

Nobel Prize in Physics for 2021

The Nobel Prize in Physics for 2021 was shared by three scientists ‘for groundbreaking contributions to our understanding of complex physical systems’¹. Half of the Prize was awarded to Prof. Giorgio Parisi for ‘the discovery of the interplay of disorder and fluctuations in physical systems from atomic to planetary scales’¹. Parisi was born in Rome in 1948. He received his Ph.D. degree under the guidance of Nicola Cabibbo from University of Rome La Sapienza in 1970 and worked at the Laboratori Nazionali di Frascati before moving to University of Rome Tor Vergata in 1981. He has been a Professor at Sapienza University of Rome since 1992.

Parisi is a highly distinguished theoretical physicist with seminal contributions on a wide variety of subjects. His initial research was in the area of high-energy physics. His most important work in this area with Guido Altarelli in 1977 gave a lucid and simple derivation of the equations for the structure functions of deep inelastic scattering based on the parton model, but consistent with the scaling description of asymptotic freedom. His work on the large N limit of quantum field theories, solvable models at large N and random matrices shaped many important ideas in the field. He also introduced new ideas to solve non-perturbative quantum chromodynamics on a lattice. In the late seventies, Parisi shifted his attention to the physics of disordered systems in which the constituting elements (atoms, molecules, spins, ...) do not form ordered structures². The most famous work of Parisi is on spin glasses – a class of magnetic systems with random interactions that do not exhibit conventionally ordered magnetic phases (in physics, the term ‘glass’ is used to denote systems with disordered structure and slow dynamics). A prototypical example of a spin glass is a dilute alloy with a small percentage of a magnetic material (such as iron or manganese), and a nonmagnetic metallic host (such as copper or silver). The magnetic atoms distributed randomly in the metallic host carry ‘spins’ which may be considered as tiny bar magnets with magnetic moments that can point in different directions³. Due to the randomness in the distances between pairs of spins, the interaction between a pair of spins in a spin glass can be either ‘ferromagnetic’ which tends to make the spins parallel to each other, or ‘antiferromagnetic’ which induces antiparallel align-

ment. This leads to ‘frustration’⁴: it is not possible to simultaneously minimize the interaction energies of all pairs of spins. The presence of strong frustration rules out the occurrence of conventionally ordered magnetic phases. However, experiments on these materials showed strong evidence for the existence of a phase transition from a high-temperature disordered phase to a new phase as the temperature is decreased. This led to many attempts to understand the nature of this low-temperature phase. It was suggested that in the low-temperature phase, individual spins are ‘frozen’ (their time averages are non-zero), but the directions in which they are frozen are random, so that the magnetic moment averaged over all the spins is zero. In contrast, the time average of each spin is zero in the high-temperature phase. Several models for spin glasses were proposed, but theoretical analysis of these models was hampered by the difficulty in treating the randomness of the pair interactions.

In a series of papers published in the late seventies and early eighties, Parisi worked out an exact solution of a spin-glass model with infinite-range interactions. The solution uses the ‘replica method’ in which n identical replicas (copies) of the system are introduced and the random interaction parameters are averaged over, leading to a non-random system with interactions between spins belonging to different replicas. Initial attempts to solve the thermodynamics of this system, assuming that all replicas are equivalent, led to unphysical results. Parisi proposed a novel way of ‘replica symmetry breaking’ and showed that it leads to physical predictions for the properties of the low-temperature spin-glass phase. Perhaps the most interesting aspect of the Parisi solution is that the low-temperature phase is not characterized by a finite number of order parameters, but by an ‘order parameter function’ defined in the interval between 0 and 1. This function is identically zero in the high-temperature phase, but takes a non-trivial form in the low-temperature spin-glass phase. The physical interpretation of this function is based on the existence of a highly complex ‘free energy landscape’ in this system. Due to the presence of random interactions that cause frustration, the free energy of this system, expressed as a function of the local magnetizations of individual spins, has a large number of local minima in the spin-

glass phase. A theoretical treatment of the thermodynamics in this phase must take into account the presence of all these free-energy minima. This, in turn, requires information about how the local magnetizations in different minima differ from one another. Parisi’s order parameter function contains the required information about the distribution of the similarity (‘overlap’) between the configurations of local magnetizations in different free-energy minima. The minima are arranged in a tree-like (‘ultrametric’) structure in which the overlap of the local magnetizations at two minima is determined by how far up the tree one must go to find a common ancestor. The presence of many free-energy minima makes the dynamics very slow at temperatures near and below the spin-glass transition temperature. This explains various phenomena associated with slow dynamics observed in experiments on spin glasses. The predictions of the Parisi solution have been verified in numerous computer simulations and rigorous mathematical analysis. It is a good example of the emergence of a regular structure in the free-energy landscape of a system with strong disorder as well as thermal fluctuations at the microscopic scale.

The work of Parisi on spin glasses paved the way for theoretical and computational studies of a variety of physical systems that exhibit complex free energy or energy landscapes. Structural glasses constitute a class of such systems in which Parisi and co-workers have made path-breaking contributions. Structural glasses (such as window glass) are obtained by cooling or compressing a liquid so rapidly that crystallization is avoided. The constituting particles form a disordered structure, similar to that of a liquid, in the glass phase. A fascinating property of glass formation is that the dynamics of the liquid becomes slow as the glass transition is approached – the viscosity increases by about 14 orders of magnitude as the temperature is decreased by a moderate amount (disorder and slow dynamics are common features of all glassy systems). Numerous studies over decades have attempted to develop a theoretical understanding of these phenomena. It was known from earlier numerical studies that glass-forming materials exhibit a large number of local minima of the energy expressed as a function of the coordinates of the constituting particles. The idea that

the presence of many distinct minima may influence both the thermodynamics and dynamics of glass-forming liquids had long been appreciated. Parisi's work on spin glasses showed the way in which the presence of a complex energy landscape can be taken into account in a theoretical analysis. Motivated by the scheme of replica symmetry breaking in the Parisi solution, several researchers studied other disordered spin models and showed that a class of such models exhibits properties similar to those observed near the glass transition. These models exhibit a first-order phase transition and a simpler form of replica symmetry breaking than that in the Parisi solution. These studies led to the formulation of the 'random first order transition' (RFOT) theory of glass transition. Parisi made many important contributions in the further development of this theoretical description. These include a theory of glass transition based on replica symmetry-breaking in the equations of liquid-state theory and a framework to study the behaviour near the glass transition and in the glass state by including a term in the energy function that tends to localize the particles near their positions in a particular equilibrium configuration. He has also contributed significantly to devising numerical tests of different theories and developing an efficient numerical method for simulating glass-forming systems at low temperatures. More recently, Parisi and co-workers have made important progress in this field: they have shown that the equilibrium and dynamic properties of a system of hard spheres can be obtained exactly in the limit of infinite dimensions. This work also led to a theory of jamming – the process of compressing a collection of hard objects into a state in which their freedom to move is lost. It has been shown that the behaviour observed in experiments and simulations on jamming in two and three dimensions is close to that found in the exact solution in the infinite dimension limit. The exact results derived by Parisi and his collaborators for idealized models (with infinite-range interactions or in infinite dimensions) of spin glass and structural glass provide a concrete basis for considering disordered systems and a theoretical framework for analysing the properties of experimentally studied systems in two and three dimensions. The extent to which the properties of real systems conform to the predictions of the idealized theories is being extensively examined in experiments and computer simulations.

The work of Parisi on systems with complex energy or free-energy landscapes has led to significant progress in several areas of research that fall outside the realm of conventional physics. These include computer science, neuroscience and biology. In the K-SAT problem in computer science, one is concerned with the question of whether M logical clauses, each involving K logical variables taken from a pool of N variables, can be satisfied simultaneously. Analysis of random K-SAT problems using tools developed in the study of spin glasses has led to many important results, such as the occurrence of a 'phase transition' as M is changed for a fixed N . The fraction of unsatisfied clauses increases from zero to non-zero values as M is increased above the value at which the transition occurs. Such analysis has also led to the development of efficient algorithms for checking satisfiability. In neural network models of associative memory, the interactions between pairs of model neurons are chosen to ensure that certain specified network states, representing stored memories, become attractors of the collective dynamics of the neurons. In some of these models, the stored memory states are local minima of a suitably defined energy function. The analogy of these models with glassy systems is obvious. Methods developed in studies of spin glasses have been helpful in answering questions such as the maximum number of memories that can be stored in a network and the typical error in the retrieval of a memory. The protein-folding problem in biology is concerned with the process of folding of a protein from a denatured state to its native state. The folded native state corresponds to the global minimum of the energy function. The question of how a protein can find the global minimum without getting stuck in local minima has puzzled biologists for a long time. Studies making use of theories of glassy dynamics have provided key insights into this question – it has been found that the energy landscape of proteins existing in nature has a 'folding funnel' that guides the protein to its native state. The notion of a rugged fitness landscape occurs in theories of evolution. These examples illustrate how the work of Parisi on glassy systems has shaped research in many fields.

Parisi has also made important contributions to other areas of statistical physics. In 1986, Parisi and co-workers introduced the famous Kardar–Parisi–Zhang (KPZ) equation for describing the stochastic growth of

interfaces. Since then, this equation has been found to describe a wide variety of nonequilibrium processes. He has also developed, together Uriel Frisch, a theory of intermittency in fluid turbulence. In a study that makes contact with climate science, Parisi and co-workers pointed out that stochastic resonance may play an important role in climate change. More recently, with colleagues in Rome, he has worked on the formation of patterns in flocks of birds. Parisi has emphasized the important role of computations in many areas of physics, and has been actively involved in the development and use of special-purpose supercomputers for high-energy physics⁵ and spin systems⁶. Last but not the least, Parisi has been the nucleus of a large group of eminent physicists, working mostly in Rome and Paris, who have played a pivotal role in the development of the field of disordered and complex systems.

In summary, the ground-breaking work of Parisi has led to the development of concepts and techniques that have made it possible to analyse and understand the properties of a wide variety of complex systems in different areas of science that appear to be random and intractable at first sight. Studies of some of these systems have revealed the emergence of beautiful patterns that could not be predicted at the outset. The award of the Nobel Prize in Physics to Parisi recognizes the importance of the field of complex systems and celebrates his pioneering contributions to the development of this field.

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1. <https://www.nobelprize.org/prizes/physics/2021/summary/>
2. https://www.nobelprize.org/uploads/2021/10/fig5_fy_en_21_disorderedSystems.pdf
3. https://www.nobelprize.org/uploads/2021/10/fig7_fy_en_21_spinGlass.pdf
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