

## TOPOLOGICAL METHODS IN ANALYSIS

IN his Presidential Address to the Mathematics Section of the 38th Indian Science Congress, Dr. Racine gave a survey of the methods developed by J. Schauder and J. Leray for the boundary value problems associated with elliptic partial differential equations of the second order. The first linear problems to be studied were those of determining harmonic functions satisfying the boundary conditions of a Dirichlet- or Neumann-problem (D- or N-problem). The D-problem in this case may be solved, under very general conditions, by the method of "balayage" due to H. Poincaré. Such a method has not been adapted so far to the general elliptic equation. By constructing appropriate Green's functions, Giraud established the equivalence of the general D- and N-problems to certain integral equations and thus made available the method of potentials and integral equations as in the classical case of the Laplace equation. Giraud also developed a method of successive approximation for treating non-linear problems. Schauder's method of solution of linear elliptic problems is extremely original and deduces an existence-theorem from an "*a priori approximation*" of the solution. Generally it excludes the possibility of any construction and gives only existence, not uniqueness, theorems. He makes use of refined forms of certain well-known inequalities in the theory of Newtonian potentials and a process of successive approximation starting from the solution for the Laplace equation. Schauder's most important contribution to the theory of partial differential equations of mathematical physics is his new method of solving non-linear problems. His first investigations deal with extensions to function-spaces

of the celebrated "fixed-point-theorem" of L. E. J. Brouwer. There exists an equivalence between a given D-problem and the problem of finding the fixed points of the transformation  $u = U(u)$ , where  $U$  is a functional operator which maps a certain function-space  $V$  or a subset of it onto itself. Schauder generalised Brouwer's fixed-point-theorem to topological spaces, but he was obliged to restrict himself to metric vectorial spaces, in particular, Banach spaces. He developed also a second method, based upon his generalisation of Brouwer's theorem on the "invariance of the domain" to Banach spaces in which he defined a weak topology in addition to the strong topology. Leray had utilised a topological invariant: "the degree of a transformation". This notion, due to Brouwer, was generalised to function-spaces by Leray and Schauder, who were thus enabled to develop a general method of proving existence-theorems in the theory of functional equations. The method of Leray and Schauder may be applied to the most general non-linear problems under certain conditions. It has been very successful in the hydrodynamical theories of viscous fluids and of wakes. Recently Leray has developed the theory of a topological invariant attached to the continuous mappings of a topological space into itself. It is valid in compact Hausdorff spaces. His new theory, based essentially on the notion of the "total index" of a transformation with respect to an open set, has enabled him to arrive at a very powerful synthesis of his and Schauder's previous results and to generalise the same considerably.

V. R. T.

## GLEEP

THE Graphite Low Energy Experimental Pile (Gleep) at Harwell, was constructed to meet two main requirements: the first was for a pile able to work at as high a power as it is possible without introducing elaborate cooling arrangements. This high power was needed for the production of the radio-active forms of elements which are proving of immense value in many branches of medical and scientific research: the second was to compare, for scientific purposes, the ways in which different elements absorb slow neutrons, the type of atomic particle emitted by the pile.

This relatively small experimental pile requires 12 tons of uranium and a further 21 tons of oxide of uranium. In addition, the pile contains over 500 tons of graphite in a very pure state. It has an output of 100 kilowatts.

By providing radio-active forms of natural elements for research purposes, the Gleep has already proved immensely valuable. In addition, the experience gained in operating this experimental pile during the last three years has proved an invaluable aid to the design of larger piles suitable for providing energy on an industrial scale.

—By courtesy to B.I.S.