ON THE RAYLEIGH'S FLOW PAST AN INFINITE POROUS PLATE—I

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THE Rayleigh's problem[1] in fluid mechanics, *i.e.*, the flow of a viscous incompressible fluid due to an impulsively moving plate has been examined for a rotating fluid by Chawla.[2] Here we consider the two-dimensional unsteady flow of a viscous incompressible fluid past an infinite porous flat plate (chosen along X-axis) at zero incidence with uniform suction $v_0 < 0$. The relevant equations of motion together with the initial and the boundary conditions are

$$\frac{\partial u}{\partial t} + v_0 \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}, \; v_0 < 0. \tag{1}$$

$$\begin{aligned} t \leqslant 0: &\quad u = 0 \quad \text{for } y \geqslant 0, \\ t > 0: &\quad u = U_0 \quad \text{for } y = 0, \\ &\quad\; < \infty \quad \text{for } y \to \infty. \end{aligned} \tag{2}$$

We introduce the non-dimensional quantities

$$\phi = \frac{u}{U_0}, \; \eta = \frac{y U_0}{\nu}, \; \tau = \frac{U_0^2 t}{\nu}, \; \beta = -\frac{v_0}{U_0}, \tag{3}$$

where β can be looked upon as the suction parameter. Then (1, 2) become

$$\frac{\partial \phi}{\partial \tau} - \beta \frac{\partial \phi}{\partial \eta} = \frac{\partial^2 \phi}{\partial \eta^2}, \tag{4}$$

$$\begin{aligned} \tau \leqslant 0: &\quad \phi = 0 \quad \text{for } \eta \geqslant 0, \\ \tau > 0: &\quad \phi = 1 \quad \text{for } \eta = 0, \\ &\quad\qquad\;\; \text{for } \eta \to \infty, \end{aligned} \tag{5}$$

The solution of (4) subject to (5) is

$$\phi = \tfrac{1}{2} \left\{ e^{-\eta\beta} \operatorname{erfc}\left(\frac{\eta}{2\sqrt{\tau}} - \frac{\beta}{2}\sqrt{\tau}\right) + \operatorname{erfc}\left(\frac{\eta}{2\sqrt{\tau}} + \frac{\beta}{2}\sqrt{\tau}\right) \right\} \tag{6}$$

The non-dimensional skin friction Γ defined by $-(d\phi/d\eta)_{\eta=0}$ becomes

$$\Gamma = \frac{1}{\sqrt{\pi\tau}} e^{-(\tau\beta^2/4)} + \frac{\beta}{2} \operatorname{erfc}\left(-\frac{\beta}{2}\sqrt{\tau}\right). \tag{7}$$

The steady state corresponds to $\tau \to \infty$; in this case ϕ and Γ are given by

$$\phi_{st} = e^{-\eta\beta}, \; \Gamma_{st} = \beta. \tag{8}$$

Tables I and II give the calculated values of ϕ and Γ given by (6, 7) for $\beta = 0, 1, 2$; $\tau = \cdot 04$, $\cdot 36$, 1, ∞ and $\eta = 0(\cdot 2)1$.

TABLE I

Velocity distribution near an impulsively moving plate

τ	β \ η	0	·2	·4	·6	·8	1
·04	0	1	·4795	·1573	·033[illegible]	·0047	·0004
	1	1	·4321	·1280	·0249	·0031	·0003
	2	1	·3861	·1[illegible]28	·0180	·0020	·0001
·36	0	1	·8133	·6377	·4795	·3455	·2388
	1	1	·7249	·5080	·34[illegible]3	·2214	·1375
	2	1	·6283	·3843	·2274	·1299	·0716
1	0	1	·8875	·7773	·6714	·5716	·4795
	1	1	·7828	·6064	·4645	·3515	·2626
	2	1	·6622	·4362	·2857	·1858	·1198
∞	0	1	1	1	1	1	1
	1	1	·8187	·6703	·5488	·4493	·3679
	2	1	·6703	·4493	·3012	·2019	·1353

TABLE II

The skin friction at the plate

τ \ β	0	1	2
·04	2·8210	3·3491	3·9330
·36	·9403	1·5237	2·2599
1	·5642	1·1996	2·0503
∞	0	1	2

From the calculated values, we infer that (*i*) as β (corresponding to the suction velocity v_0) increases, for a given time, the velocity at any point of the fluid decreases and the skin friction at the plate increases and (*ii*) for any given β, as the time advances, the velocity at any point of the fluid increases and the skin friction decreases. Ultimately ϕ and Γ settle down to the steady state values corresponding to $\tau \to \infty$.

1. Schlichting, H., *Boundary Layer Theory*, Pergamon Press, 1955, p. 64.
2. Chawla, S. S., *J. Phys. Soc. Japan*, 1967, **23**, 663.

ON THE RAYLEIGH'S FLOW PAST AN INFINITE POROUS PLATE—II

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IN the previous article referred to as (I), the effect of suction on Rayleigh's flow in fluid mechanics has been considered. Equations (6) and (7) of I are the expressions for the non-dimensional velocity ϕ_s and the skin friction Γ_s at the plate.

Here the same problem is considered in the framework of hydromagnetics. It is assumed that the fluid is weakly conducting and that a uniform magnetic field H_0 is applied along Y-axis perpendicular to the flow. When the magnetic lines of force are fixed relative to the fluid, the equation of motion[1] is

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} - mu, \; m = \frac{\sigma \mu^2 H_0^2}{\rho}. \tag{1}$$

Solving (1) subject to the initial and boundary conditions (2) of I, we obtain, in terms of the non-dimensional quantities (3) of I,

$$\frac{\partial \phi}{\partial \tau} = \frac{\partial^2 \phi}{\partial \eta^2} - \alpha^2 \phi, \; \alpha = \frac{\sqrt{m\nu}}{U_0}, \tag{2}$$

where α is looked upon as the Hartmann number in the unsteady motion. The solution of (2) is

$$\phi_M = \tfrac{1}{2}\left\{e^{-\eta\alpha} \operatorname{erfc}\left(\frac{\eta}{2\sqrt{\tau}} - \alpha\sqrt{\tau}\right) + e^{\eta\alpha} \operatorname{erfc}\left(\frac{\eta}{2\sqrt{\tau}} + \alpha\sqrt{\tau}\right)\right\}, \tag{3}$$

$$\Gamma_M = \frac{1}{\sqrt{\pi\tau}} e^{-\alpha^2\tau} + \alpha \operatorname{erfc} \alpha\sqrt{\tau}. \tag{4}$$

as $\tau \to \infty$,

$$\phi_{M,\,st} = e^{-\eta\alpha}, \; \Gamma_{M,\,st} = \alpha. \tag{5}$$

If the magnetic lines of force are fixed relative to the plate,[2] (1) is to be replaced by

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - m(u - U_0). \tag{6}$$

In this case,

$$\phi_R = 1 - e^{-\alpha^2\tau} \operatorname{erfc} \frac{\eta}{2\sqrt{\tau}}, \tag{7}$$

$$\Gamma_R = \frac{e^{-\alpha^2\tau}}{\sqrt{\pi\tau}}. \tag{8}$$

As $\tau \to \infty$,

$$\phi_{R,\,st} = 1. \; \Gamma_{R,\,st} = 0. \tag{9}$$

Figure 1 shows that, for equal values of α and β, $\phi_S < \phi_M < \phi_R$ for a given η at $\tau = 1$. This is found to be true, for all values of τ $(0 < \tau < \infty)$. Thus the fluid velocity is retarded more by applying suction than by the magnetic field. Similarly from Fig. 2, we infer that the skin friction at the plate is more in the case of suction than in the magnetic case.

Again, a study of Fig. 1 and Fig. 2 indicates that, (1) as β (corresponding to the suction velocity v_0) increases, the velocity at any point of the fluid decreases and the skin friction at the plate increases. (2) As α (corresponding to the strength of the magnetic field H_0) increases, (*i*) when the magnetic lines of force are fixed relative to the fluid, the velocity at any point decreases while the skin friction at the plate increases and (*ii*) when the magnetic lines of force are fixed relative to the plate, the velocity at any point increases while the skin friction at the plate decreases.

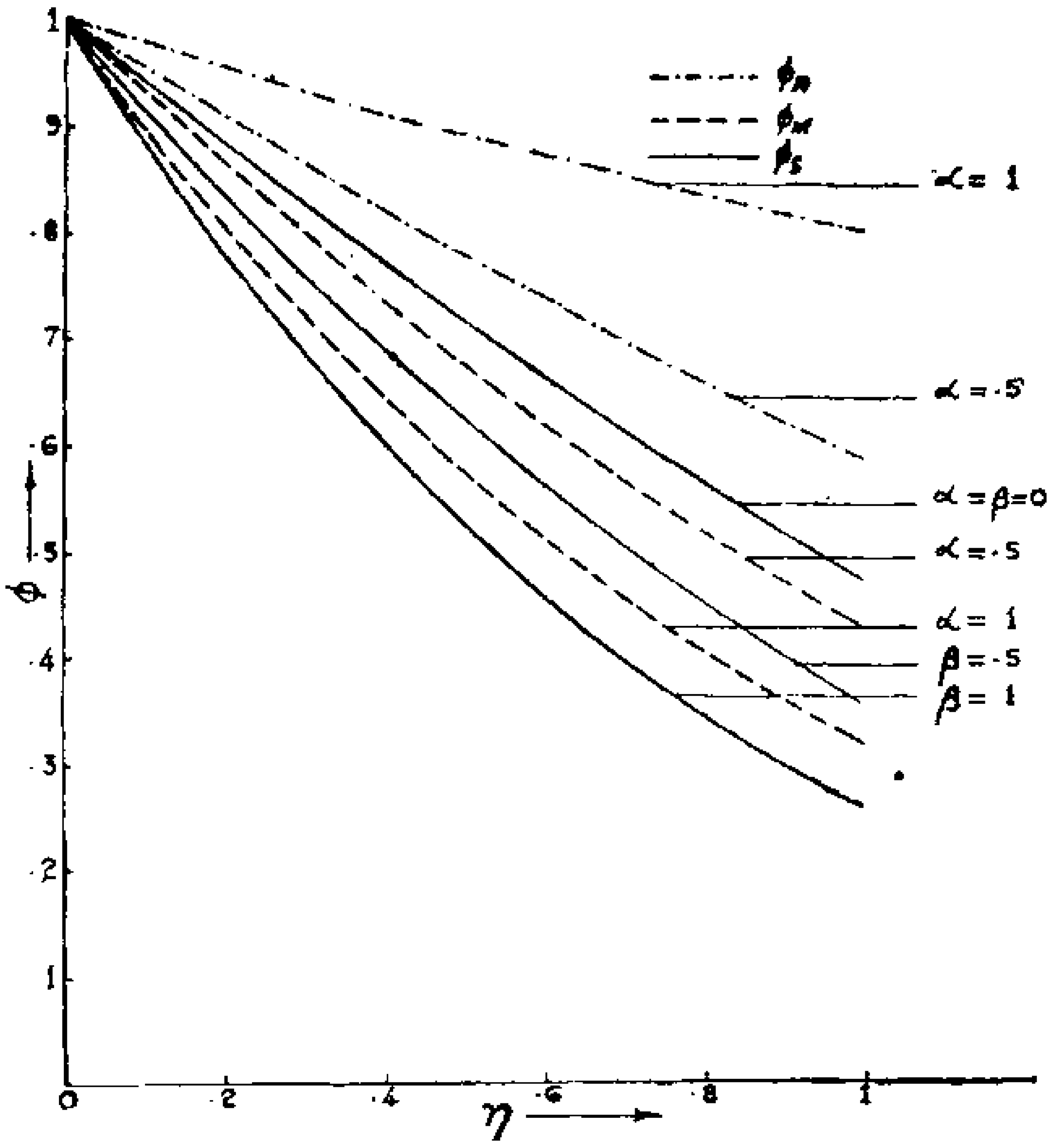

FIG. 1. Non-dimensional velocity ϕ *versus* η at $\tau = 1$ for different values of α and β.

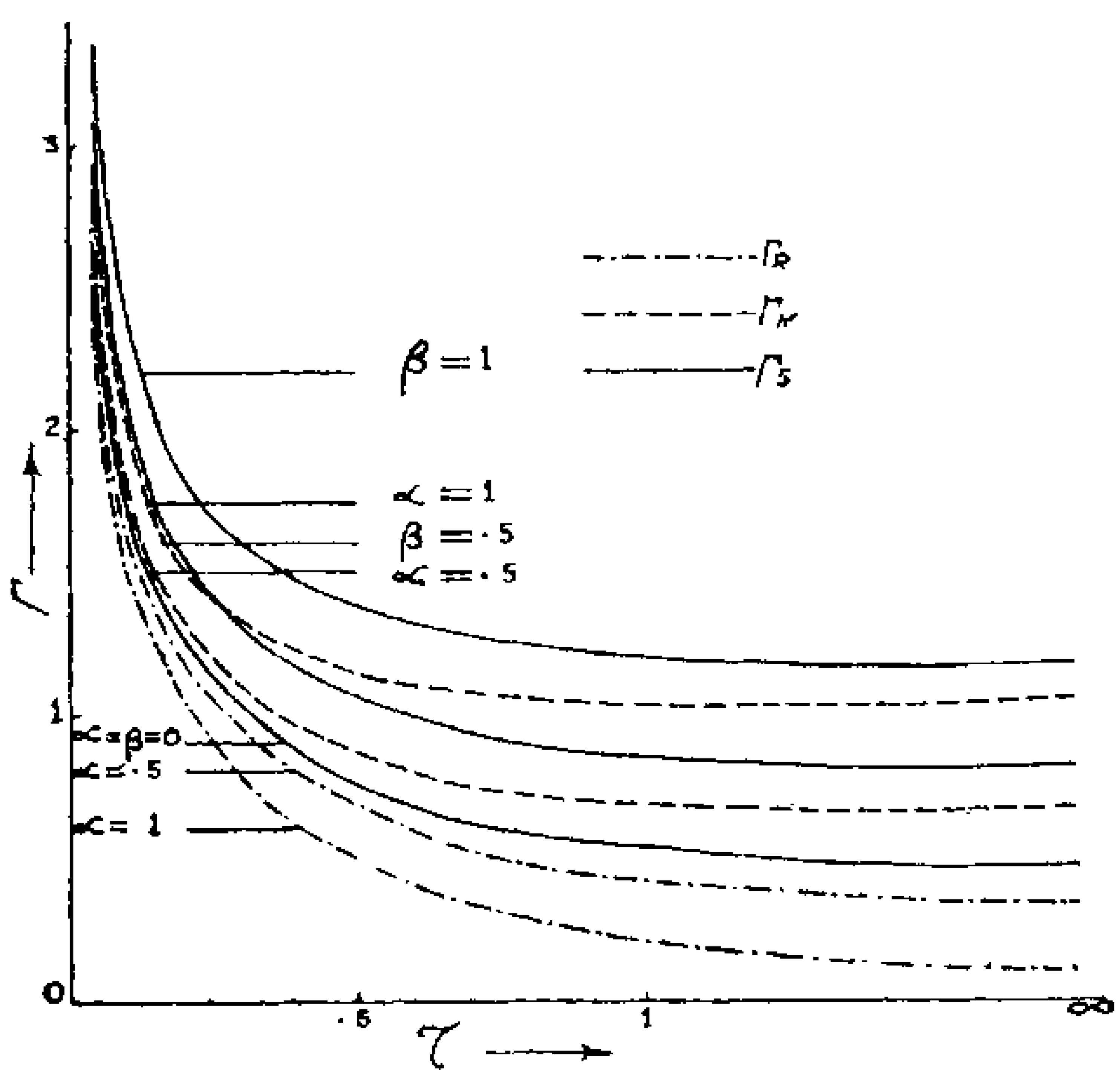

FIG. 2. Non-dimensional skin friction Γ *versus* τ for different values of α and β.

Also, numerical calculations of ϕ for various values of τ show that, for any given α or β, as the time advances, the velocity at any point of the fluid in the corresponding case increases and the skin friction decreases. Ultimately ϕ and Γ settle down respectively to their steady state values corresponding to $\tau \to \infty$.

1. Pai, S. I., *Magnetogas Dynamics and Plasma Dynamics*, Springer-Verlag, 1962, p. 65.
2. Soundalgekar, V. M., *Appl. Sci. Res.*, 1965, **12 B**, 151.