LETTERS TO THE EDITOR

A NOTE ON ROTATING UNIVERSES

The first rotating universe was invented by K. Gödel in 1949. This universe is filled with a static distribution of perfect fluid with vanishing pressure and non-zero angular velocity, the stream-lines being geodesics. There have been several attempts at generalizing Gödel's universe. Synge has given a general scheme of static distribution of imperfect fluids with geodesic stream-lines and non-vanishing twist, such that when pressure becomes isotropic, the distribution reduces to Gödel's universe. Raval and Vaidya have given a non-static generalization of Gödel's universe on the lines of Synge's generalization. In both the cases, the universe is filled with imperfect fluid, with geodesic stream-lines and non-vanishing twist. If \( v^i \) is the four-vector of velocity satisfying

\[ v^i v_i = 1 \]  

one can define the acceleration vector \( f^i \) and the twist or angular velocity tensor \( w_{ik} \) as

\[ f^i = v^i, v^k \]  
\[ w_{ik} = v_{ik} - v_{ki} \]  

a semicolon denoting covariant differentiation. From (2) and (3) one easily finds

\[ f_i = w_{ik} v^k \]  

Therefore \( w_{ik} = 0 \) implies \( f^i = 0 \). But the converse is not necessarily true. In the generalizations of Gödel's universe mentioned above, \( f_i = 0 \) and \( w_{ik} \neq 0 \). So that this converse is not true for these generalizations. However for perfect fluid \( f_i = 0 \) would always imply \( v^i_{ik} = 0 \). This restricts the possible perfect fluid distribution too severely. This can be seen as follows:

The energy momentum tensor \( T^i_k \) for perfect fluid is given by

\[ T^i_k = (p + \rho) v^i v_k - p g^i_k \]  

The conservation law of general relativity leads to

\[ (T^i_k)_; i = 0 \]  

From (5) and (6) we obtain the "equation of motion" in the form

\[ (p + \rho) f_i = p_\cdot v_i - \frac{dp}{ds} v_i \]  

where

\[ \frac{d}{ds} = v^k \frac{\partial}{\partial x^k} \]  

and comma denotes partial differentiation. With the help of (7) we can say that \( f_i = 0 \) implies

\[ v_i = p_\cdot \left( \frac{dp}{ds} \right)^{-1} \]  

provided \( dp/ds \neq 0 \). Substituting this \( v_i \) in \( v^i; k v_i = 0 \), we can ultimately simplify it to the form,

\[ p_\cdot \left( \frac{dp}{ds} \right)_k - p_\cdot \left( \frac{dp}{ds} \right)_i = 0 \]  

(9) states that \( dp/ds \) is a function of \( p \), say \( \phi(p) \), and thus from (8) we conclude that \( v_i \) is the gradient of a scalar which further implies \( w_{ik} = 0 \). (8) further implies that the vector \( v_i \) is proportional to the gradient \( p_\cdot \), i.e., that the covariant velocity vector is along the normal to the hypersurface \( p = \) constant. Thus a perfect fluid distribution with geodesic stream-lines is a very restricted type of distribution, the two severe restrictions being (1) the stream-lines are normal to equipressure hypersurfaces and (2) the twist tensor \( w_{ik} = 0 \). Thus in case of perfect fluids if \( f_i = 0 \), it follows that \( w_{ik} = 0 \) provided \( dp/ds \neq 0 \). Therefore, if we wish to have perfect fluid distribution with geodesic stream-lines and non-zero twist, we must consider \( p = 0 \) (Gödel). On the other hand, if we wish to retain geodesic stream-lines with non-zero-twist and non-constant pressure we must forego the requirement of perfect fluid. This is what has been done in reference (2). Now in the early stage of the evolution of the universe it is expected that the radiation will dominate over matter and therefore the fluid pressure cannot be taken as zero. In that event in order to generalize Gödel's static model to the non-static case the result which we have proved in this note requires that for a model of a rotating universe filled with perfect fluid (and not dust), we must forego the requirement of geodesic stream-lines.

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