THE OPTICS OF MIRAGES

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1. Introduction

IN describing and interpreting our visual impressions, it is natural to adopt the geometric approach provided by ray-optics, since this is the most familiar and therefore the most easily understood approach. But this does not mean and it would indeed be incorrect to assume that even the most familiar optical effects which we can perceive with the unaided vision can always be explained on the basis of the ideas of geometrical optics. The propagation of light in various circumstances needs in general the concepts of the wave-theory for its correct and complete elucidation. Only in the particular case of a set of waves of constant type and the rays associated with them are the wave-optical and ray-optical descriptions completely equivalent.

The foregoing remarks have a bearing on the phenomena of atmospheric optics, in other words of the effects noticed when distant sources of light are viewed through great depths of air. The atmosphere of the earth is an inhomogeneous medium and though the refractivity and its variations are both small, this is set off by the very long optical paths often involved. Hence, the deformations of the wave-fronts in their passage from the light-source to the observer are by no means negligible and in particular circumstances, their consequences may be very conspicuous. The familiar phenomenon of the twinkling of the stars may be cited as an example. In the case of terrestrial objects, we are concerned with the lower levels of the atmosphere where the refractivity is a maximum and also liable to large variations by reason of a variety of circumstances. Hence, a variety of effects may arise which are readily observable. As remarked above, it would not be correct to assume that these effects can all be understood or explained on the basis of the ideas of geometrical optics. It is, on the other hand, to be anticipated that the aid of the wave-theory would be needed for the purpose.

2. The Origin of Mirages

Mirages constitute a remarkable group of effects arising from variations of the refractive index of the atmosphere. They are generally described as of two kinds, being respectively the so-called superior and inferior types of mirage. The latter are quite common and may be described as the manifestation above the level heated surface of the earth of a reflection of the sky and other elevated objects : the latter appear as inverted images against the background of the reflected sky and thus simulate the reflection of terrestrial objects at the surface of a pool or lake. The superior type of mirage arises when the thermal conditions of the atmosphere are the reverse of those which give rise to mirages of the inferior type ; they are observed when the atmosphere rests on a cold level surface above which there lies a hot stratum of air. Objects at or near the level of the cold surface are usually visible to the observer, and in addition inverted images of them are also seen higher up in the atmosphere. Pictures of both types of mirage are to be found reproduced in many treatises on optics and meteorology. A famous example of the superior type of mirage in which objects thus seen reflected were ships at sea was that described and figured by Vince in the year 1799. His drawings are reproduced or referred to in many accounts of the subject. We shall have occasion to refer to them again later in this article.

3. Mirages and Ray-Optics

The theory of mirages usually accepted purports to base itself on geometrical optics. This explanation is said to have been first put forward by Monge, but it was later elaborated and discussed by many other authors. The interested reader will find a very full review of the literature covering 80 pages in the Second Edition (1922) of Pernter's treatise on meteorological optics as revised by Exner. The explanations usually given are illustrated by drawings which show the curved path of the rays from the source which reach the observer and are perceived by him as a reflected image of the source. These drawings seem very plausible, but when one examines them with care, it becomes evident that the explanations put forward do not really come to grips with the problem and may indeed be described as a kind of make-believe. That a geometric theory is fundamentally incapable of explaining the phenomenon of the mirage becomes clear on a critical examination of the subject.

The circumstances in which mirages are observed bear a superficial resemblance to those in which the familiar phenomenon of total reflection occurs, *viz.*, light travelling in a medium of higher refractive index meets a medium of lower index at an incidence exceeding the critical angle and is then turned back. There is, however, a fundamental difference between the problem of the mirage and the circumstances of total reflection referred to above. We do not have in the atmosphere anything in the nature of a discontinuous change of refractive index; what we are actually concerned with is a progressive change of index. In the latter circumstances, a pencil of rays travelling obliquely through the stratified medium would, according to Snell's law of refraction, be progressively deviated until it reaches a layer at which its course becomes tangential to the plane of the stratifications; thereafter, it would continue on a course parallel to the stratifications. No question of total reflection can therefore arise.

4. Mirages and Wave-Optics

If it be assumed that the wave-optical and ray-optical descriptions of the behaviour of light are completely equivalent, it would follow that the rays and wave-fronts in an isotropic but inhomogenecus medium are everywhere normal to each other. Accordingly, if the course of the rays is known, the wave-fronts form a set of surfaces cutting them orthogonally. It was stated above as a consequence of Snell's law of refraction that a ray of light initially making an angle with the plane of the stratifications would be progressively deviated from its course until it becomes tangential to that plane and would then continue on a course parallel to the stratifications. This would happen to every one of the rays of an incident beam of light. Accordingly, if we take two adjacent rays at a finite distance apart, the part of the wave-front between them would swing round and at the same time contract in its extension and ultimately become a point which moves parallel to the stratifications in a plane whose position can be specified exactly for the particular circumstances of the case.

An approach to the problem of the mirage based on the ideas of geometrical optics thus leads us to conclude that the energy of the incident radiation would be concentrated at a limiting plane which it reaches but is unable to penetrate. The intensity of illumination in that plane would therefore be infinite. Since such a result is physically inadmissible, it follows that the approach which leads to it should be given up in favour of a different and more rigorous treatment based on the first principles of the wave-theory.

5. The Analytical Theory and Its Results

To make the problem tractable, it has to be idealised to some extent. We consider the optical behaviour of a slab of the medium which is assumed to be of finite thickness and bounded by plane parallel faces extending to infinity. The material is also assumed to be stratified in planes parallel to its faces, the refractive index being μ_1 at the front surface and μ_2 at the rear, μ_1 being greater than μ_2. Plane waves of light are assumed to be incident on the slab at a glancing angle ϕ_1. It is clear that if $\mu_1 \cos \phi_1$ is greater than μ_2, the incident waves cannot emerge from the rear face of the slab. We are interested in ascertaining the nature of the disturbance both within the slab and in front of it in the circumstances stated.

The solution of the problem is contained in a paper by S. Pancharatnam and the present author published in the *Proceedings of the Indian Academy of Sciences* for May 1959. We must be content here with a brief statement of the results. The paper quoted also contains a full account of the results of experimental studies on the subject. This will be drawn upon very freely in the latter half of the present article. It should be mentioned that the experiments, besides confirming the results of the analysis have also revealed other unsuspected features which assist in the elucidation of the phenomena actually noticed in mirages on a large scale in the open-air. The illustrations accompanying the present article, Figs. 1, 2 and 3 are also taken from the paper under reference.

Briefly stated, the analysis indicates that when $\mu_1 \cos \phi_1$ is greater than μ_2, the light incident on the front face of the slab is returned from its interior with full intensity and in a direction justifying its description as a regularly reflected disturbance. The analysis also shows that within the slab, the intensity of illumination attains a large value in the vicinity of the limiting plane at which the refractive index μ_L has the value $\mu_1 \cos \phi_1$. The intensity falls off rapidly to small values in the rear of that plane, while in front of it, the intensity diminishes gradually at the same time exhibiting a series of maxima and minima of which the separation falls off progressively, finally approaching a constant value.

The situation described above closely resembles the effects well known to all students of optics which are observed in the vicinity of *caustics*. Accordingly, we may in the present case state that the incident and reflected

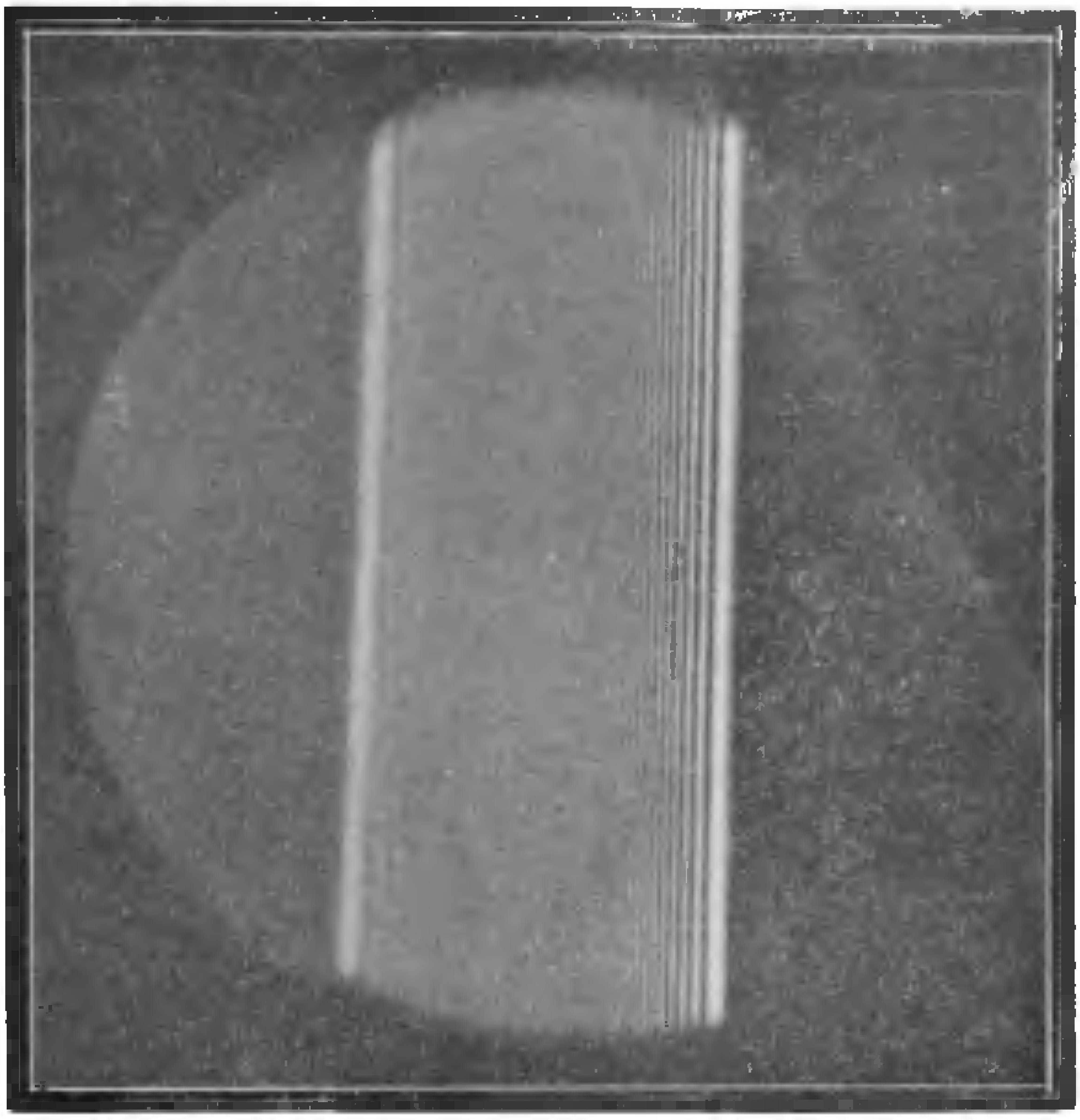

FIG. 1

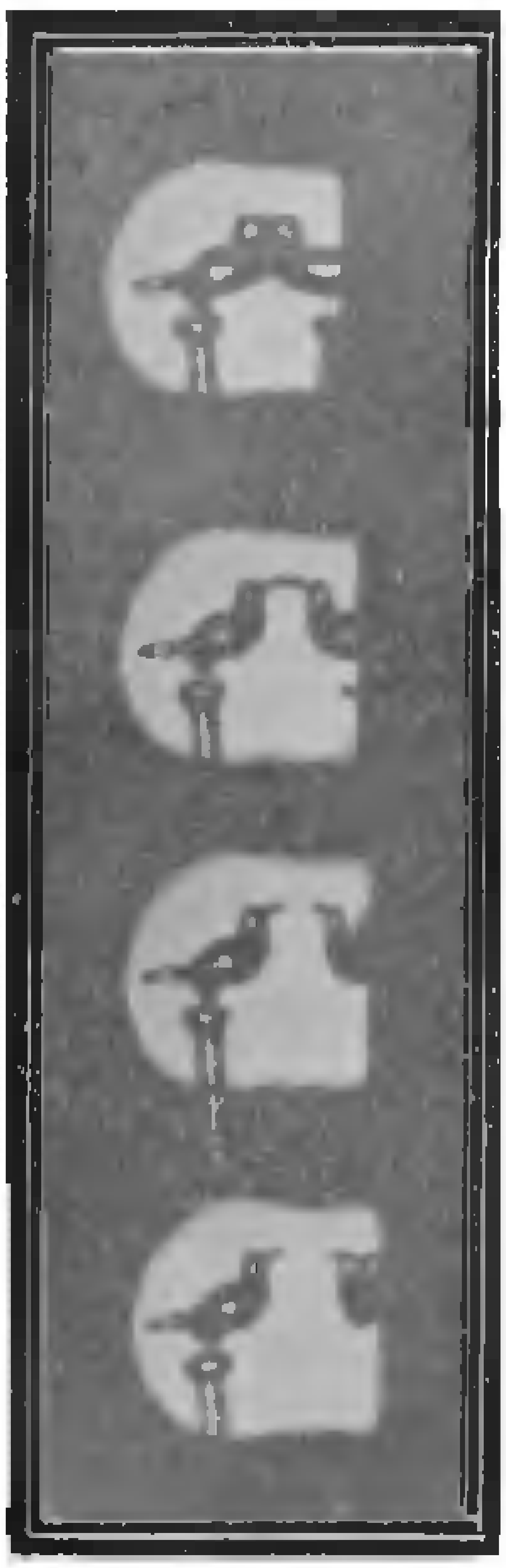

FIG. 2

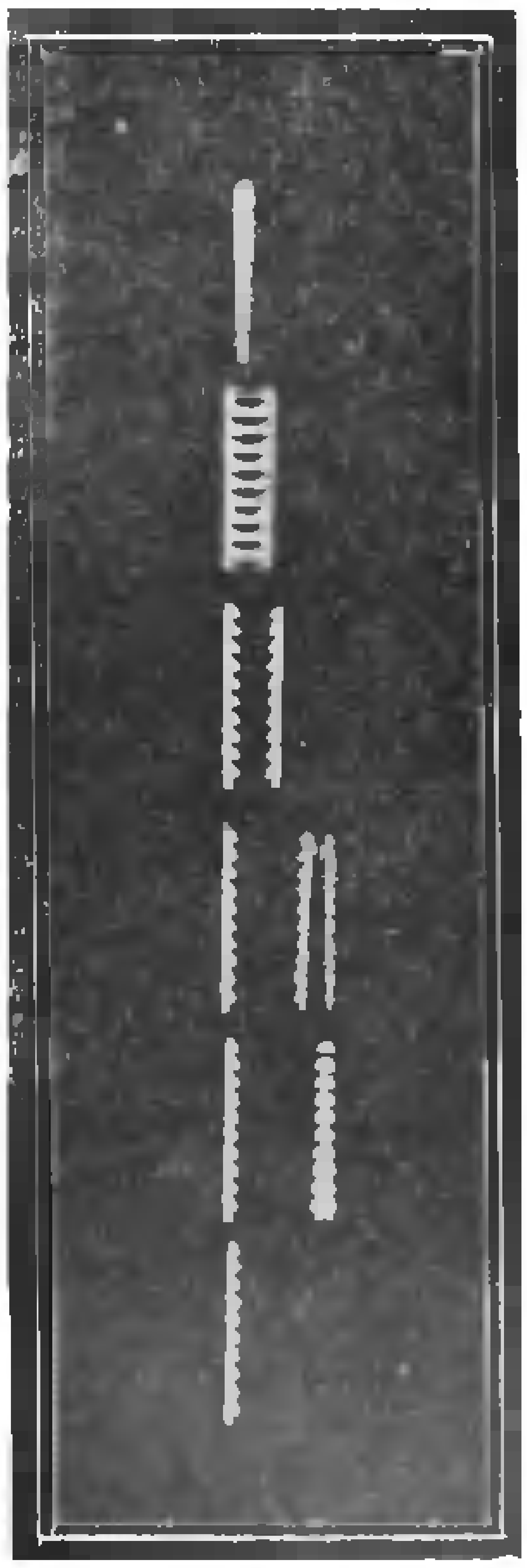

FIG. 3

disturbances join up and form a cusped wave-front; the cusp rests upon the limiting plane referred to above and the entire wave-front moves along that plane in a direction parallel to the plane of incidence. At and near the cusp, the two branches of the wave-front are sensibly parallel to each other, besides being normal to the plane of the stratifications. Further out, however, they separate and diverge, ultimately becoming normal to the incident and reflected rays in the sense of geometric optics. The progressive diminution in the spacing of the successive maxima of illumination is readily understood on this basis.

6. Observation of the Caustic and Accompanying Interferences

Mirages can be produced and observed on a small scale in the laboratory as has been shown by R. W. Wood and others. In the arrangements generally used, the hot plate above which the mirages are observed is held horizontally. Though in some respects this arrangement is very convenient, it is not useful for critical studies owing to the thermal instability of the air above a heated plate. This difficulty is minimised by holding the heated plate edgewise so that its surface is vertical while the length remains horizontal. The object studied is an illuminated slit kept at some distance from the heated plate parallel to its surface. The light diverging from the slit is rendered parallel by a collimating lens and then allowed to fall obliquely on the heated plate. The beam is allowed to cover the whole length of the plate, the angle of incidence being adjustable by a lateral movement of the illuminated slit.

Photographs of the field of illumination in the rear of the hot plate could be secured with these arrangements and with very short exposures, provided that sunlight is used to illuminate the slit. With such short exposures the effects of thermal instability are eliminated completely. A typical example of a photograph thus obtained is reproduced in Fig. 1 accompanying. The use of a red filter to monochromatise the light greatly improves the results obtained. It will be seen that the features indicated by the theory and other features as well are very clearly exhibited.

The field of illumination appearing in Fig. 1 consists of three parts. To the right of the bright caustic (*i.e.*, towards the heated surface) the field is dark, while to its left lies an illumined strip containing a large number of interference fringes whose separation narrows down to a constant value as we move away from the caustic; since the heated plate is necessarily of finite extension, the reflected part of the cusped wave-front does not extend to infinity but is terminated. This manifests itself in the field of view by the occurrence of a second edge to the left of which the intensity is considerably less (though not zero), the edge being bordered by some broad fringes.

7. Relation of the Mirage to the Cusped Wave-Front

The actual mirage is observed when the eye is kept at any point which lies on the bright strip of light lying to the left of the caustic, the eye being focussed on the plane containing the object, *i.e.*, at infinity. It is to be expected that two images would then be seen whose positions lie respectively along the directions of the normals drawn from the eye to the two branches of the cusped wave-front leaving the nearer edge of the plate. In fact, the fringes observed to the left of the caustic in Fig. 1 may be regarded as due to the interference between the light from two such virtual sources, the progressive narrowing of the spacing of the fringes to the left of the caustic corresponding to the increasing separation of the sources. That the separation of the two images observed depends on the position of the eye or aperture through which the phenomenon is viewed is illustrated in Fig. 3. In order to make the nature of the image evident, the serrated edge of a hacksaw blade has been used to form one of the edges of the slit. An aperture was kept before the camera lens and the succession of photographs exhibit the alteration in the phenomena as the aperture is gradually moved to the left. When the aperture is on the bright caustic, a single image is seen, while as it is moved to the left this separates into two images, one of which is a direct (or more properly, a refracted) image, the second being an inverted reflected image, the separation between the two gradually increasing. A remarkable feature of the sequence of phenomena illustrated in Fig. 3 is the occurrence of a third erect image close to the reflected image in the fourth and fifth photographs of the sequence; this image starts developing when the aperture has been moved towards the outer edge of the central illumined strip (where broad fringes start appearing in Fig. 1) and becomes coincident with the reflected image when the aperture is exactly at the edge mentioned. As the aperture is moved further left, only the 'direct' image continues to be visible, as is shown in the last photograph of the sequence; this is to be expected since the reflected part of the cusped wave-front is no longer received through the aperture.

The third image mentioned above could be cut off by inserting an opaque screen near the farther end of the heated plate and adjusting it so that its edge protrudes a little beyond the surface of the plate. This shows that the image is due to the ordinary refraction of rays directly entering the edge of the heated layer at the farther end of the plate. The main features of the path of such rays may be deduced from the experimental observations described in the previous paragraph. The terminus of the reflected part of the cusped wave-front corresponds to certain limiting rays entering the region at the farther end of the plate. Rays which are able to enter the edge of the heated layer at a closer distance to the plate than these limiting rays proceed a longer distance before emerging from the heated stratum and also suffer a larger deviation. These rays give rise to the erect third image ; in fact, the second refracted wave-front, obtained by drawing the surfaces orthogonal to these rays, meets the termination of the refracted part of the wave-front so as to form a second cusp.

Till now we have dealt mainly with the case when the distant object is of negligible angular dimensions. When an object of finite angular dimensions is used, the point on the image which corresponds to any particular point on the object is to be determined as before for each setting of the eye. In this case there will be a distortion of the images because the position of the limiting layer as well as the inclinations of the cusped wave-fronts corresponding to any particular point on the object varies with the position of the object-point. Figure 2 shows the photographs taken using as the object a small model of a bird made of glass. This was placed near the focal point of the collimating lens. The sequence of photographs show the variation in the appearance of the phenomena as the eye is moved away from the plane of the plate. The appearance of a third erect image in addition to the usual reflected image may be discerned in the last two photographs of the sequence. It is worthy of note that a third erect image adjoining the inverted image which is the principal feature of the mirage has actually been noticed by various observers in mirages as seen in a large scale in the open air. It was noticed, for example, by Hiller in his study of mirages of the inferior type produced by a long vertical wall which had been warmed by the sun's rays. Hiller's photograph showing this effect is reproduced in Pernter's treatise already mentioned and also elsewhere. The third erect image is a conspicuous feature in Vince's drawings of ships at sea exhibiting the phenomenon of the superior mirage. It may safely be presumed that the explanation of its appearance is analogous to that of the effect noticed in the laboratory experiments and illustrated in Figs. 2 and 3 above.

COMMONWEALTH EDUCATION CONFERENCE

THE report of the Commonwealth Education Conference, presented at the final session at Oxford on July 28, 1959, adds an important chapter to the history of Commonwealth endeavour and co-operation.

The Conference has helped to carry a step forward the scholarship scheme originated by Canada and approved last year at Montreal. As a result not only will the scheme come into force in the year 1960-61 but the target of 1,000 scholarships may well be exceeded. Of these United Kingdom has offered to provide 500 and Canada 250. In the main the Commonwealth Scholarships will be for post-graduate study or research. A limited number of awards will be made to senior scholars of established reputation and achievement and called Commonwealth Visiting Fellowships.

The report points out that over the first five years Commonwealth Governments will spend at least £10,000,000 in addition to their present expenditure, on education.

One of the major problems to be solved is about the acute shortage of trained teachers. While the long-term problem of Teacher Training needs must be solved by the respective countries themselves, the report shows that a number of practical suggestions have been made at the Conference to meet the immediate needs.

On the supply of teachers the report estimates that 500 teachers are wanted immediately for training institutions, well over 1,000 a year for secondary schools, and 200 a year for technical schools. Universities also need staff, often in highly specialized subjects. In this respect the attention of Governments is being drawn to the need for satisfactory arrangements for the reception and welfare of the scholars and teachers on which much of the success of the plan will depend.

The report suggests that funds are to be allocated for teaching English as a second language, and a group of Commonwealth experts will shortly consider the problems involved in teaching this subject.