

Research Notes.

Metrical Problems of Continued-Fraction-Theory.

KHINTCHINE (*Comp. Math.*, 1, 361-382) has analysed the general nature of the continued fraction development of an irrational number in a very interesting way,—i.e., those properties which hold good for almost all numbers. Suppose α represents any number between 0 and 1; and let

$$\alpha = \frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \dots + \frac{1}{a_n} + \dots$$

or

$$= [a_1, a_2, \dots, a_n, \dots]$$

say in any abridged form. Let $m_n(x)$ be the measure of the aggregate of values α for which $Z_n(\alpha) = [a_{n+1}, a_{n+2}, \dots] < x \leq 1$; then it has been shewn by Gauss that

$$\text{Lt } m_n(x) = \frac{\text{Log}(1+x)}{\text{Log } 2}.$$

This bit of Gauss's work was forgotten until Kuzmin [*Atti. del. Congr. Intern. Bologna*, 1928, 6, 83] gave a proof of the result and also extended it. Some more important results had been obtained by Bernstein and Borel; the chief result being—If $\phi(n)$ is an increasing function then the result $a_n = 0$ [$\phi(n)$]

is true or false according as $\sum \frac{1}{\phi(n)}$ is convergent or divergent. (It is to be noted that a set of values whose measure is zero is always excluded.) Some more results were obtained by Khintchine himself during 1923-25. These results have now been extended and beautifully precised. After proving some lemmas he obtained a general result which is of great interest. The first lemma is this:—Let $E \left(\begin{smallmatrix} n_1, n_2, \dots, n_k \\ r_1, r_2, \dots, r_k \end{smallmatrix} \right)$ be the measure of the aggregate of values α for which

$$a_n = r_t [t = 1, 2, 3, \dots, K].$$

If the n_t 's are all different then we have

$$\frac{E \left(\begin{smallmatrix} n_1, n_2, \dots, n_{k+1} \\ r_1, r_2, \dots, r_{k+1} \end{smallmatrix} \right)}{E \left(\begin{smallmatrix} n_1, n_2, \dots, n_k \\ r_1, r_2, \dots, r_k \end{smallmatrix} \right)} < \frac{C}{r^2},$$

where C is an absolute constant. Now Gauss has shewn that

$$m_n(x) = \sum_{v=1}^{\infty} \left[m_{n-1} \left(\frac{1}{v} \right) - m_{n-1} \left(\frac{1}{v+x} \right) \right] \text{ and}$$

$$m'_n(x) = \sum_{v=1}^{\infty} \frac{m'_{n-1} \left(\frac{1}{v+x} \right)}{(v+x)^2}$$

By constructing similar functional equations and using a lemma of Kuzmin he generalises his earlier result into

$$\left| \frac{E \left(\begin{smallmatrix} n_1, n_2, \dots, n_{k+1} \\ r_1, r_2, \dots, r_{k+1} \end{smallmatrix} \right)}{E \left(\begin{smallmatrix} n_1, n_2, \dots, n_k \\ r_1, r_2, \dots, r_k \end{smallmatrix} \right)} - \frac{\text{Log} \left[1 + \frac{1}{r(r+2)} \right]}{\text{Log } 2} \right| < B e^{-\beta \sqrt{n_{k+1} - n_k}}$$

where $n_1 < n_2 < \dots < n_k < n_{k+1}$, and B and β being arbitrary constants. Utilising these results he proves the following important result:—Let $f(r)$ be a positive function of r such that $f(r) < kr^{1-\delta}$ where k and δ are two absolute positive constants. Then

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n f(a_k) = \sum_{r=1}^{\infty} f(r) \frac{\text{Log} \left[1 + \frac{1}{r(r+2)} \right]}{\text{Log } 2}$$

for almost all values of α . If we take $f(r) = \text{Log } r$ then we get the interesting result that

$$\text{Lt } (a_1, a_2, \dots, a_n)^{\frac{1}{n}} = \prod_{r=1}^{\infty} \left(1 + \frac{1}{r(r+2)} \right)$$

$$\frac{\text{Log } r}{\text{Log } 2} = 2.6 \dots \text{ for almost all values of } \alpha.$$

It is of course obvious that we cannot obtain in a similar way a corresponding result for the arithmetic mean. The difficulty of this problem was already pointed out by Borel and Bernstein. By means of these methods alone the following result has been obtained:—

For every $\epsilon > 0$,

$$E \left\{ \left| \frac{S_n \log 2}{n \log n} - 1 \right| > \epsilon \right\} \rightarrow 0 \text{ as } n \rightarrow \infty$$

where $S_n = \sum_{k=1}^n a_k$. This is not really equal

to the result that $\frac{S_n}{n \log n} \rightarrow \frac{1}{\log 2}$ for almost all

α . In fact it is known that $\lim_{n \rightarrow \infty} \frac{S_n}{n \log n}$ is infinite. Another interesting result is that

$\sum_{n=1}^{\infty} S_n^{-1}$ is divergent for almost all values of α .

K. V. I.

A Problem concerning Orthogonal Polynomials.

SZEGO (*Trans. Am. Math. Soc.*, **37**, 1, pp. 196-216) has proved certain interesting results in connection with the existence of two Jordan curves possessing a common system of orthogonal polynomials. Particular cases of such curves are (1) concentric circles for which $1, z, z^2, \dots, z^n, \dots$ are the corresponding orthogonal polynomials with weight-function unity, and (2) confocal ellipses with foci at ± 1 for which the Tchebecheff polynomials have the orthogonal property with weight-function $|1 - z^2|^{-\frac{1}{2}}$. Although he has not solved the general problem he has obtained the following two results which forms a very decisive step in the solution of the general problem. The first theorem runs as follows:

Given two analytic Jordan curves C_1 and C_2 and two continuous positive functions $n_1(z)$ and $n_2(z)$ as the respective weight-functions and if they possess the same system of orthogonal polynomials then one curve say C_1 must contain the other C_2 and C_1 is a level curve obtained through the conformal transformation of the outer region (C_2) into the exterior of a circle, the point at ∞ being a fixed point; and there exists an analytic function $D(z)$ regular and $\neq 0$ outside C_2 ($z = \infty$ is to be considered as an inside point) such that—

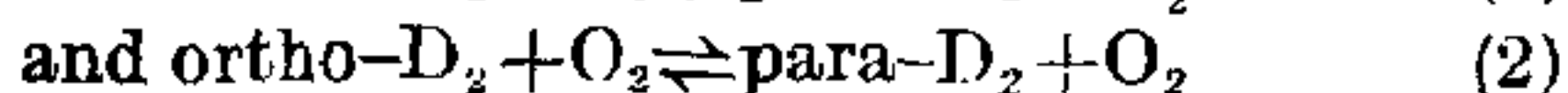
$$\lim_{Z \rightarrow Z_0} \begin{cases} |D(z)|^2 = n_1(z) & \text{for points on } C_1, \text{ and} \\ |D(z)|^2 = n_2(z_0) & \text{for } Z_0 \text{ on } C_2. \end{cases}$$

It is also easily seen that the method of proof applies for more general curves also. He has also determined all Jordan curves C and all analytic functions $D(z)$ regular and $\neq 0$ outside C which possess the property that if C_r be a level curve (the transformation being the same as in the previous theorem) then the set of orthogonal polynomials with weight-function $|D(z)|^2$ are independent of r . He has proved that there are only five essentially different cases (ignoring multiplication by means of a certain constant and linear transformation). He has also given short and elegant proofs of the orthogonality property in these particular cases.

Ratio of the Magnetic Moments of the Proton and the Deuteron.

THE magnetic moments of the proton and the deuteron have been determined by the molecular ray method by Stern and by

Rabi, but there is some discrepancy between their results. The ratio of the magnetic moments of the two particles is not therefore known with certainty. Now J. Farkas and A. Farkas (*Nature*, 1935, **135**, 372) have calculated the ratio of the magnetic moments of the two particles in question by comparing the rates of the reactions



The calculation is made according to the formula of Kalckar and Teller:

$$\left(\frac{\mu_H}{\mu_D} \right)^2 = \frac{a k_{H_2}^{(2T)}}{k_{D_2}^{(T)}}$$

where a is a constant $= 1.12$ for $T > 120^\circ \text{K}$.

and $= 1.18$ at $T = 83^\circ \text{K}$ and $k_{H_2}^{(2T)}$ and $k_{D_2}^{(T)}$ are the velocity constants for the reaction (1) at $2T$ and for (2) at T . The values obtained are $\mu_H/\mu_D = 3.85, 4.03$ and 4.07 respectively at $83^\circ, 193^\circ$ and 293°K . The variation is within the experimental error which is less than 5%.

T. S. S.

The Electronic Charge.

THERE is still an unsolved difficulty regarding the correct value of the charge on the electron. The oil-drop method of Millikan and its results have been discussed by Birge and yield the value $4.768 \times 10^{-10} \pm 0.005 \times 10^{-10}$ e.s.u. for the electronic charge. The value obtained by using the accepted structure and constants of calcite and the wavelengths of X-rays determined by means of a grating is much higher. Bäcklin has recently repeated his measurements with greater accuracy and arrives at a value 4.805×10^{-10} e.s.u. by this method. Now Schopper has determined e by finding the total charge carried by a counted number of α -particles and finds that $e = 4.768 \times 10^{-10}$ e.s.u. in very good conformity with the oil-drop value. Birge and McMillan (*Phys. Rev.*, 1935, **47**, 320) have rediscussed the results of Schopper and come to the conclusion that $e = 4.780 \times 10^{-10}$ e.s.u. This is 0.25% above the oil-drop value but is far lower than the value obtained from the grating measurements. A. E. Ruark (*Phys. Rev.*, 1935, **47**, 316) discusses the discrepancy between the crystal and ruled-grating wavelengths and shows that if Bäcklin's value $e = 4.805 \times 10^{-10}$ e.s.u. (which is in very good agreement with Bearden's value

$4.806 \times 10^{-10} \pm 0.003 \times 10^{-10}$) is used together with the value of $\frac{e}{m} = (1.7579 \pm 0.0003) \times 10^7$ (e.m.u./g) obtained by Shane and Spedding, the discrepancy between the measured energy of photoelectrons and that calculated from X-ray wavelengths vanishes. This higher value of e leads to $1/\alpha = 137.04$ while Eddington's theory requires it to be 137. It thus seems as if the higher value for e is more satisfactory, but then the lower values obtained by the direct methods of Millikan and Schopper remain unexplained.

T. S. S.

Inter-molecular Compounds and Raman Spectra.

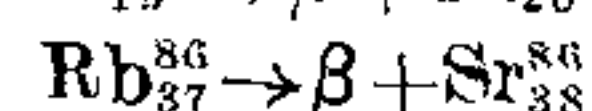
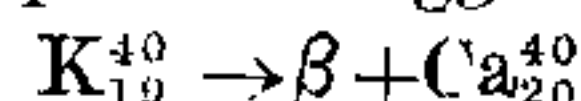
It is well known that alcohol, ether, ketone and aldehyde molecules form addition complexes with inorganic acids and metallic halides, but regarding the nature of the complexes, whether they are valency compounds (quadrivalent oxygen, oxonium form), coordinative compounds (trivalent oxygen, onium form) or merely molecules held together by Van der Waal's, dipole, and such forces, only indefinite qualitative notions exist. Generalisations and analogy considerations are dangerous in this field of chemistry, and extensive physical investigations are greatly needed. G. Briegleb and W. Lauppe (*Zeit. physikal. Ch. (B)*, 1935, 23, 154) show that as Raman spectra furnish an insight into the symmetry and binding state of molecules, a comparison of the spectra of a molecular compound with those of the components should give information about the changes in the symmetry and the binding state of the components caused by the molecular compound formation. Investigations carried out hitherto however show that no marked changes in the spectra are to be found when complexes are formed between like or unlike molecules solely on account of weak secondary valency forces. In the present paper a study of two definite molecular compounds HBr-ethyl ether, and SnCl_4 -ethyl ether is reported. At room temperatures, a solution of HBr in ether gave the characteristic frequencies of ethyl bromide and thus showed that here the component molecules have actually reacted with each other. At -40° however, a new spectrum different to that of the component molecules was obtained, corresponding to the existence of a definite inter-molecular

compound. In the second case investigated, the frequencies of SnCl_4 were found to be considerably influenced possibly on account of a distortion caused by the neighbouring ether molecule. Further details and other studies are to be reported later.

M. A. G.

Radioactivity of Potassium and Rubidium.

THE emission of β -rays by Potassium and Rubidium has been known for some time, but the problem of deciding the exact nature of the disintegration process has been offering various theoretical and experimental difficulties. The long life period of Potassium and Rubidium and the high velocity of the β -rays emitted by them are not to be expected from theoretical considerations. Klemperer (*Proc. Roy. Soc.*, 1935, 148, 638) has critically examined the different schemes suggested by Gamow for the disintegration process. (1) Simultaneous emission of 2 β -rays for each decaying atom. (2) An α -ray change followed by a fast β -ray change. (3) The Ca and Sr formed as a result of slow β -ray change from K and Rb respectively, may decay rapidly giving fast β -rays. Klemperer has finally arrived at the conclusion that the radio activity is not due to K_{40}^{40} and K_{41}^{41} but to K_{40}^{40} , and in the case of Rubidium to Rb_{88}^{88} . The process suggested is as follows:—



Of the two groups of β -rays emitted, one of them may be connected with γ -ray emission. It has been shown that K_{19}^{40} has a resultant nuclear spin of 4 or 5 while Ca_{20}^{40} has zero spin. Since according to the Fermi theory the life-time of a β emitting radio element depends upon the initial and final spin of the nucleus, the contradiction between half life period and β -ray activity of Potassium and Rubidium is cleared up.

M. P. V.

Distillation with Mercury Vapour.

DISTILLATION with steam is a well-known and important method used widely in the separation and purification of organic compounds. For substances of boiling point 400° or over, however, this method cannot be employed. H. Decker (*Ber.*, 1934, 67, 1636) has now carried out some successful experiments using mercury in the place of steam. He finds that many high boiling compounds go over with mercury nearly

a hundred degrees below their boiling point, the volume of mercury coming over, in the cases investigated being but 1/10 of the distillate. The distillation can be carried out in a perfectly smooth manner. Indigo, chrysen and pyren are among those tried. It should be possible to distil out the high boiling compounds from such substances as resins and pitch, and the various other possibilities of this method have yet to be worked out.

M. A. G.

Action of Water on the Latent Photographic Image.

FORMATION of a flat image, if considerable time elapses between the exposure and development of photographic emulsion kept in a warm humid climate is a common experience. Howard James and co-workers (*J. Phys. Chem.*, 1934, 38, 1211) have made systematic investigations on the rôle played by water in the above effect. If Azo emulsion is exposed to light so as to produce a considerable developable density, the latent image is either reduced or completely destroyed by the action of water vapour for several hours. Bromide emulsion requires longer treatment with water vapour. The dried emulsion gives a satisfactory print on second exposure. Formic acid, acetic acid vapours and liquid ethylene glycol are even more effective than water vapour. Ether, ethonol (absolute), carbon tetrachloride, carbon disulphide, benzene and nitro-benzene are however without any appreciable effect, in the vapour phase. The softening action common to the first group of substances on gelatin, probably facilitates the reversal of the exposed grain.

According to photographic theory, exposure of the emulsion to light results in the liberation of equivalent quantities of silver and halogen and the latter taken up by "the halogen acceptor". Gelatin in gelatin emulsion usually plays the rôle of the halogen acceptor. A more powerful halogen acceptor, like silver nitrate in the gelatin emulsion, will prevent the action of water vapour on the reversal. Complete destruction of the image is not possible unless the light exposure is small, and the amount of residual density increases with the period of exposure. On strong exposure some of the halogen is probably removed from the sphere of action and consequently the image cannot be completely destroyed.

Water vapour has also sensitising action on the emulsion. The unexposed emulsion on treatment with water vapour and drying acquires increased sensitivity. Water vapour treatment over several days produces fog which resembles very closely that due to age (age mottle).

K. S. R.

The Cotton Wilt Disease in Bombay.

A full and connected account of the ten year research work on the Wilt Disease of Cotton carried out as one of the research schemes financed by the Indian Central Committee by Mr. G. S. Kulkarni and his assistants on the Dharwar Experiment Station appears in the *Indian Journal of Agricultural Science*, Vol. 4, Part VI. The studies have related to practically every relevant factor but from the point of view of controlling the disease they have yielded no useful results. In the last resort the breeding of resistant types appears to be the most promising line of work. Resistant types evolved by selection alone proved low yielders and therefore commercially of little value. The need for combining resistance with high yield by cross-breeding and also of a study of the different physiological strains of the fungus itself together with the reaction of the types of plants evolved to each of these strains is indicated. The observation that soil temperatures between 20° C. and 27° C. constitute the optimum for the development of the disease is made use of in a technique for testing types for wilt resistance rapidly. Provincial research to evolve types suited to the different important cotton tracts appears to be the only means of solving the difficulty.

The Farm Cart with Pneumatic Dunlop Tyres.

RESULTS of comparative tests regarding the performance of farm carts fitted with pneumatic tyres as against the ordinary steel tyred carts are summarised in the *Journal of Agriculture and Live-stock in India*, Vol. 5, Part 1. The summary relates to trials conducted in Lyallpur in the Punjab and at the Agricultural College Farm, Poona. The draft with the rubber-tyred cart is greatly reduced, the reduction varying from 26 to 39 per cent. depending upon the load carried and the kind of road traversed. With a draft ranging between 200 lbs. and 250 lbs. the country cart pulls a load of one

ton while the rubber-tyred cart pulls about two tons. Tests relating to durability are still wanting and the relative costs for the new equipment over a reasonably long working period have still to be worked out.

Production of Canesugar in India in 1933-34.

THE SUGAR Technologist to the Government of India gives an account of the progress of sugar making in India during 1933-34 in the *Journal of Agriculture and Live-stock in India*, Vol. 5, Part I. The number of factories that worked during the year was 112 as against 57 during the previous year, a truly astonishing advance due certainly to the protection which the sugar industry now receives. About 15% of the factories worked between 150 and 174 days, 20% worked between 125 and 149 days, about 21% worked between 100 and 125 days and 25% worked between 75 and 100 days. Only one factory worked up to 200 days and one likewise over 200 days. The working season of the remainder was under 75 days. The maximum daily crushing for any factory was 1,604 tons of cane, while the lowest was only 15 tons. The outturn of sugar per cent. cane crushed taking all-India figures advanced only slightly over those of previous years, being only 8.80, which compared with the 11 to 12% outturns of Java should be deemed to be low. The damage due to frost, floods and insect pests, and the disastrous earthquake which occurred in Bihar, were responsible for the low outturn and the shortened crushing season of many factories. The Technologist strongly stresses the importance of well-qualified technical experts in the factories to ensure efficient working. The total production of sugar for the year was 453,965 tons as against 290,177 tons in 1932-33.

The Utilisation of Cane Molasses as Cattle Feed.

LABH SINGH records some further observations on these trials in the *Journal of Agriculture and Live-stock in India*, Vol. 5, Part I. These further trials disclose the rather important result that cattle fed on molasses in the summer months are injuriously affected. It is recommended that molasses feeding be confined to the winter months and up to a quantity of 2 lbs. per animal.

The Pollination of the Apple.

VERY interesting observations of much practical importance made as the result of a four years' study are recorded in *Bulletin* No. 162—New Series, of the Canadian Department of Agriculture. All varieties of apples produce better crops when cross-pollinated with another suitable variety with the exception of "Baldwin" which is self-fruitful. There is much difference in respect of the suitability of varieties as cross-pollinators, and a list of those suitable and unsuitable among the ordinary commercial varieties is given. Among the former are Alexander, Cox's orange, Jonathan, Rome Beauty, while among the latter figure Blenheim, Gravenstein and Ribston. A second point brought out is that insect pollinators are required by all varieties, wind pollination alone giving unsatisfactory results. The need for the provision of colonies of bees in apple orchards is emphasised.

Entamoeba Kamala n. sp.

UNDER the title "Etiology of Enzootic Bovine Hæmaturia" Captain S. C. A. Datta, B.Sc., M.R.C.V.S., has contributed a very valuable article to the *Indian Journal of Veterinary Science and Animal Husbandry*, Volume IV, Part IV. Although Bovine Hæmaturia has been known to occur in countries as widely separated as Australia, Great Britain, and parts of Europe and America and India, yet very little is definitely known as to its nature and cause. In this article which is profusely illustrated with plates definite evidence has been furnished to prove that it is a parasitic disease due to a large protozoan organism which seems to belong to *Phylum Rhizopoda*. It is similar to but larger than *Entamoeba histolytica* and affects bovines. The author proposes therefore the name *Entamoeba Kamala* for this new species of parasite. As this knowledge is bound to be of great value in the control of the scourge, Captain Datta deserves the gratitude of the Veterinary Profession and the stock-owning public. His further notes on the subject will be keenly awaited.

S. D. A.

Effect of X-Rays on Chromosomes.

C. L. HUSKINS AND A. W. S. HUNTER (*P.R.S. B*, 1935, 117, No. 802) have described a few examples of breaks and translocations of

either whole chromosomes or of their constituent chromatids caused by X-irradiation in the nuclei of the microspores of *Trillium*. Lateral translocations of fragments on to a broken chromatid were found with a high frequency in the first mitotic prophase after irradiation. The six anthers of a bud in this species are usually at the same stage of mitotic cycle and thus it was possible to determine with accuracy the exact stage of division at the time of irradiation, by immediately fixing one of the anthers. Apparent constrictions of chromosomes were found upon destaining to be merely chromatid breaks. Cases of *de novo* origin of trabents and an example of ring-formation have been recorded. The "delayed action" of X-rays, so commonly observed by several authors to become apparent first in the anaphase of the first division following irradiation, is explained on the basis that chromatid breaks will not be obvious before anaphases in chromosomes which are not stained to show the internal structure and that they may, owing to the surrounding matrix which holds them together, not be seen until the next division. The arguments of Darlington, and Mather and Stone on the time of chromosome splitting are critically analysed and the conclusion is reached that the somatic chromosomes of *Trillium* microsporocytes, including the "attachment constriction" are longitudinally double at all stages except just prior to anaphase separation when they are 4-partite.

Somatic Synapsis in *Chironomus*.

R. L. KING AND H. W. BEAMS have described (*Journal of Morphology*, 1934, 56, 527) the somatic chromosomes of *Chironomus*. The diploid number is 8. In the spireme nuclei of Salivary glands they find four segments, each representing a pair of homologous chromosomes in intimate somatic synapsis. Each pair could be recognised by its characteristic distribution of chromatin discs. The somatic synapsis in *Chironomus* is not a simple approximation of homologous chromosomes, but can be compared to meiotic synapsis.

Development of *Cheyletus eruditus*.

OUR knowledge of the developmental history of *Acarina* is very meagre and the interest-

ing article by H. A. Hafiz [*Proc. Roy. Soc. Lond. (B)*, 803, 1935] is certainly welcome. Earlier investigators like Kramer, Claparède, Michael, Neustead and Duval have restricted themselves to the description of a few stages. The cellular development in a parthenogenetic individual like *Cheyletus* clears some very abstruse points in the organogeny of the mites. He has studied from the blastoderm formation (which takes place from 1-4 hours) up to the final stage, i.e., the emergence of the hexapod larva (92-96 hrs.). A single layer of blastodermal cells is formed; this differentiates itself into a median and two lateral plates. The ventral plate elaborates endoderm cells; the middle plate gives rise to mesoderm cells. Five pairs of thickenings form the larval appendages. The absence of anus is characteristic of not only the adult but also the embryonic stages. "Salivary gland" cells arise in association with the trachæ.

Diorite-Limestone Reaction, a Study in Contamination.

IN the current issue of the *Geological Magazine* (March 1935, No. 849) Miss Joplin of Cambridge has contributed a very instructive article on the reaction between diorite and limestone and consequent contamination. The area studied is situated in New South Wales and is made up of shales, quartzites and limestones. The limestones have been invaded by a series of tongues of diorite, and these have been contaminated by the assimilation of lime and has given rise to definite and sharply marked off mineral assemblages. This reaction has produced well-marked zones which can be differentiated into diorite core, zone of turbid felspar, clinozoisite zone and garnet zone. By a detailed study of the chemical analyses of these different rocks, she has been able to deduce certain physical conditions controlling contamination. By comparing these deductions with the well-known works of Eskola, Read and Tilley she concludes that the degree of concentration of the foreign material is the most important factor in contamination and that assimilation takes place at a low temperature in the presence of abundant volatiles of which water is the most important.