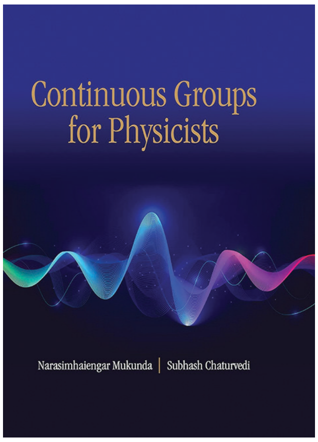

Continuous Groups for Physicists. Narasimhaiengar Mukunda and Subhash Chaturvedi. Cambridge University Press, University Printing House, Cambridge CB2 8BS, UK. 2023. xviii + 280 pages. Price: £49.99.

Group theory is an important part of mathematics. Over the years, it has found applications not only in mathematics but also in physics, chemistry, material science and is even central to public key cryptography. The first ideas of group theory were developed in the 19th century. Group theory has three main historical sources: number theory, the theory of algebraic equations and geometry. Early results in group theory were obtained by Euler, Lagrange, Ruffini and Abel. Galois was the first to employ groups to determine the solvability of polynomial equations.

Sophus Lie laid the foundations of the theory of continuous groups around 1870. Some of his ideas were developed in collaboration with Klein. The initial application that Sophus Lie had in mind was to the theory of differential equations. A major stride in the development was the work of Killing and Cartan, which led to the classification of semi-simple Lie algebras. Over the years, a vast amount of work has been done in the area of group theory, with important contributions from Weyl, Wigner, Bargmann and Harish-Chandra. Group theory describes the symmetries that the laws of physics seem to obey. Some examples of these are the non-compact Lorentz and Poincaré groups. In quantum chemistry, space groups are used to classify crystal structures. Cryptographic protocols use braid groups. The braid group is intimately connected to the Yang–Baxter equation, which was first introduced in statistical mechanics. The braid group is also connected to the knot theory. Group theory has also found important applications in algebraic coding and automata. Remarkably, the problem of 'Hearing the Shape of a Drum' was partially solved using the symmetry group of the Laplacian operator.

In the last 50 years or so, there have been remarkable developments in physics where a major role was played by group theory. One major development in the last fifty years was the development of the standard model of particle physics, where group theory is a key element. The concept of group has been generalized to supergroup. A supergroup is like a Lie group, but where functions may have even and odd parts. A supergroup has a super Lie algebra, which plays a role similar to that of Lie algebra for Lie groups in that they determine most of the representation theory, which is the starting point for their classification. Extending these ideas, the super Poincaré group and super-conformal group have also been discussed.

Perhaps one of the most profound developments in the last fifty years is the application of conformal groups and associated scale and conformal symmetry. The idea of scale transformation and scale invariance is old. In fact, scaling arguments were commonplace for the Pythagorean school, Euclid and Galileo. They became popular again at the end of the 19th century, perhaps the first example being the idea of enhanced viscosity of Reynolds as a way to explain turbulence. In recent years, one major application of these ideas has been the development of the renormalization group. The gist of the renormalization group is that as the scale varies, the theory presents a self-similar replica of itself, and any scale can be accessed from any other scale by group action. It is fair to say that the renormalization group is now one of the most important tools in modern physics. In particle physics, it has led to the concept of asymptotically free theories. Perhaps, the most interesting application of the renormalization group is the deep understanding of the critical phenomena in condensed matter physics.

Other equally important applications of conformal groups are towards the development of conformal field theory in 2 space-time dimensions and anti-de Sitter (ADS)/conformal field theory (CFT) correspondence in higher space-time dimensions. In two dimensions, the transformations of the conformal group are conformal transformations, and there are an infinite number of them. One important application of these is in the development of quantum field theory, which is invariant under conformal transformations. The conformal field theory in two dimensions has found important applications in statistical mechanics, string theory and condensed matter physics. On the other hand, the conformal field theories in higher dimensions have become more popular with the ADS/CFT correspondence. This is a conjectured relationship (duality) between two kinds of physical theories. This duality represents a major advance in understanding string theory and quantum gravity. Much of the usefulness of the duality is because it is a strong–weak duality. This fact has been used to study many aspects of nuclear and condensed matter physics. The CFT in higher dimensions has also helped develop numerical conformal bootstrap techniques.

The book on the continuous groups by Mukunda and Chaturvedi is based on the lectures given by Mukunda at the Institute of Mathematical Sciences, Chennai, in 2007 to post-graduate and doctoral students. As the authors themselves have said, the book is not intended to be a textbook in the traditional sense but a reference book. One of the strong points of the book is the discussion of 'several nonstandard' topics which are not normally found in the textbooks on group theory. The book is divided into 10 chapters. While the first 7 chapters have been devoted to the discussion of several aspects of compact continuous groups, the last 3 chapters discuss some aspects of non-compact groups. Some problems have been given at the end of each chapter. I wish they had included more problems.

Chapter 1 is a good introduction and summary of the basics of finite groups and their representations. The authors take the reader from groups and their examples to homomorphisms and (semi)direct products, representations and their semi-simplicity, characters and their orthogonality. One deviation from the standard notation is the use of 'direct products' and ×, where 'tensor products' and ⊗ are more common/standard.

Chapter 2 offers a good crash course on the permutation group and its sub-groups and then discusses Young tableaux and symmetries. The well-known classes of symmetric functions – monomial, elementary/homogeneous symmetric, power sum and Schur functions – are introduced, and the Murnaghan–Nakayama rule is carefully explained. The authors show the connection of these symmetric functions to the representations of the symmetric group and provide multiple recipes to construct the latter. A small peeve is that at the end of Section 2.2, the authors lead up to the phrase 'Hecke algebra' but do not mention it.

Chapter 3 has a detailed discussion about rotations in 2 and 3 dimensions. Chapter 4 does for Lie algebras what chapter 1 does for finite groups. It introduces the basics, including nilpotent, solvable and (semi)-simple Lie algebras and the adjoint representation. It also relates the 'equivalent' (or at least closely related) notions one encounters while studying Lie groups and their Lie algebras. In particular, the commutators of group elements and vector fields are related via exponentiation. Chapter 5 extends the discussion of chapter 4 to complex Lie Algebras and then discusses in some detail the four infinite classical families. The five exceptional groups are only briefly mentioned. Chapter 6 discusses the 'non-standard' topic of spinor representations of real orthogonal groups in both even and odd dimensions. Chapter 7 discusses the 'non-standard' topics like 'Schwinger representation' of a group with examples, induced representations and systems of generalized coherent states.

The last three chapters have been devoted to discussing a few aspects of non-compact groups. In chapter 8, the authors have discussed the properties and uses of symplectic groups and their metaplectic covering group in a quantum mechanical setting. Chapter 9 discusses the 'non-standard' Wigner theorem on the representation of symmetry operations in quantum mechanics and the Unitary–Antiunitary Theorem. In the last chapter, the authors briefly discuss the Euclidean group $E(3)$, the Galilean group, the homogeneous Lorentz group $SO(3,1)$ and its double cover $SL(2,C)$, and finally, the Poincaré group – and look at their finite-dimensional non-unitary representations. I wish the authors had done more justice to the non-compact groups. I found the omission of any discussion about the conformal group rather glaring, considering the several remarkable developments that have taken place in recent years.

Overall, the book will serve as a good reference book on continuous (Lie) groups for physicists.

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