*, 1964, **13 B**, 625.

FORCE FIELD, MEAN SQUARE AMPLITUDES AND CORIOLIS COUPLING COEFFICIENTS OF DICHLOROFLUOROAMINE

DICHLOROFLUOROAMINE belongs to Cs point group having 6 normal modes of vibration $4a' + 2a''$. Detailed analysis of the infrared spectrum of $NFCl_2$ was done by Hirschmann *et al.*[1] The present investigation deals with the calculation of potential constants, generalised mean square amplitudes and Coriolis coupling coefficients of this molecule. Force constants are obtained by the method of Wilson[2] and generalised mean square amplitudes by that of Morino and Hirota.[3] According to Jahn's Rule,[4] $NFCl_2$ has one active Coriolis coupling $a' \times a''$ corresponding to both ζ^x and ζ^y as per the co-ordinate system followed. The Coriolis coupling coefficients are calculated by the method of Meal and Polo.[5]

Force constants, generalised mean square amplitudes and Coriolis coupling coefficients are given in Tables I, II and III respectively.

TABLE I

Force constants (md/Å) of $NFCl_2$

f_R	3·2140	f_α	0·5678	f_β	0·4353
f_r	2·6770	$f_{R\alpha}$	0·1688	$f_{R\beta}$	−0·1377
f_{rR}	0·2693	$f_{r\alpha}$	−0·0423	$f_{r\beta}$	0·0959
f_{rr}	0·0004	$f_{\alpha\alpha}$	0·0923	$f_{\alpha\beta}$	−0·0220
$f_{R\alpha}$	0·3514				

Where N − F = R = 1·37 Å, N − Cl = r = 1·70 Å, FNCl = α = 102°, ClNCl = β = 106°.

TABLE II

Generalised mean square amplitudes (Å)² at 300° K. of $NFCl_2$

Atom pair	N-F	N-Cl	F...Cl	Cl....Cl
$(\Delta z)^2$	0·002717	0·002883	0·003367	0·006830
$(\Delta y)^2$	0·003792	0·002400	0·004898	0·001658
$(\Delta x)^2$	0·005310	0·003899	0·000211	0·001185
$(\Delta z \Delta y)$	0·000000	0·002580	−0·000326	0·000000
$(\Delta z \Delta x)$	−0·000302	0·001535	−0·000684	0·000000
$(\Delta y \Delta x)$	0·000000	−0·000690	0·000579	−0·000302

TABLE III

Coriolis coupling coefficients of $NFCl_2$

Coupling	ζ^x	ζ^y	Coupling	ζ^x	ζ^y
ζ_{15}	−0·4125	0·6417	ζ_{16}	0·3472	0·4874
ζ_{25}	−0·6777	−0·4942	ζ_{26}	0·1136	0·1288
ζ_{35}	0·3927	0·2343	ζ_{36}	−0·1832	−0·5967
ζ_{45}	0·3246	0·4192	ζ_{46}	−0·0338	0·6289

The generalised parallel mean square amplitude of N-F bond is slightly smaller than that