

Research Notes.

Almost Periodic Functions in a Group I.

THE theory of almost periodic functions built up by Bohr, Wiener, Besicovich and others has been extended in a remarkable manner by Neumann (*Trans. Am. Math. Soc.* 36, No. 3). The theory of representations of continuous groups developed chiefly by Weyl (for an account of the theory see Weyl 'Group Theory and Quantum Mechanics') was shewn to include the theory of periodic functions as a special case. The determination of a canonical system of representations by means of which every representation of the group can be expressed corresponds with the problem of determination of a set of orthogonal functions by means of which every orthogonal function can be expressed. The notion of almost periodicity which was defined by Bohr for the case of functions $f(x)$ which are defined in $-\infty < x < \infty$ and which are continuous has now been extended by Neumann to the case of functions defined in any group G of an extremely general type. The definition he gives is this. Let $f(x)$ be a function defined in G (x denotes an element of the group). Next let M be the aggregate of all functions $f_a(x) = f(xa)$, where a denotes the various elements of the group. Now given any sequence of functions $f_1, f_2, \dots, f_n, \dots$ of M if we can find a subsequence $f_{\lambda_1}, f_{\lambda_2}, \dots, f_{\lambda_r}, \dots$ such that the least upper bound of $|f_{\lambda_\nu} - f_{\lambda_\mu}| \rightarrow 0$ as ν and μ tend to ∞ then the function is said to be right almost periodic. In a similar manner the left almost periodicity is given.

The chief difficulty that was encountered was to find the analogue of the Bohr-mean

i.e., $\text{Lt.}_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T f(x) dx$ as the groups con-

sidered are not assumed to be topological. Neumann overcomes this difficulty by defining a new mean in Part I. Neumann uses his results to deduce fundamental theorems concerning the representation of groups, which were given in case of various special groups by Weyl and Haar. It is also shewn that this theory of almost periodic functions is the widest range over which this theory of representations holds without any loss of generality. Part IV deals with the relations of this theory when the group has a topological structure and

with the determination of all almost periodic functions belonging to the group. In Part V it is proved that the maximal amount exists in abelian groups (subject to certain topological restrictions). We await with great interest his further memoirs on the subject.

K. V. I.

Asymptotic Partition Formulae III.

PARTITION into k^{th} powers by Maitland Right. *Acta Mathematica*, 63, pp. 113-91. Hardy and Ramanujan have determined the order of $p(n)$, i.e., the number of partitions of n by means of the celebrated method of approximating the contour integral for $p(n)$. The author here proposes to determine the orders of $p_k(n)$, i.e., the number of partitions of n into k^{th} powers. Hardy and Ramanujan had to deal with the singularities of the elliptic modular function and the known transformation theory of the function was of great help in finding out the nature of the function near a boundary point. In the case considered by the author the generating function did not possess any general transformation theory and therefore a good deal of his paper is devoted to the development of a transformation theory of the corresponding generating function. He obtains a formula for $p_k(n)$ which generalises the Hardy-Ramanujan conjecture, viz., $p_k(n) = B_0 n^{b-3/2} e^{\Delta n^b}$ and also gives us the order of the error.

K. V. I.

Equilateral and Equiangular Hexagons in Space (The Carbon Six Ring).

K. R. GUNJIKAR published about a year ago in the *Journal of the University of Bombay* an investigation of the construction of equilateral and equiangular polygons in space. He only considered those cases for which $n \leq 6$ where n is the number of the sides of the polygon. It is only when $n=6$ there is any mathematical interest in the problem. However the case $n=6$ is of special interest to the organic chemist. The investigation of the same problem has been recently published by P. C. Henriquez in the *Proceedings of Koninklijke Akademie van Wetenschappen te Amsterdam* on the very same lines of Gunjekar without noticing

his work. We will, however, give here the solution of the problem as given by Gunjkar. Let ABCDEF be the polygon, θ denote the complement of the angle between two adjacent sides and let α, β, γ be the angles made by the planes ABC, CDE and EFA with the plane ACE. (i) If $\alpha = \beta = \gamma$, α is imaginary for $\theta < 60^\circ$, there is one hexagon for each θ if θ lies between 60° and 120° and two such hexagons for each θ if $\theta \geq 120^\circ$. (ii) If α, β, γ are not all equal, then α and β can be determined satisfying certain conditions, if γ is known. This is really a very interesting result. For real values of $\alpha, \beta, \gamma, \theta$ should be $\geq 60^\circ$. If θ lies between 60° and 120° , γ can vary between $\pm \cos^{-1} \left[\frac{4 \cos \theta - 1}{\sqrt{3}} \cot \frac{\theta}{2} \right]$ and if θ is greater than 120° , γ can vary between $\pm \cos^{-1} \left(\sqrt{3} \cot \frac{\theta}{2} \right)$. Henriquez has given in his paper the above results of Gunjkar. The solutions (1) correspond to the 'non-deformable' ring and the solutions (2) correspond to the deformable ring. Henriquez has also given in his paper a discussion of the double six ring (two six rings with one side common) which is new.

N. S. N.

The Structure of Crystals.

THE recent issue of *Zeitschrift für Kristallographie* is devoted to papers on crystal structure. The subject has become, of late, a controversial one. The controversy lies in the understanding of the nature of real crystals occurring in nature. For example, if a real crystal possesses a perfect lattice so that the crystal planes responsible for the X-ray reflection are all parallel, then the reflection of a parallel incident beam of X-rays should occur only within a few seconds of the arc in the neighbourhood of the Bragg angle. Actually it is not so. To explain this discrepancy, Darwin introduced the idea of a mosaic structure with the assumption that the crystal planes which should be strictly parallel in an ideal crystal are not so in the case of a real crystal.

Later on, other mosaic theories were formulated by Smekal and Zwicky. According to Smekal's theory, a real crystal contains many pores and cracks running throughout the crystal in a manner such that the total volume of the pores and cracks is by far negligible compared to the volume of the

crystal. According to Zwicky's first theory cracks will develop in a growing real crystal due to the contraction of the surfaces which are shown to bear great stresses. According to his second theory, a real crystal is a space lattice with a secondary structure throughout the volume. According to Zwicky, the crystal with the secondary structure is more stable than the perfect crystal. The *Zeitschrift* contains two of its papers by Buerger. Buerger has levelled a caustic criticism over Zwicky's theory of the secondary structure citing many experimental evidences. Apart from his criticism, he has presented in his other paper an account of his theory of the lineage structure of crystals. According to his theory, a real crystal is a continuous one but branched. Each branch is called a lineage and each lineage is an almost straight line lattice. He shows that his theory is supported by many experimental evidences including the X-ray results. The *Zeitschrift* contains also a paper by Buckley who has criticised the existing mosaic theories. In his paper he has given a brief account of each theory and has pointed out that they need correct interpretations of experimental evidences supported by right theories. He has criticised the Darwin mosaic on the ground that it assumes inhomogeneity in a real crystal just to interpret the X-ray results while there are many more experimental facts to support the idea that a real crystal is a homogeneous one. The *Zeitschrift* contains also many more important papers by Smekal, Goetz, Taylor, James and others.

N. S. N.

Artificial Radioactivity produced by Neutron Bombardment.

IN the *Proceedings of the Royal Society* (1934, 146, 483), E. Fermi and his collaborators have given a collected account of the results obtained by them by bombarding various elements with neutrons. In this process new isotopes of elements have been obtained which do not correspond to known stable isotopes, but are unstable and disintegrate with definite half-value periods. This induced radioactivity was first discovered by Curie and Joliot and has been described in a previous note in this *Journal*. While Curie and Joliot used α -particles as missiles and were thus limited to a study of the light elements only, Fermi and his

co-workers have used neutrons and have been able to observe induced radioactivity in almost all the elements with only a few exceptions. The paper contains a table at the end which gives all their results at a glance. The probable active isotope produced by the neutron bombardment, its half-value period, the intensity of the activity, mean energy of the β -rays emitted and the presence or absence of γ -rays, are all listed. The presence of an active product having a half-value period of 13 minutes in the case of Uranium had been previously interpreted on the assumption that a new element of larger atomic number than 92 was responsible for this particular activity. Another component of half-value period 90 min. is also found to give reactions similar to those of the 13 min. product and both are concluded to be elements of higher atomic number than 92, probably being isotopes of one such element. The other results obtained are (1) that a large percentage of chemical elements can be activated by neutron bombardment; (2) that most of the neutrons that meet the nucleus produce an active atom; (3) that the active product is sometimes an isotope of the bombarded atom, but in other cases its atomic number is less by one or two units (the former alternative was observed with five heavy elements, while the light elements conform to the latter alternative); and (4) that only electrons have been found to be emitted under the radioactivity but no positrons have been detected. These interesting results will no doubt prove valuable for understanding the constitution of atomic nuclei.

The Liquefaction of Helium by an Adiabatic Method.

EXPERIMENTS with liquid helium have led to discoveries of great importance such as supra-conductivity, but so far only four or five laboratories are equipped with the costly apparatus necessary for liquefying helium. Even these use a method that has a very low efficiency, viz., the method which utilises the Joule-Thomson effect. It is well known that an adiabatic expansion method would be very much more efficient, but expansion engines working at such low temperatures have not been so far designed. The piston has to be air-tight and yet move without friction—a process which is only possible when a suitable lubricant can be found. In

the case of air Claude made use of the liquid air itself as the lubricant, but even such an artifice is impossible in the case of liquid helium because it would have very low lubricating properties. But Prof. P. Kapitza has now devised an expansion engine which overcomes these difficulties and can produce liquid helium at 2 litres per hour using $1\frac{1}{2}$ litres of liquid nitrogen per litre of liquid helium for preliminary cooling. The new design is fully described in *Proc. Roy. Soc.* (1924, 147, 189). Compressed helium enters through the inlet tube into a heat exchanger A and is then cooled to 65° K. by passing round a ring-shaped container having liquid nitrogen. It then goes through another heat exchanger B to the expansion engine and thence to a third heat exchanger C and back through B to the compressor. After a few circuits the cooling is such that part of the helium leaves the expansion engine in the liquid state, but the liquid helium is not separated out in this way. Instead, part of the high pressure helium, after passing through B, goes through C where it is cooled by the up-coming helium from the expansion engine, and then passing through a fourth heat exchanger D, it passes through an orifice into the liquefaction vessel where part of it liquefies on account of the Joule-Thomson cooling. The unliquefied gas goes back through D and C and goes back to the compressor. A throttle valve is used between D and the liquefaction vessel from which the liquid helium can be drawn off. Details of the design of the expansion engine and the heat exchangers are given in the paper.

The Statement of the Third Law of Thermodynamics.

ATTEMPTS have been made to make a simple and rigorous statement of the third law of thermodynamics ever since its formulation. One of such statements associates zero entropy with the lowest energy state (lowest quantum state). The criterion is often found to be either unnecessary or insufficient. A crystal of diamond is of zero entropy at the absolute zero of temperature, but is not in the lowest energy state. Crystalline solutions, on the other hand, are presumably in their lowest vibrational levels though the entropy cannot be taken as zero. Another statement of the third law ascribes zero entropy to a perfectly ordered

condition. Perfect order is undoubtedly a sufficient criterion though not a necessary one, as for instance, in the case of mosaic crystals, degenerate gases or electrons in metals obeying Fermi statistics. The principle of the unattainability of the absolute zero suffers from similar defects. Thus a concise statement of the third law has often proved a bit elusive.

The discussions of Eastman and Milner (*J. Chem. Phys.*, 1933, **1**, 411) and of Rodebush (*J. Chem. Phys.*, 1934, **2**, 668) make it clear that a system which is an exception to the third law (e.g., glasses, solid solutions) differs significantly from other phases at the absolute zero only in the fact that the exemplars of the system differ among themselves and comprise of themselves a large number of distinguishable states. It is this indefiniteness, which is imposed by the non-selective character of the process of formation of the system that contributes towards a finite entropy at absolute zero. So it is suggested that a simple, comprehensive and sufficiently restrictive statement of the third law may be made in the form that "The entropy of any phase of sharply specifiable energy is zero at the absolute zero."

K. S. G. D.

Cataphoresis of Proteins.

WORKERS in the field of Colloid Chemistry and allied branches of Science will be deeply interested in the contribution by Kemp and Rideal (*Proc. Roy. Soc.*, 1934, **147A**, 1-21) on the Cataphoresis of Gliadin. They have by employing the microcataphoretic method, studied the mobility of Gliadin adsorbed on a fine suspension of Quartz. The Langmuir concept of adsorption of gases by surfaces has been applied to the solid-liquid interface. The cataphoretic mobility of quartz suspension is found to be a function of the concentration of the protein. The rate of adsorption of Gliadin by quartz is found to vary with the sign and magnitude of the charge on the protein, which in turn is governed by the pH. The charge on the quartz surface, however, is shown to depend upon the ion environment and not on pH. The effect of strong electrolytes on the mobility of Gliadin adsorbed on quartz has been studied from the theoretical and practical standpoint. It has been shown that the isoelectric point of proteins is profoundly influenced by the ionic strength of the medium. The acidic and basic dissociation

constants have been determined by potentiometric titrations and the isoelectric point has been evaluated. The Debye-Huckel expression for cataphoretic migration has been found to hold good for not too high nor too low ionic strengths. The deviation from the formula, for the Cataphoretic velocity of Gliadin at higher ionic concentrations (using Acetate buffers) has been shown to be due to the adsorption of acetate ions. At low ionic concentrations, Donnan equilibrium between the protein particles and the intermicellary liquid results in an unequal hydrogen-ion activity between the two phases, which in turn brings about a change in the surface charge of the proteins. The importance of these considerations in the interpretation of Cataphoretic data can thus be easily seen.

M. P. V.

Investigations on the Nature of Hæmopoietin, the Antianæmic in Hog's Stomach.

L. KLEIN AND J. F. WILKINSON (*Biochem. J.*, **28**, 1684) in continuation of their work on the preparation of concentrates of hæmopoietin from the press juice of hog's stomach (*Biochem. J.*, **27**, 600), have come to several interesting conclusions with regard to the nature of the antianæmic factors present in stomach and liver. It is found that when concentrates of the hog's stomach extract which contain the antianæmic thermolabile hæmopoietin are incubated *in vitro* with beef muscle, a relatively thermostable hæmopoietically active substance is obtained. The product resembles very closely the active principle present in the liver and it can also be further concentrated into a form suitable for injections.

It is considered that the relationship between the antianæmic principles in stomach and liver is that of an enzyme to the end-product, the necessary substrate being provided by the constituents of beef. The action of this enzyme, hæmopoietin results in the production of an end-product which is also active in producing the red blood-cells. This latter principle is gradually stored up in the liver until it is required by the body. It is shown that this enzyme is different from pepsin because pepsin itself is not only clinically inactive but also cannot act similarly on beef muscle.

The results of Klein and Wilkinson are significant because they reveal that it is the stomach which is really the seat of the

production of the active hæmopoietic substances, the liver merely functioning as a storehouse. This observation is in conformity with the findings of Bence (*Wien Med. Woch.*, 2, 1055) that if the stomachs of pigs were completely removed and the animals killed several months later, the antianæmic principle could not be found in the livers. In view of these findings the liver therapy in the remission of pernicious anemia might be considerably altered.

H. B. S.

Supplemental Light and Blooming in Tropical Plants.

THE effect of additional day length produced by artificial illumination during the winter months upon 100 species of green-house grown (chiefly annual) plants is reported by Francis Ramaley (*The Botanical Gazette*, 1934, 96, No. 1). Among the plants which were not affected in their blooming or which were actually retarded by increased day length there is a rather large proportion of tropical species; while very few, perhaps not any, tropical species are much hastened in blooming by increased length of light exposure. Of the plants hastened in blooming by supplemental light, most are natives of the temperate zone.

Standardization of Index Liquids.

THE identification of minerals under the microscope by making use of index liquids is becoming more and more common both among mineralogists and chemists. The liquids to be used should be colourless, chemically stable and should have as low a volatility as possible. Many mineralogists have been deterred from attempting to prepare their own set of liquids by the impression that it requires an elaborate technique to fulfil the above requirements. But J. J. Glass (*American Mineralogist*, 19, No. 10, Oct. 1934) has shown that the United States Geological Survey prepare 50 sets of standard liquids every year. Since the determination of a large portion of minerals involves the use of liquids between 1.470 and 1.740, the three components which are made use of to prepare index liquids are Government oil (acid free, colourless, tasteless and odourless oil), monochlor-naphthalene and methylene iodide. The properties of these liquids under varying conditions of temperature have been tested

and it has been shown that they maintain a constant index. Since these liquids are miscible with one another, they can be conveniently used for determining the index range from 1.470 to 1.740. In further discussing the standardization of these liquids, he has shown that by using the prism designed by C. S. Ross, a chart can be prepared showing the relationship of angle of minimum deviation and the desired index of refraction. Since some of the liquids that are used at the present time are likely to change their properties including refractive index, when exposed to light and air, the suggestion of J. J. Glass might be usefully tried in geological laboratories.

Differentiation in Basalt Lava.

A DETAILED petrological study of plateau lavas of Antrim by S. I. Tomkeieff (*Geological Magazine*, No. 845, Nov. 1934) has revealed that the vast area of basalt covering nearly 1,550 square miles can be divided into two sets—the lower and the upper—separated by an inter-basaltic bauxito-lateritic zone. The lower layer is made up of olivine basalt (dolerite), a typical representative of hebridian plateau magma type. The upper layer is made up of two contrasted types, viz., the Tholeiitic type (Non-porphyrific central magma type) and olivine basalt (dolerite) same as the lower lavas. These lavas are packed with zeolites and the most prominent of them are thomsonite, chabazite, levynite and gmelinite. The rocks have been analysed which show a gradual increase in the Niggli values of fm, mg, and k downwards and a decrease in al, c, alk. From this important observation Tomkeieff has been able to show that there was gravitational sinking of olivine without remelting. This is further supported by the occurrence of idiomorphic grains of olivine in the lower zone as contrasted with the allotriomorphic relationship of this mineral with the felspar in higher zones. He has further shown that this sinking of olivine was accompanied by rise of volatiles which is responsible for the formation of abundant zeolites. Since a great portion of the basalt is of the vesicular variety full of zeolites, he has suggested that the lava contained a considerable quantity of water.

Asphyxiation of Air-breathing Fishes of Bengal.

THE last number of the *Journal of the Asiatic Society of Bengal* (October 1934, 29, No. 4, pp. 327-332) contains an account of "An Experimental Study of the Asphyxiation of some Air-breathing Fishes of Bengal" by the late Dr. Ekendranath Ghosh. The air-breathing fishes of India, such as *Amphipneustes*, *Clarias batrachus*, *Heteropneustes fossilis*, *Ophicephalus* spp., *Anabas testudeneus*, *Fluta alba*, etc., etc., have, since a very early time, been the subject of considerable biological, experimental and morphological investigation. Usually a small vessel was used for experimental work, but Dr. Ghosh used a large tank for his experiments and thus reduced considerably the chances of the water becoming foul. He found that in his experiments most of the fishes (*Anguilla anguilla*, *Clarias batrachus*, *Ophicephalus striatus*, *O. punctatus*) survived under water for much longer periods than was hitherto possible, while *Heteropneustes* (= *Saccobranthus*) *fossilis*, *Mastacembelus pancalus*, *M. armatus* and *Rhynchobdella aculeata* could not be 'drowned'. In view of these results the author concludes that the "fish in the earlier experiments were asphyxiated as a result of insufficiency of normal water rather than for want of free air for aerial respiration". Dr. Ghosh has thus thrown considerable light on the bionomics of the air-breathing fishes of India and has shown that in experimental work on the air-breathing fishes all possible sources of errors should be eliminated to obtain satisfactory and reliable data.

S. L. H.

The Structure of the Corpus Luteum in Lower Vertebrates.

J. T. CUNNINGHAM AND W. A. M. SMART have examined the condition of the ovary in the Amphibia and the reptiles after ovulation induced by the injection of anterior extracts (*Proc. Roy. Soc. Lond.*, Nov. 1934, 116, No. 798). It is seen that in *Xenopus*, where alone ovulation occurred after injection, the follicle cells were rapidly absorbed and no corpus luteum was formed. In *Lacerta*, chosen as a type of oviparous lizard, the same result is seen while in the viviparous lizards like *Anguis* and *Zootoca*, a distinct and well-developed corpus luteum is seen, comparable with that in mammals.

The functions of the corpus luteum in various vertebrates are also discussed.

The Brain of *Echidna aculeata*.

A. A. ABBIE has made a signal contribution to our knowledge of the mammalian brain in his work on *Echidna* (*Phil. Trans. Roy. Soc. Lond.*, 1934, 224 B, No. 509). Working under the direction of Prof. A. Kappers, the foremost authority on the structure of the mammalian brain, he has emphasised the primitive characters of the brain of *Echidna* and shown its similarities with its ally, *Ornithorhynchus*. The brain of *Echidna* is typically mammalian but the primitiveness of some of its traits is undoubted. The motor system, the dorsal situation of the nucleus ambiguus, the hypoglossal nucleus and the facial nucleus and the small pyramidal tracts are all primitive features. The hypertrophy of the trigeminal apparatus is a feature which *Echidna* shares with *Ornithorhynchus*. Indeed, in the latter there is a more pronounced development of this system. This has brought about a great expansion of the ventral nuclei of the thalamus. The cochlear connections are not well developed. The cerebellum is very greatly specialised; the cerebral peduncles are large but have no fronto-pontine tracts, a character with which *Echidna* agrees with *Ornithorhynchus* but differs from all other mammals. The presence of a temporo-trigeminal tract is peculiar to *Echidna*. Another primitive trait is the rudimentary fornix-mamillare-thalamic system. The epithalamus is well developed and is connected with the pars medius of the ventral nucleus of the thalamus.

Embryology of a Psocid.

W. FERNANDO in an interesting paper (*Quart. Journ. Micros. Soc.*, 1934, 77, Pt. 1) described certain features in the development of the viviparous insect, *Archipsocus fernandi*, n. sp. The author points out that the yolk cells and the chorion are wanting. The ventral plate resolves itself into a median and two lateral plates. The former gives rise to mesoderm while from the latter the ectoderm is derived. The anterior and posterior rudiments give rise to endoderm. The usual insectan larval envelopes like the amnion and the serosa are formed.