

seems to be built up of leucocyanidin and epicatechin. Further details could not be studied due to lack of material.

To sum up the results, the leaves of *Rhododendron formosum* contain the following known compounds: dihydrotaraxerone, ursolic acid and taxifolin. It also contains two possibly new triterpenoids, and a proanthocyanidin built up of leucocyanidin and epicatechin. Dihydrotaraxerone is a substance having a saturated ring system and has not been known so far to occur in any natural source; very few compounds of this type are known to occur in nature so far.

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THE ASYMPTOTIC THEORY OF THE BLUNTED WEDGE AT HYPERSONIC SPEEDS

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AS a logical extension of the asymptotic theory for the blunted flat plate and the circular cylinder at hypersonic speeds, which has been treated in great detail by Guiraud *et al.*, Roberto Vaglio Laurin, Freeman and others, an attempt is made to treat the direct problem of the blunted slender wedge at $M_\infty = \infty$, by the method of matched asymptotic expansions. It is assumed that the asymptotic shock wave shape corresponds to the sharp wedge solution and a first correction to the shock shape has been obtained in the presence of blunting.

It is found that the matching could be effected only if the displacement thickness of the entropy layer is zero. A global energy and mass flow criteria have been employed to verify that the first correction is the only one compatible with the assumed shock shape and the given body and that this correction is found to be due to the body and not the entropy layer. There are two regions present in the flow, *viz.*, an 'outer region' where small perturbation solutions are valid and an 'inner boundary-layer-like region' called the entropy layer (hot, low-density gas that crosses the strong portion of the bow shock near the nose) where different approximations are required. The entropy layer plays an important role in the description of flows over hypersonic speeds and renders the associated asymptotic patterns non-similar by its presence. An asymptotic behaviour of the shock wave is written *a priori* with undetermined parameters which are determined finally, so that the body corres-

ponding to the shock is the one we wished to obtain asymptotically.

An analytical scheme has been developed to obtain a uniformly valid solution of the complete flow field. The equations for the steady flow of an inviscid, perfect gas with constant specific heats in Von Mises co-ordinates (x, ψ) are

$$\frac{\partial y}{\partial x} = \frac{v}{u} \quad (1.1.1)$$

$$\frac{\partial y}{\partial \psi} = \left(\frac{1}{\rho u} \right) \quad (1.1.2)$$

$$\frac{\partial v}{\partial x} + \frac{\partial p}{\partial \psi} = 0 \quad (1.1.3)$$

$$u^2 + v^2 + \frac{2\gamma}{\gamma - 1} \frac{p}{\rho} = 1 \quad (1.1.4)$$

$$\frac{p}{\rho^\gamma} = G(\psi) = \left(\frac{2}{\gamma - 1} \right) \left(\frac{\gamma - 1}{\gamma + 1} \right)^\gamma \sin^2 \beta(\psi) \quad (1.1.5)$$

where β is the shock wave angle. All the flow variables have been non-dimensionalised with respect to the freestream quantities, velocities being referred to U_∞ , density to ρ_∞ , and pressure with respect to $\rho_\infty V_\infty^2$. The distance variables have been non-dimensionalised with respect to the bluntness parameter 'd'. The shock wave slope is assumed to be the unknown and expressed in the form

$$h(x) = (\sin \beta)^{2/\gamma} \quad (1.1.6)$$

and $h(x)$ is developed asymptotically along the lines given by Guiraud² as

$$h(x) \approx \frac{h_0}{x^{\alpha_0}} \left\{ 1 + \frac{h_1}{x^{\alpha_1}} + \dots \right\} \quad (1.1.7)$$

The leading exponent α_0 of this expansion is taken to be zero based on the fact that the

solution should tend to the sharp wedge solution asymptotically for downstream.

Boundary condition.—

$$\frac{v}{u} = \tan \alpha \quad (1.1.8)$$

on the body where α is the semiwedge angle. The outer variables are chosen as (x, ω) where $\omega = y/x$ and functions are denoted by an asterisk. $\omega = 0(1)$ near the shock wave. The inner variables are taken as (x, ψ) where $\psi = 0(1)$ near the wedge and functions are denoted by $\tilde{\sim}$. The flow variables are expanded asymptotically as follows:

$$f^*(x, \omega) \sim f_0^*(\omega) + \frac{h_1 f_1^*(\omega)}{x^{\alpha_1}} + \dots \quad (1.2.1)$$

$$Y^*(x, \omega) \sim x Y_0^*(\omega) + \frac{h_1 Y_1^*(\omega)}{x^{(\alpha_1-1)}} + \dots \quad (1.2.2)$$

The general equations of motion written in outer variables (x, ω) are as given below.

$$\frac{\partial Y}{\partial x} = \frac{v}{U} \frac{\partial Y}{\partial \omega} \quad (1.3.1)$$

$$\frac{\partial Y}{\partial \omega} = D U \quad (1.3.2)$$

$$x \frac{\partial v}{\partial x} + \omega \frac{\partial v}{\partial \omega} + \frac{\partial P}{\partial \omega} = 0 \quad (1.3.3)$$

$$U^2 - V^2 + \frac{\gamma P}{(\gamma - 1) D} = 0 \quad (1.3.4)$$

$$\frac{P}{D^\gamma} = \left(\frac{v}{\gamma - 1} \right) \left(\frac{\gamma - 1}{\gamma + 1} \right)^\gamma \times \left[h_0 \gamma \left(1 + \frac{h_1}{x^{\alpha_1}} + \dots \right)^\gamma \right] \quad (1.3.5)$$

substituting the expansions for the flow and distance variables in the equations of motion and collecting coefficients of powers of x , we respectively get zeroth outer, first outer, etc., system of equations. The zeroth outer system corresponds to the sharp wedge solution as given by Yakura and they can be solved with the help of the boundary conditions. It must be remarked here that although the flow variables are uniformly valid for all ' ω ', this is not true for the distance variable, Y , because the body condition is not satisfied. Hence an inner expansion has to be constructed, taking the entropy layer and the body condition into consideration. From a study of the outer equations near $\omega = 0$, it can be inferred that the leading terms of the inner expansions for the flow variables must be of the following form

$$\tilde{f}(x, \psi) \sim \tilde{f}_0(\psi) + \frac{\tilde{f}_1(\psi)}{x^{\beta_1}} + \dots \quad (1.4.1)$$

$$\tilde{y}(x, \psi) \sim A x + \tilde{y}_0(\psi) + \frac{\tilde{y}_1(\psi)}{x^{\beta_1-1}} + \dots \quad (1.4.2)$$

The differential equation for $Y_1^*(\omega)$ is of second order with a non-homogeneous term of order $(\omega^{-\alpha_1-1})$ on the right-hand side. In order to construct the inner expansion for the flow variables, it is imperative to know the behaviour of the outer functions near $\omega = 0$. Since the perturbation in the shock shape is dependent upon the streamline displacement, the behaviour of the distance variable Y is studied near $\omega = 0$. The second order differential equation for $Y_1^*(\omega)$ is as follows:

$$\frac{d^2 Y_1^*}{d\omega^2} (A\omega^2 + B\omega + C) + \frac{dY_1^*}{d\omega} (D\omega + E) + F Y_1^* = G\omega^{-\alpha_1-1} + H \quad (1.4.3)$$

where A, B, \dots, H are functions of P_0^*, D_0^*, \dots etc. The general solution for Y_1^* can be written in the form

$$Y_1^*(\omega) \sim (P + Q \log \omega) \omega^{-(\alpha_1+1)} \quad (1.4.4)$$

near $\omega = 0$. The constants of integration can be obtained by matching.

The choice of the index in the first correction is decided from a consideration of the global mass flow conservation and energy balance. It is found that $\alpha_1 = 1$ leads to a logarithmic term for the drag which necessitates a 'log term' in the shock wave shape and matching becomes difficult. $\alpha_1 = 2$ is found to be the right index. It must be remarked here that this first correction to the shock shape is due to the body development and that the entropy layer effects are of a higher order. As it is not possible to guess the complete outer expansions *a priori* the procedure adopted is to start from the zeroth outer system of equations, then go to the zeroth inner and afterwards first outer and so on to effect matching.

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