MODULATED BIREFRINGENCE *

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THE subject of my paper is closely connected with that in the papers presented earlier by the scientists from the Soviet Union and from Italy. But they were concerned mainly with circular birefringence and circular dichroism shown by a medium which has acquired a net magnetisation due to spin-orientation. Whereas, I shall be dealing also with ordinary linear birefringence which can occur even when there is no net magnetisation—provided there is an alignment of spins parallel or antiparallel to some axis.

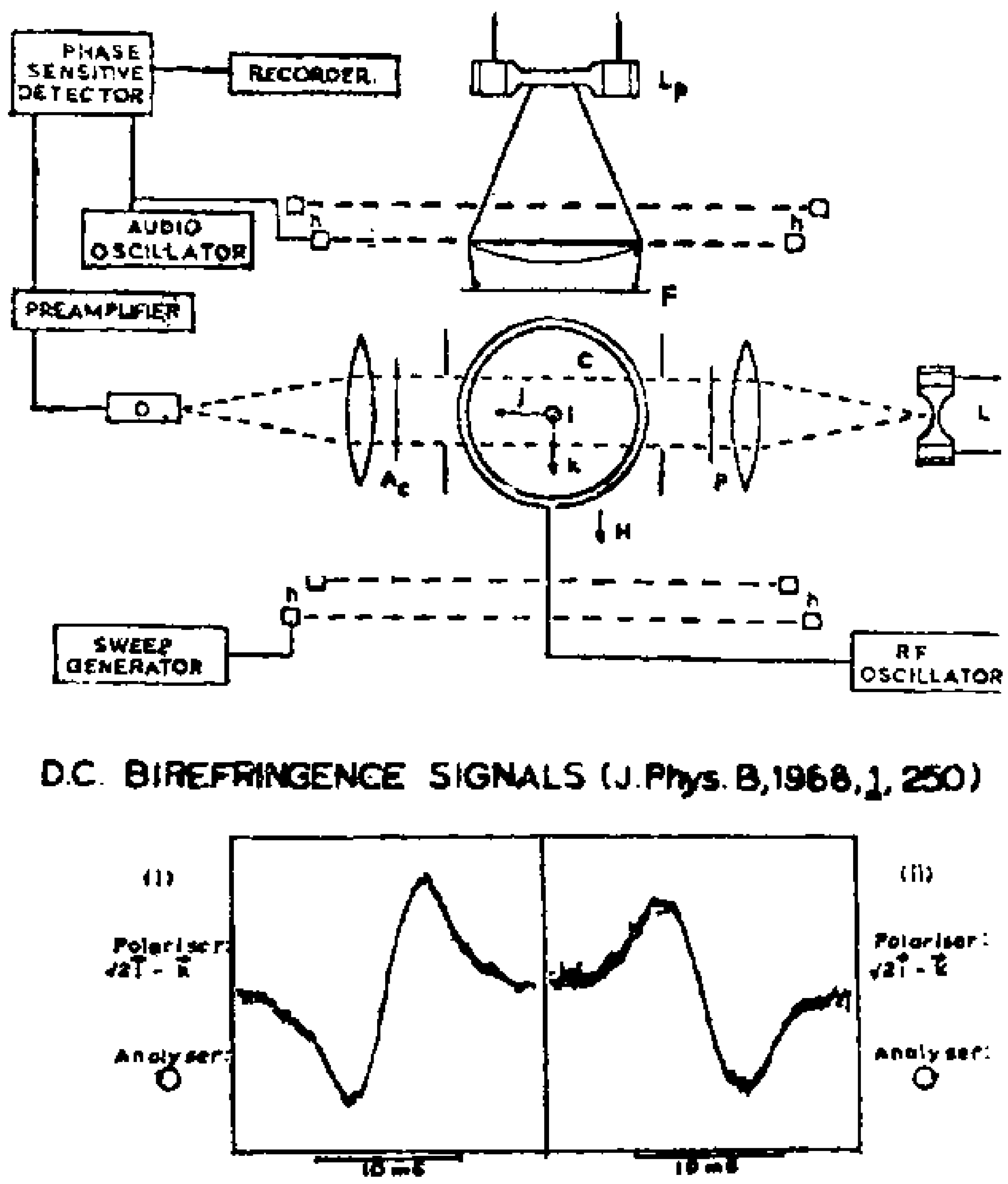


FIG. 1

Figure 1 gives an idea of the apparatus for observing *d.c.* or time-averaged birefringence signals. The sample here consists of He^4 atoms in the metastable state 2^3S_1. It is optically pumped with unpolarised 1μ resonance radiation so that the spins are *aligned* rather than oriented in line with the *k*-axis. The alignment makes the medium optically uniaxial. And this is detected in a transverse beam, linearly polarised. A monitoring lamp providing off-resonant radiation and a circular analyser are also essential. These traces show in effect a plot of the derivative of the intensity received by the photodetector D as a function of a magnetic field applied along *k*. Near magnetic resonance, transitions between Zeeman levels are induced because of the radio-frequency field; this disaligns the spin-assembly, reduces the birefringence and gives rise to these signals.

For other important features of the *d.c.* birefringence I can only refer you to the original paper. In that paper I had commented only briefly on the fact that the radio-frequency field does not only produce transitions. The expression for the polarisability, derived by Series, on which this work is based, shows oscillatory terms, reflecting the coherence between Zeeman levels. Thus in addition to *d.c.* birefringence, we should also have *modulated birefringence* about which I shall speak.

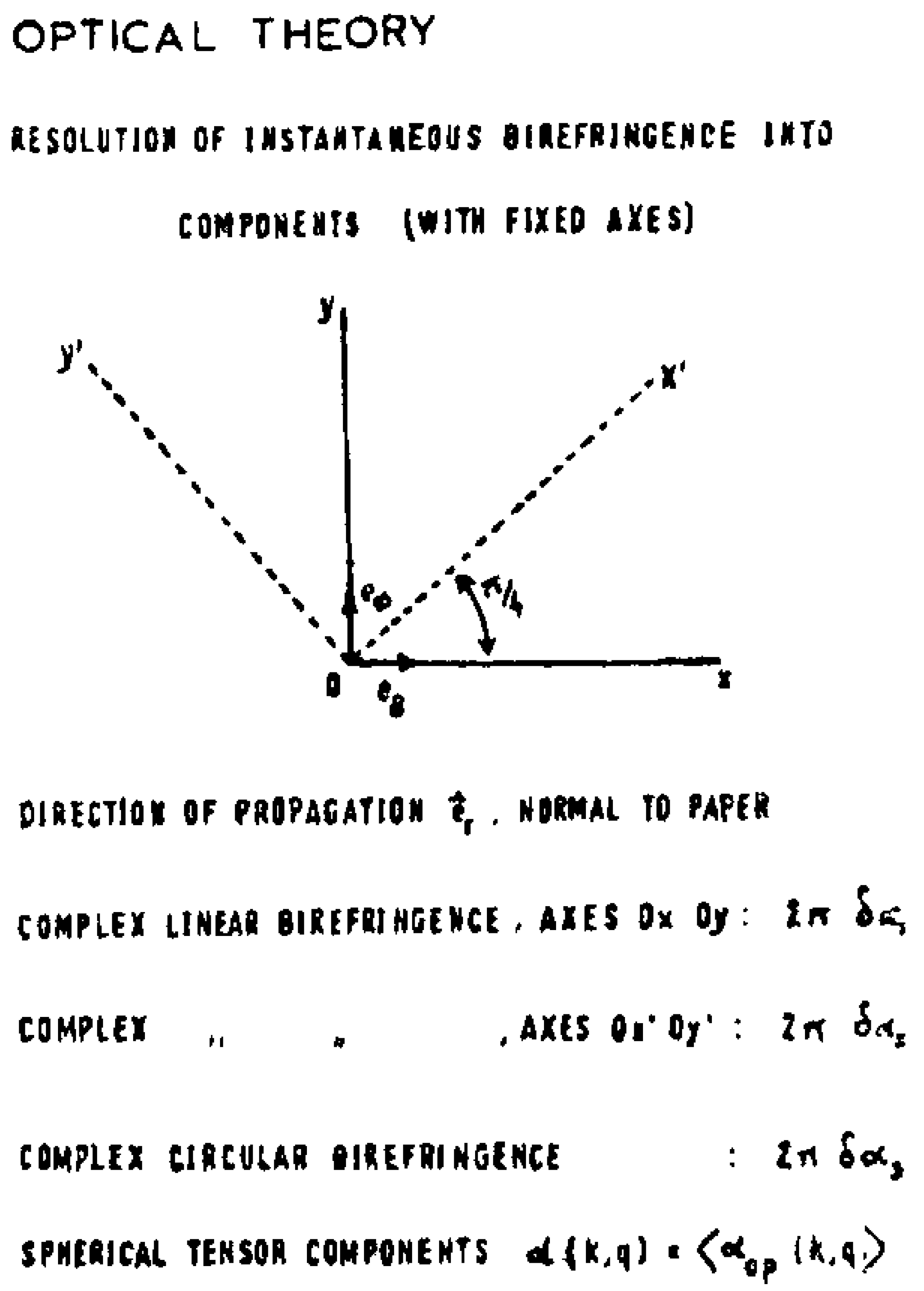


FIG. 2

A fairly comprehensive theory for the optical monitoring of optical pumping signals has been developed for this purpose and I can only touch on some points. Our approach, in contrast to that in Cohen-Tannoudji's recent work, is to solve Maxwell's equations for light

* This paper reproduces the text of a lecture by the late Dr. S. Pancharatnam to a Conference at Warsaw in June 1968.

propagation in an anisotropic medium. We consider an arbitrary direction of propagation which we take to be normal to the plane of the screen. The principal planes of linear birefringence may be changing in time, and instead of following them around we resolve the linear birefringence into two components with respect to fixed axes: First is the linear birefringence with respect to axes Ox Oy as principal planes, and second that with principal axes Ox′ Oy′ turned by 45° with respect to the first. Lastly, we have of course complex circular birefringence. These anisotropy parameters can be quite simply related to the expectation values of polarisability operators as in Happer's work, but I shall not go into this. The question is what property of the spin-assembly do the real parts of the birefringence parameters measure, and what effect do they have on the polarisation of the incident beam ?

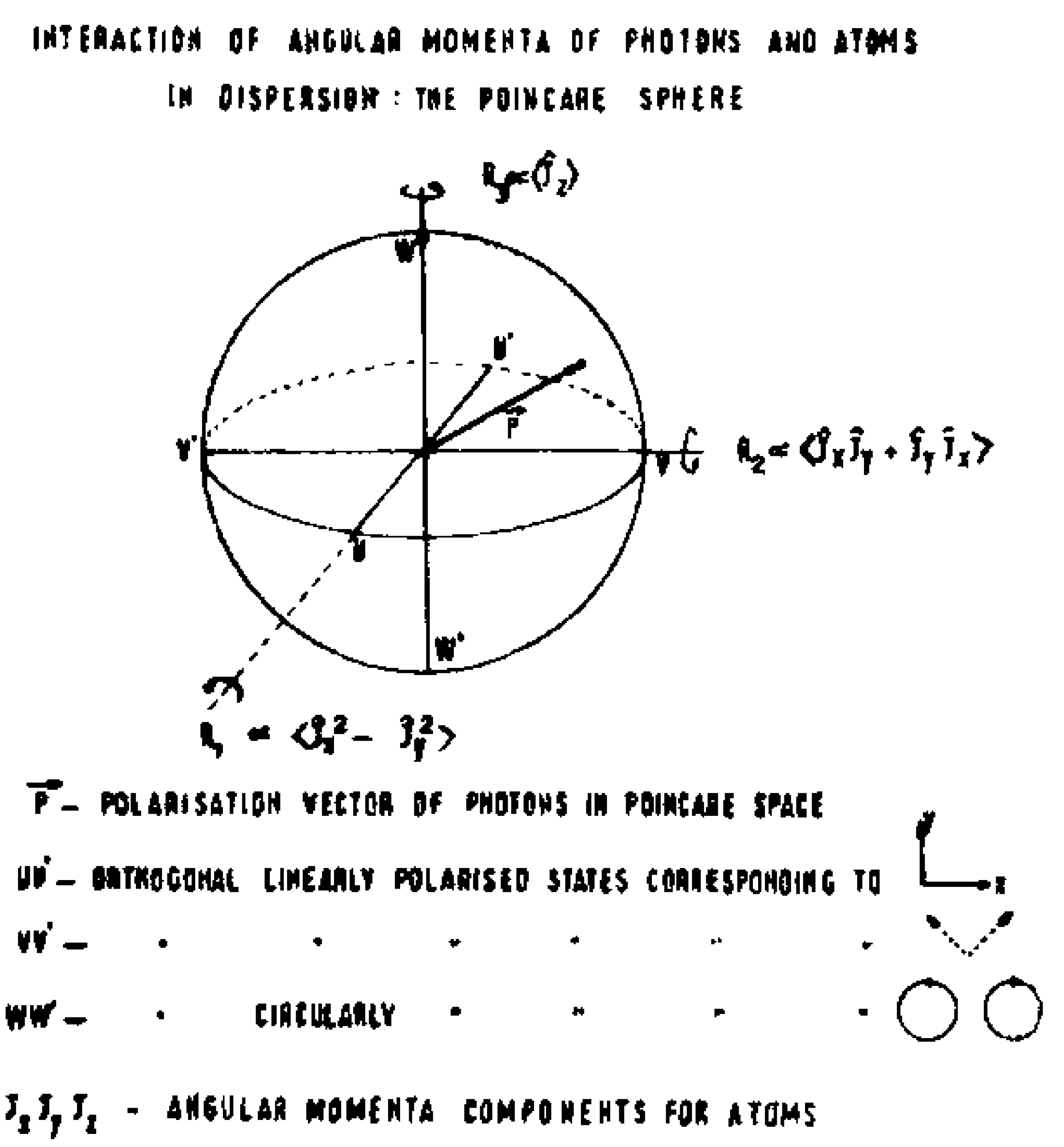
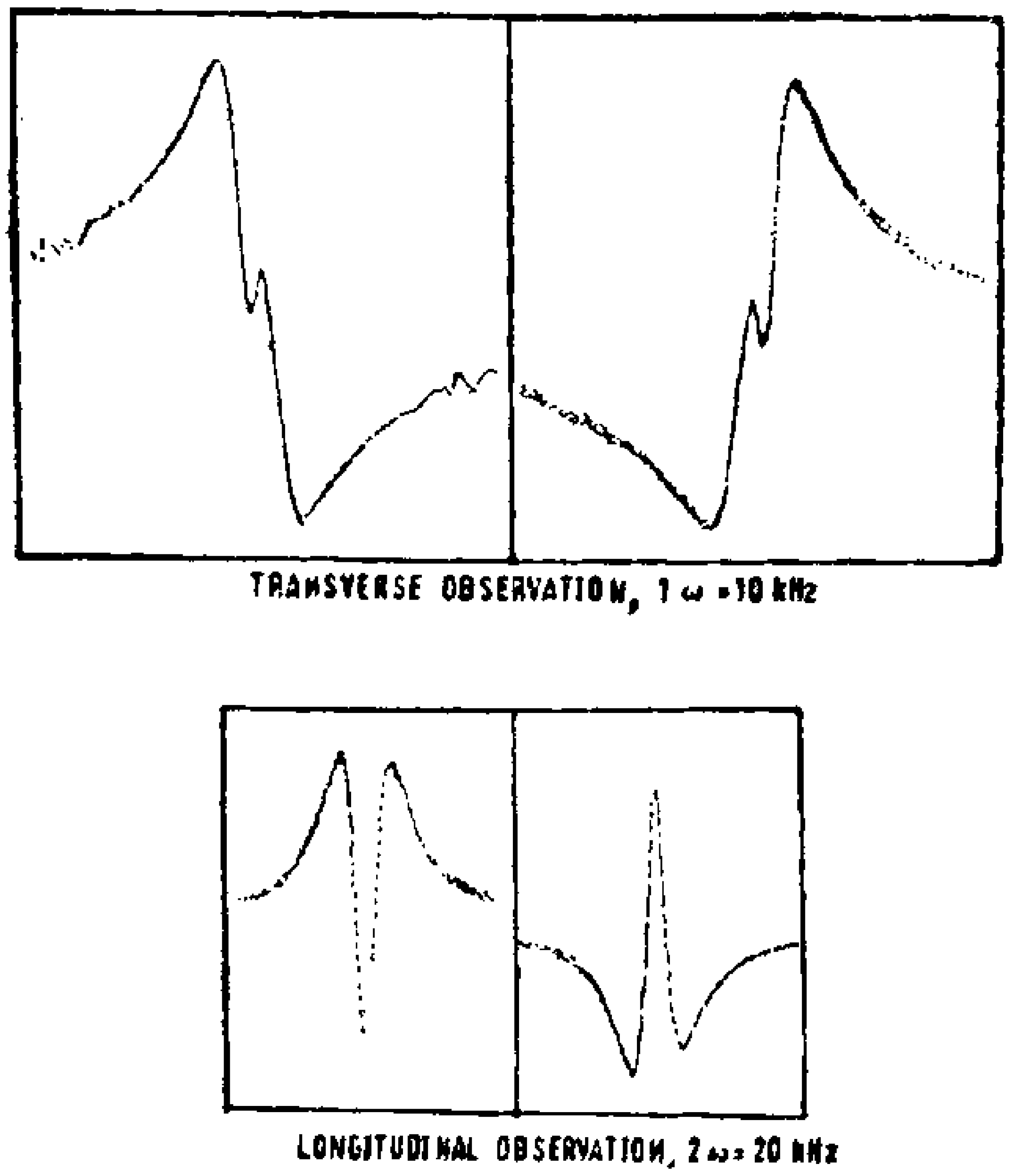


FIG. 3

Figure 3 illustrates the answer to this question which really concerns the interaction of the angular momenta of photons and atoms in dispersion. We use Poincare's representation of the state of polarisation of light by a point on the surface of a unit sphere. Alternatively, if we join the origin to the representative point, the radius vector $\vec{P}$ represents the polarisation vector of the incident photon in Poincare space. This Poincare representation is very closely related to both the classical and quantum mechanical description of light by Stokes parameters, as has been emphasised by Fano. Orthogonally polarised states are represented by opposite points on the Poincare sphere. Thus, the points U and U′ on the equator represent, as indicated in Fig. 3, the orthogonal linearly polarised states Ox Oy. The points V and V′ represent orthogonal linearly polarised states turned by 45° to the first set as drawn here. Lastly, the poles WW′ represent left and right circular polarised states. These are precisely the three pairs of states with respect to which we had resolved the birefringence in the previous figure. The subject of angular momenta in quantum mechanics is intimately connected with rotations. So it is particularly satisfying to note that the effect of the 3 real types of birefringence is to induce respectively proportional rotations R_1, R_2 and R_3 about these mutually orthogonal axes. What about the magnitudes of these rotations ? Let J_x, J_y, J_z be the components of the spin angular momentum of the atomic assembly along Ox, Oy, Oz in real space. Then the rotation R_3 about the polar axis, which represents circular birefringence, is proportional to the expectation value of $(\hat{J}_z)$. The rotation R_1 representing the real x, y birefringence is proportional to the expectation value of $(\hat{J}_x^2 - \hat{J}_y^2)$; while the rotating R_2 representing the 45° birefringence is proportional to the correlation between the x and y components of the angular momentum. It should be noted that even when the average values of the angular momentum components vanish—that is when we have no orientation, these two average values which are quadratic in the components may not vanish—that is when we have alignment. In fact, it is convenient to express these two in terms of the five alignment components, which are the second rank multipole components of the density matrix.

Under magnetic resonance conditions, these expectation values contain not only d.c. components but also oscillatory components; for example, Manuel and Cohen-Tannoudji have monitored the oscillation of J_3 averaged. We know that Bloch's equations, which may be regarded as equations of motion for the orientation components, determine the time-dependence and resonance functions for J_3. In exactly the same spirit we have derived separate equations of motion for the alignment components alone, under magnetic resonance; these are solved assuming that some mechanism pumps one of the components, *viz.*, that with ordinal number zero. Also 3 phenomenological relaxation times are introduced. The equations of motions and the resonance functions for alignment apply to spin-systems of any angular momentum J, just as Bloch's equations apply to any J. When we set the 3 relaxation times equal, the resonance functions reduce to the functions A to E first derived by Series.



MODULATED BIREFRINGENCE SIGNALS

FIG. 4

Figure 5 illustrates examples of modulated birefringence signals in magnetic resonance. The experimental set-up using He^4 is basically similar to that in the first figure. But now we look at *a.c.* signals, phase locked to a rotating transverse field, and use different combinations of polariser and analyser. Here we have in transverse observation, essentially the function B of Series, arising from the 45° birefringence which is modulated at the fundamental frequency. Here we have in longitudinal observation essentially the resonance function D of Series in a birefringence signal modulated at twice the frequency of the rotating field.

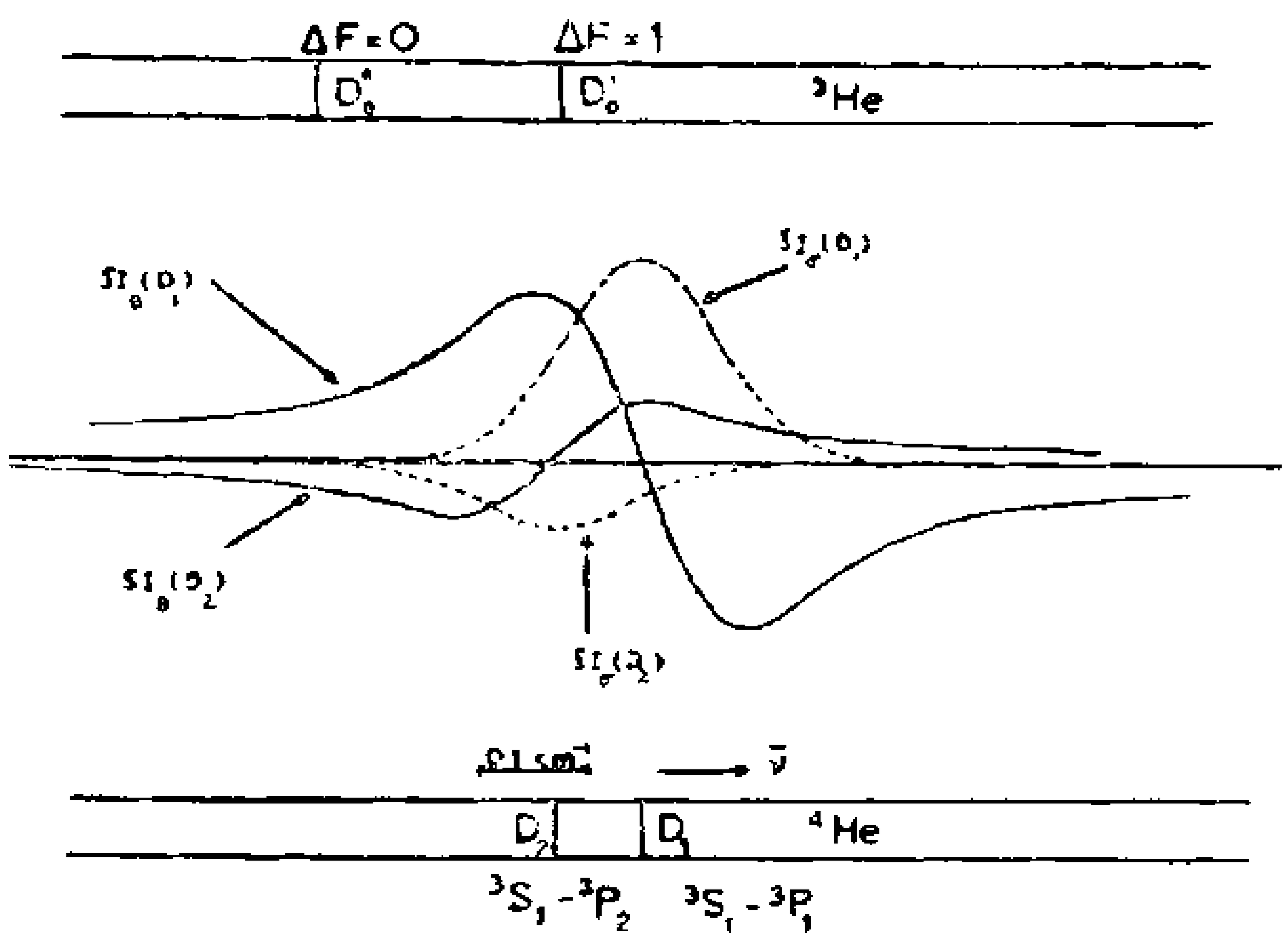


FIG. 5

Note by the Editor: Dr. S. Pancharatnam whose lecture at Warsaw is being published in this issue of *Current Science* was a Fellow of St. Catherine's College at Oxford at the time of his death on the 28th May 1969, when he was only 35 years of age. Pancharatnam went to Oxford in 1964 to work at the Clarendon Laboratory. During his last five years, his main interest was on the interaction of photons and atoms, optical pumping and related experiments, to which subject he made significant contributions. During the last few months of his life, he wrote extensively on many new theoretical ideas he had developed on the subject. These manuscripts are being sorted out and it is hoped that they will, in due course, find publication.

Dr. Pancharatnam started his research career at the Raman Research Institute, Bangalore, where his early investigations on the optical phenomena of crystals led him to develop his fundamental work on the generalized theory of interference and the description of partial polarisation and partial coherence. Immediately prior to his move to Oxford, he was for two years at the University of Mysore as a Reader in Physics, where he took a major part in the organisation of the Physics Department.