C. S. Seshadri (1932–2020)

C. S. Seshadri was a deep mathematician and a major figure in the field of algebraic geometry. His contributions to research and education in mathematics in India have been invaluable. In particular, he played a key role in the development of the School of Mathematics of the Tata Institute of Fundamental Research (TIFR), Bombay, as a world-renowned centre of mathematical research, and in the founding of the Chennai Mathematical Institute (CMI).

Seshadri was born in Kanchipuram on 29 February 1932. His childhood and schooldays were spent in Chengalpet. He studied in Loyola College, Madras, from 1948 to 1953, and obtained his BA (Hons) degree in mathematics. Father Racine, a French mathematician teaching at Loyola College, had a decisive influence on his early mathematical formation (and, indeed, on that of many young mathematicians of that time). Racine was in touch with many of the modern developments in mathematics and played an important role in introducing modern fields of mathematics to Indian students and mathematicians. For instance, he was one of the first in India to introduce modern algebra at the undergraduate level. Association with him at the formative stage was crucial for the future mathematical development of Seshadri. Seshadri said about Racine: 'His influence was in creating awareness in students of what is important in mathematics and encouraging them to pursue it. He had a knack of identifying and nurturing good students.'

On Racine's advice, Seshadri joined Tata Institute of Fundamental Research (TIFR), then newly established by Homi Bhabha, as a research student in mathematics in 1953. I should say something about the atmosphere and infrastructural facilities in TIFR at that time, which made young research students flourish.

As a theoretical physicist, Bhabha was aware that mathematics had an important role to play in any programme of fundamental research. It was in 1949, when K. Chandrasekharan (K.C.), at the age of 29, joined TIFR, that a plan for organizing mathematical research at the highest international level was initiated, and K.C. set up the School of Mathematics. This was the turning point for the remarkable progress of mathematics in the post-independence era in India. In the course of 15 years, K.C. succeeded in creating in India an institution which was recognized internationally as a leading centre of mathematical research, thereby transforming the mathematical landscape of India. Seshadri was one of the early beneficiaries of this development.

K.C. had spent three years in the Institute for Advanced Study in Princeton, before joining TIFR. He and K. G. Ramanathan (K.G.R.), who was in Princeton at the same time, and who helped K.C. later in TIFR, were in close contact with famous mathematicians such as Hermann Weyl and Carl L. Siegel. They understood how mathematical research was organized and carried out in the USA and in Germany. Based partly on this input, K.C. created conditions in TIFR for students to be exposed to first rate mathematics, and to equip them with the tools and means to do high level creative research. In order to introduce young mathematicians in fields in which there was no expertise in India, foremost experts from abroad were invited to TIFR to give structured courses. The students wrote up the lecture notes and they came into direct contact with outstanding mathematicians.

There was an excellent library in mathematics with a large collection of books and journals including back volumes. The access to such a vast collection of classical and current mathematical literature, like the Cartan seminars and the notes of Bourbaki seminars, played an important role in the work of mathematicians at TIFR.

Another important aspect was that students were given complete freedom to read and work on whatever topic they wanted, as long as K.C. and K.G.R. were convinced that the student was capable and serious. Fortunately, there was an influx of talented young Indian students too. Brilliant students like Seshadri profited by this 'stimulating atmosphere', as Seshadri called it, and produced outstanding work, which in turn established TIFR as a renowned centre of mathematics.

Seshadri obtained PhD degree in 1958 from the Bombay University. His thesis adviser was K. Chandrasekharan.

Seshadri spent three years (1957–1960) in Paris, on deputation from TIFR. His move to algebraic geometry took place in Paris, under the definitive influence of Claude Chevalley. During the first year, Chevalley gave a course on algebraic geometry, which Seshadri followed 'thoroughly and meticulously'. He took part and gave some lectures in the Chevalley seminar next year. Towards the end of that year he solved the problem of constructing Picard varieties of non-normal varieties, a problem posed by Chevalley. Seshadri would call Chevalley his guru and published a note 'My apprenticeship with Chevalley'. Apart from Chevalley, Jean-Pierre Serre was the person from whom he benefitted most.

During his stay in Paris, Seshadri also cultivated a taste for French culture, food and drink, and acquired fluency in the French language.

He returned to TIFR in 1960. At TIFR, he was a member of the mathematics faculty till 1984, when he moved to the Institute of Mathematical Sciences, Chennai. During his stay in TIFR, an active group in algebraic geometry was established. Some of his memorable and deep research work was done during this period. (This will be described further below.)

Seshadri was also a good organizer and a successful academic entrepreneur. To many people, this came as somewhat of a surprise, as he gave the impression of being an extremely absent-minded person; he was also well-known as someone who would often not complete his sentences when speaking, but instead allow them to trail off, leaving one to deduce the rest! However, for those who knew about the enormous amount of concentrated effort and energy he invested in his mathematical endeavours and the tenacity with which he would pursue a problem till a solution was found, this would not have come as a surprise.

In 1989, he established the Spic Mathematical Centre, later renamed the Chennai Mathematical Institute (CMI). CMI has been a success story, and Seshadri was proud of his role in its formation. Apart from being a first-rate research centre, CMI runs excellent undergraduate and graduate education programmes in mathematics and computer science. One of the motivations for the creation of CMI was Seshadri's strong conviction that an undergraduate programme intended for training people for research should be taught by researchers, and that young undergraduates should directly come into contact with those doing research. He had, I think, as a model the Ecole Normale Superieure, Paris. In my opinion (shared by many), CMI is now among the best undergraduate institutions in the world in the fields of mathematics and computer science.

Seshadri retired as Director of CMI in 2010, but continued to pursue research actively even after retirement.

I knew Seshadri from the days we were students in Loyola College, Madras. We joined what was known as the 'Intermediate class' in 1948, and studied for BA (Hons) from 1950 to 1953. The mathematics students in the Intermediate class were divided into two sections; there was a special and difficult examination in mathematics, and the person who got the first place in each section was awarded a prize called the Racine Prize. Seshadri and I were studying in different sections, and having topped our respective sections, we were both awarded this prize. I believe this is how we first came to know about each other. The prize was named after Father Racine, who, as I noted above, was to have a decisive influence on Seshadri's early mathematical formation. It was Racine who suggested that we apply to the newly established TIFR to do our PhD studies. Racine knew K.C. and K.G.R. when they were in Madras, and was aware that they were trying to do something potentially very significant in TIFR.

Our close friendship started when we joined TIFR as research students in 1953 (on the same day!) From then on, we had parallel academic careers in TIFR, progressing up each step of the academic ladder together. Already as research students we interacted very closely, reading and learning together a considerable amount of mathematics; this played an important role in our future mathematical development. We spent three years together in Paris in the late 1950s. Although we were working in completely different fields of mathematics when we were in Paris, we used to discuss mathematics regularly.

We returned to Bombay and TIFR in 1960. By that time, both in topology and differential geometry, fibre bundles were very well understood, and it looked as though the time was just ripe to develop the theory of vector bundles in algebraic geometry. Around 1963, Seshadri and I started thinking about the problem of holomorphic vector bundles on compact Riemann surfaces, which resulted in the Narasimhan–Seshadri theorem. This is considered to be a major breakthrough. In retrospect, it looks as though we were unconsciously and independently equipping ourselves during our Paris days with all the knowledge and tools which would play a part in this work. We were also very lucky, because in our student days in Bombay, even before going to Paris, K.G.R. had told us about the work of Andre Weil on the generalization of abelian functions. He had heard about this work from his teacher Carl L. Siegel. For no particular reason at all, Seshadri and I had immediately started a seminar on this paper, and Seshadri gave some lectures. (So, in some sense, we were aware of this problem from our student days.) This paper of Weil (which was not then so well known) dealing with unitary bundles, and the work of David Mumford on stable vector bundles, were important inputs in our work.

Although we did not formally collaborate after this work on bundles, I always profited by his scholarship in algebraic geometry; he shared his insights generously with me. I benefitted also by his thoughtful advice on practical matters.

The following brief description of his work gives an idea of the depth and breadth of Seshadri's work.

Algebraic geometry is basically the study of systems of polynomial equations and their solutions (and the geometric property of the space of solutions). Algebraic geometry is a central field in mathematics, with intimate connections with other major areas like number theory and complex analysis, and also with mathematical physics. The subject has been revolutionized during the second half of the last century by the introduction of new concepts and techniques by Grothendieck and others. Seshadri was a leader in this field.

In response to a question of Serre's, as to whether every vector bundle on the affine space is trivial (equivalently, whether every finitely generated projective module over a polynomial ring over a field is free), Seshadri proved in 1958 that vector bundles on the affine plane are trivial. This work attracted considerable attention and made Seshadri's name first known in the mathematical community. (The general case was settled by D. Quillen and A. A. Suslin about 15 years later.)

In 1965, M. S. Narasimhan and Seshadri proved the fundamental result relating stable vector bundles (an algebraic concept defined by David Mumford) on a compact Riemann surface to unitary representations of the (orbifold) fundamental group of the surface. This celebrated and influential result, known as the Narasimhan–Seshadri theorem, has served as a model for a large amount of literature intertwining algebraic geometry, differential geometry and topology, and was generalized in various directions. This result opened up a whole new field of moduli of algebraic vector bundles.

Seshadri (who had earlier constructed the Picard variety of a complete variety and proved its universal property) also constructed the compact (projective) moduli spaces of vector bundles on curves. This work, which introduced the notion of what is now known as S-equivalence (Seshadri equivalence), inspired and served as a model later for several constructions of moduli spaces.

He also introduced and studied the notion of parabolic bundles.

Seshadri worked extensively on the construction of quotients in algebraic geometry, and on Geometric Invariant Theory, originally motivated by its connection with moduli problems. He proved Mumford's Conjecture (that a linear representation of a reductive group is geometrically reductive) first in the case of GL(2) and then, using global geometric techniques, in the case 'stable = semi-stable'. In this work he also proved a powerful and very useful criterion for ampleness of a line bundle, now known as Seshadri's criterion of ampleness, leading to the recent vast literature on 'Seshadri constants'. The Mumford conjecture was proved in full by W. Haboush. More recently, Seshadri has given a geometric proof of this theorem.

One important series of works, by Seshadri in collaboration with V. Lakshmibai and C. Musili, is the theory of Standard Monomials, which began with a goal of giving characteristic-free descriptions of homogeneous coordinate rings of homogeneous spaces for classical groups, and ultimately became an important technique in representation theory. The problem was to find canonical bases of finite dimensional irreducible representations, or more generally canonical bases of the coordinate ring of Schubert varieties, using the combinatorics of the Weyl group. Classical Standard Monomial Theory (SMT) was created by Hodge and the modern SMT, by Seshadri.

Seshadri received several prestigious awards and honours. He was elected a fellow of the Royal Society, London, in 1988, and a Foreign Associate of the US National Academy of Sciences, in 2010. He was a Fellow of the Indian National Science Academy, the Indian Academy of Sciences and the American Mathematical Society. He received the Bhatnagar Prize in 1972, and the TWAS Trieste Science Prize for his distinguished contributions to science in 2006. He was awarded the *Padma Bhushan* by the President of India in 2009. He received Docteur Honoris Causa from Université Pierre et Marie Curie (UPMC), Paris in 2013, as well as Honorary Doctorates from the University of Hyderabad and Banaras Hindu University.

In addition to mathematics, his other great passion was music: Seshadri was deeply interested in Carnatic music, and was himself an accomplished musician. He had a genial personality, and enjoyed interacting with people.

He passed away on 17 July 2020. He leaves behind two sons.

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