Dynamic pricing and markdown timing policies for fashion goods with strategic behaviour of consumers

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In this article, we discuss the optimal pricing strategy and best markdown timing in two sale periods for a monopoly seller, when faced with strategic consumers. Based on the Stackelberg game theory, a mathematical model is constructed to maximize the seller's revenue when the markdown timing is certain or uncertain. Consumers are heterogeneous with different valuations for the same product. Moreover, after retailer decision-making, consumers would determine their purchase policies about the time and price, by comparing the prices and individual valuations in the two sale periods. Finally, a numerical example is considered to illustrate the optimal pricing strategy and best purchase policy.

Keywords: Fashion goods, markdown timing, pricing strategy, strategic consumers.

THE concept of perishable products, such as vegetables, newspapers, food, fashion, airline tickets and electronic products was first proposed by Weatherford and Bodily[1]. Here we focus on fashion goods, for which customers' reservation prices decrease over the sales season. Consequently, the seller should fix different prices at different periods of the sales horizon to adjust demand, in order to maximize his/her profits. However, some consumers realize that the seller would carry out price reduction promotion. So they usually compare the consumer surplus in different sales seasons to determine the best purchase price and time. Accordingly, the seller should consider the effect of consumers' strategic behaviour when choosing pricing policy.

Dynamic pricing and revenue management has gained increasing popularity in retail settings, and has engendered a growing body of academic research in recent years[2]. The recent growth in internet-based marketing has stimulated widespread experimentation with dynamic pricing, the practice of varying prices for the same goods over time or across customer classes, in an attempt to increase total revenue for the seller.

According to their willingness to wait and their sensitivity price, consumers can be divided into two types: myopic customers, also called 'angel' by the sellers, who will immediately buy an item when the current price is lower than their willingness to pay; the other is strategic customers, also called 'devil', who predict the possibility of buying at a lower price in the future and choose to wait the discounted price, even though the current price is lower than their reservation prices[3].

There is extensive literature on dynamic pricing research that includes mathematical analysis of optimization and game-theoretic models, empirical examination of field data, and laboratory experiments. Many surveys have focused on the study of dynamic pricing with strategic consumers[2,4–9]. Elmaghraby[10] studied optimal markdown mechanisms in the presence of strategic consumers who have fixed valuation throughout the selling season.

Whether markdown timing is certain or uncertain, the arrival of myopic and strategic customers is random. In these circumstances, we discuss how a monopolist seller makes dynamic pricing policy at two sales periods to gain maximum revenue. Based on Stackelberg game theory, we first develop the optimal dynamic pricing policy model when markdown timing is certain. Next, the extension model is established to make the optimal joint decision about dynamic pricing and purchasing when markdown timing is uncertain. Then, an example analysis is employed to illustrate the optimal pricing and markdown timing of the seller, and to find more interesting conclusions for the practices.

Base model descriptions

Problem statement

Consider a monopoly seller, who sells a single fashion product over a finite sales horizon: $[0, H]$ in the market without competition. The sales horizon is split into two periods of the sales season: $[0, T]$ and $[T, H]$. The markdown time T is decided by the seller, when he/she will change the full price to a discounted price. T is known by the consumers in the base model. In this game, the seller

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has a stock of Q units, and gives a fixed initial price p_1 during the first period of the sales horizon; he/she selects the markdown timing and offers a discounted price $p_2(p_2 < p_1)$ during the second part of the entire season[11]. The seller chooses the optimal pricing and discount timing strategy in order to maximize the expected profits collected over the entire season.

In the market, there are such characteristic customers who are price-sensitive and time-sensitive over the selling season. There are two types of consumers: those who may purchase at a high price immediately when they arrive at the store early, or they will wait until time T to purchase at a discounted price, and low-valuation consumers who may purchase at a highly discounted price or not buy at all, no matter when they arrive at the store. If the percentage of myopic consumers is β, the percentage of strategic consumers is $1 - \beta$. The purchasing decisions made by both populations are described in greater detail below.

We suppose that customers come to the store at a decided flow of constant rate per unit of time[12]. Each customer only buys one product. Because customers are heterogeneous, they have different basic valuations of the same product, which decline with the sales season. To highlight this, we use a multiplicative valuation function of the type:

$$V(t) = V * e^{-\alpha t},$$

where the base valuation V of each customer is drawn from a given continuous density distribution form f (ref. 11). The customer's valuation reduces at a rate α ($\alpha \geq 0$). The assumption that valuations decline over the course of the season seems to be prevalent in the sale of fashion and seasonal items[13]. Here, we assume that the rate of reduction of α is equal for the population of customers. For example, the case $\alpha = 0$ indicates that customers are not time-sensitive in their valuation. In other words, customer valuation for the products does not change during the entire season. If α value is high, the customer valuation decreases faster. The second heterogeneity is the different arrival times of customers. Additionally, we assume that every consumer only buys a unit of the product and then leaves the store immediately.

Our model is characterized by the set of parameters $\{\lambda, \alpha, \beta, H, Q, p_1, f\}$, assumed to be known to the seller and all consumers. Additionally, each consumer only observes the current price, but he/she does not see other consumers or the current level of inventory.

Purchasing decision of consumers

In our model, myopic customers will buy a product immediately if their valuation at the arrival time is larger or equal to the current price announced by the seller. However, strategic customers may decide to postpone their purchase if they believe that a later purchase at a lower price may bring a higher expected surplus than what they can gain by an immediate purchase. Let us now analyse the purchase decision-making process of strategic consumers.

Strategic customers who arrive prior to time T behave according to the following: a given strategic customer i, arriving at time t, will purchase immediately upon arrival (if there is inventory), if two conditions are satisfied about his/her current surplus $V_i e^{-\alpha T} - p_1$: (1) it is non-negative; and (2) it is larger or equal to the expected surplus he/she can gain from a purchase at time T (when the price is reset to p_2). This decision has a premise that the customer believes that the stock will still be available at markdown time T. Also, at time T, all existing customers take a look at the new price p_2 and if they can obtain a non-negative surplus, they request a unit of the remaining products (if any). If there are fewer units than the number of customers who wish to buy, the allocation is made randomly and equally. After time T, new strategic customers buy according to whether or not they can gain a non-negative surplus. Nevertheless, a myopic customer, arriving at time t before markdown time T, will buy a unit if his/her current surplus is non-negative; otherwise he/she will wait until time T. At or after time T, myopic customers make a purchase decision according to whether or not they can obtain a non-negative surplus.

When markdown time T is fixed, the seller must increase the consumer surplus gained at the first period or reduce the consumer surplus gained at the second period of the season, in order to attract more strategic consumers purchasing a unit in the first period to maximize his/her expected profits. Furthermore, there are two main strategies for the seller to reduce the initial price p_1 or to postpone the markdown timing. We compare the total revenue of the seller in case of different markdown timings. (i) When markdown timing is early, the initial price p_1 will be higher. Only a few strategic consumers purchase in the first period. Most of them purchase in the second period. (ii) When markdown timing is later, the initial price p_1 will be lower. In this case, the total number of consumers who purchase will be more than in (i). Our results show that the total revenue of the seller is lower when markdown timing is delayed. The reason is that the initial price p_1 and discount price p_2 are lower in (ii), although there are more consumers. Therefore, it is helpful to improve the economic performance of the seller by choosing the right time to cut down the price.

The seller at first announces the full price and the discount price and the markdown timing, and then consumers decide whether to buy or not; so a dynamic game of complete information is constructed. The game process is as follows: first, when entering the market, the seller announces the prices (p_1, p_2) in the two periods of sales season and markdown time T. Second, consumers arrive at the market, and the arrival process is according to a Poisson distribution with a mean of λ per time unit; third, at the first sales season, myopic customers decide to buy or not depending on whether their current valuation is larger than or equal to the current price, and then leave the market. Nevertheless, strategic customers decide to buy or to wait through trading-off the current and the expected future surplus in the second period of the sales horizon. Finally, at the second period of the season, all population of consumers decide whether to buy or not by comparing the current valuation with the discount price.

This is a dynamic game in two periods, with backward induction method. The consumers employ the purchasing strategy comparing the current and expected surplus, under the condition of a given seller's pricing and markdown timing strategy. Then, according to the analysis of the trade-off results, the seller selects optimal pricing and markdown timing strategy.

In the base model, given a markdown time T, the seller's strategy is to give the full price p_1 and discounted price p_2, where p_2 is dependent on the remaining inventory Q_T at the markdown time T and then forms the discounted price menu $\{\{p_2(Q_T)\}_{Q_T=1}^{Q}\}$. There is a competitive situation among consumers, which arises due to the fact that an individual consumer's decision impacts the product availability for others. Theorem 1 shows the existence of a threshold which is dependent on the price and time. Then, consumers purchase the product immediately upon arrival, if and only if their valuation is higher than this threshold.

Theorem 1: For any given pricing policy $\{p_1, p_2(Q_T)\}$ and markdown time T, it is optimal for all consumers to implement their purchasing decisions according to a threshold function $\theta(t)$. Actually, a consumer arriving at any time t of the sales horizon will buy a product immediately upon arrival if $V(t) \geq \theta(t)$. Otherwise, if $V(t) < \theta(t)$ and $t < T$, the consumers will revisit the store at time T, and buy an available unit if $V(t) \geq \theta(t)$.

For the two different types of consumers, the threshold function $\theta(t)$ is defined in two different forms. The threshold function $\theta(t)$ for strategic customers (Figure 1) is given by

$$\theta(t) = \begin{cases} \delta(t) & 0 \leq t < T, \\ p_2 & T \leq t < H. \end{cases} \tag{1}$$

Specifically, $\delta(t)$ in eq. (1) is the unique solution to the implicit equation.

$$\delta(t) - p_1 = E_{Q_T}[\max\{\delta(t)e^{-\alpha(T-t)} - p_2(Q_T), 0\}] \times 1 \cdot \Pr(N \mid Q_T)]. \tag{2}$$

The left-hand-side of eq. (2) represents the current surplus that the strategic customer can gain by immediately buying a unit, whereas the right-hand-side of eq. (2) represents the expected surplus that will be gained by postponing the purchase to time T. The value of the latter expected surplus takes into account two conditions. The first condition is that the discounted price $p_2(Q_T)$ must bring the customer to a non-negative surplus, that is $\delta(t)e^{-\alpha(T-t)} - p_2(Q_T) \geq 0$. The second condition is that the customer can be assigned to a product with the remaining inventory Q_T. The variable N represents the number of consumers strategically waiting to buy until time T. Given a specific inventory Q_T, the allocation probability depends on the distribution of the number of other consumers waiting to buy at markdown time T. Here, we assume that there is an equal probability for the customers to be allocated a unit. Moreover, the threshold function $\theta(t)$ in the range $[0, T]$ will be defined as $\delta(t)$ in the following study.

Theorem 1 not only proves that the threshold policy is optimal for every consumer, but also takes into account the impact of purchase strategies of other customers.

Furthermore, the threshold function $\theta(t)$ for myopic customers is defined by

$$\theta(t) = \begin{cases} p_1 & 0 \leq t < T, \\ p_2 & T \leq t < H. \end{cases} \tag{3}$$

Pricing strategy of the seller

Here, we discuss the seller's optimal pricing policy in response to a given purchasing strategy and a given initial price p_1. We will divide our customers into four categories: customers who buy immediately at the premium price; customers wait who strategically for markdown time T to purchase a product at the discounted price; the customers who wait nonstrategically for the markdown time T to purchase a unit, and customers who purchase a product immediately at the discounted price. The word 'purchase' means a desire to buy. Despite that, the desire is not sure to be implemented.

The first type of customers (denoted by I) arrive during $[0, T)$ and purchase immediately at price p_1. The expected number of customers of this type can be calculated as follows

$$N_1(\delta, p_1) = (1-\beta) \cdot \lambda \cdot \int_{t=0}^{T} P[V \geq \delta(t)e^{\alpha t}]dt + \beta \cdot \lambda \cdot \int_{t=0}^{T} P[V \geq p_1 e^{\alpha t}]dt. \tag{4}$$

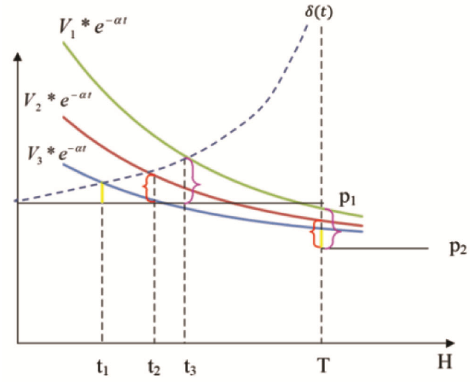

Figure 1. The decision model of strategic customers.

The second type of customers (denoted by S) arrive during $[0, T)$ and strategically postpone their purchase to time T at price p_2 due to anticipation of higher expected surplus. The expected number of customers of this type can be calculated as follows

$$N_S(\delta, p_1, p_2) = (1-\beta)\cdot\lambda\cdot\int_{t=0}^{T} P[\min\{\max\{p_1 e^{\alpha t}, p_2 e^{\alpha T}, \delta(t)e^{\alpha t}\}\} \le V \le \delta(t)e^{\alpha t}]\,dt. \quad (5)$$

The third type of customers (denoted by N_W) arrive during $[0, T)$ and wait for time T to buy a unit because their valuation upon arrival is lower than the initial price p_1. The expected number of customers of this type can be calculated as follows

$$N_{N\_W}(p_1, p_2) = \lambda\cdot\int_{t=0}^{T} P[\min\{p_1 e^{\alpha t}, p_2 e^{\alpha T}\} \le V \le p_1 e^{\alpha t}]dt. \quad (6)$$

The fourth type of customers (denoted by L) arrive during $[T, H]$ and have higher valuation upon arrival than the discounted price p_2. The expected number of customers of this type can be calculated as follows

$$N_L(p_1, p_2) = \lambda\cdot\int_{l=T}^{H} P[V \ge p_2 e^{\alpha t}]dt. \quad (7)$$

The total revenue collected from the four groups of customers is given by

$$\begin{aligned}\pi(p_1, p_2) &= p_1\cdot\min\{N_I(\delta, p_1), Q\}\\ &+p_2\cdot\min\{N_S(\delta, p_1, p_2)+N_{N\_W}(p_1, p_2)\\ &+N_L(p_1, p_2), Q_T\}.\end{aligned} \quad (8)$$

The optimal discounted price p_2 is chosen to maximize the seller's total expected revenue. Specifically

$$\begin{aligned}\{p_1, p_2\} &= \arg\max_{p_1,p_2}\{p_1\cdot\min\{N_I(\delta, p_1), Q\}\\ &+p_2\cdot\min\{N_S(\delta, p_1, p_2)+N_{N\_W}(p_1, p_2)\\ &+N_L(p_1, p_2), Q_T\}\}.\end{aligned} \quad (9)$$

Extended model analysis

In the extended model study, the markdown time T is the seller's decision variable. Next, we discuss the impact of markdown time on the seller's revenue. We verify each practical markdown time based on the total revenue function of the seller, and find the optimal markdown time T^* to maximize the seller's revenue.

The objective function of the seller based on the base model is given by

$$\max_T\left\{\max_{p_1, p_2}\{\pi(p_1, p_2, T)\}\right\}. \quad (10)$$

The partial derivative of each variable is obtained by the extreme value theorem of multivariate function as follows

$$\frac{\partial\pi}{\partial p_1}=0,\ \frac{\partial\pi}{\partial p_2}=0,\ \frac{\partial\pi}{\partial T}=0. \quad (11)$$

Nevertheless, it is difficult to find the explicit expression of the two prices when revenue of the seller is maximized because of the complexity of the solution. Next, we search the optimal joint decisions about equilibrium pricing (p_1^*, p_2^*) and markdown timing T^* through the analysis of numerical simulation.

Numerical analysis

We employ a numerical study to highlight the optimal joint decisions about pricing and markdown timing in the presence of heterogonous customers characterized by different base valuations. In this study, in order to represent the level of heterogeneity in the base valuation of customers, we assume that it follows a uniform distribution[14,15]. The probability density function of base valuation of customers is denoted as f, and defined as follows

$$f(V)=\begin{cases}1, & 0<V<1\\ 0, & \text{otherwise.}\end{cases} \quad (12)$$

Without loss of generality, the length of sales horizon H is normalized to 1. Therefore, T should be interpreted as fractions of the whole selling season. Similarly, we assume that the average number of customers arriving at the store is 50 during the whole horizon.

Due to the complexity of the objective function, the expression of the optimal price (p_1^*, p_2^*) cannot be directly written, but all possible solutions can be calculated (using Matlab) to search for the maximum profit and the corresponding optimal price policy. In the numerical study, five parameters are assigned the following values: $\{\lambda = 50, \alpha = 0.2, \beta = 0.5, T = 0.5, H = 1\}$, and the inventory is always well stocked. Figure 2 *a* shows that the total revenue of the seller increases first and then decreases with increase in p_2 under the condition that p_1 is given; when p_2 is fixed, the profit of the manufacturer decreases with increase in p_1. Therefore, the optimal price is $p_1^* = 0.4525$, $p_2^* = 0.2353$, and the maximum revenue is 39.1312. There is no dimension in all the parameters in the text.

Figure 2 *b* shows that at a given value of the markdown time T, the seller's revenue increases first and then decreases as p_2 increases continually, in the case that p_1 is fixed at 0.4524. That is, the total revenue is a convex function of p_2. In addition, the profit of the manufacturer increases with the time delay of price reduction. When p_2 is fixed at an arbitrary value, the profit of the seller increases with the delay of the discount timing T, that is, the objective function of the seller strictly increase with the markdown time. In this numerical example, the seller

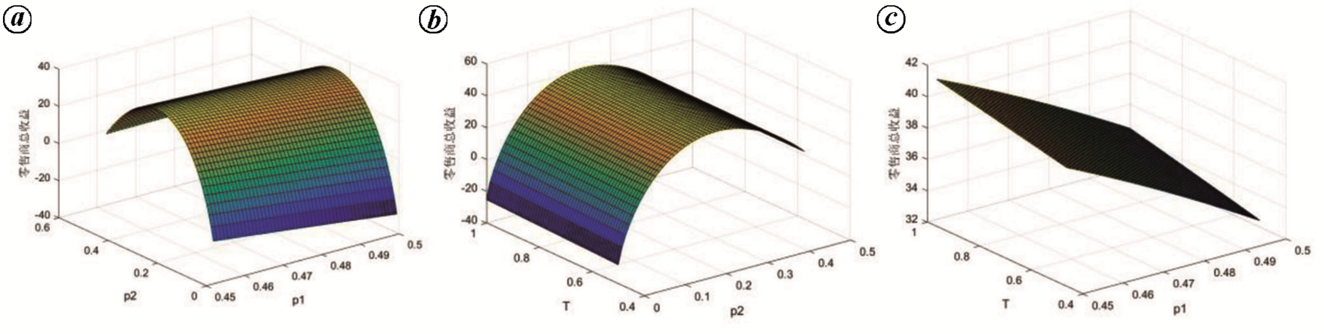

Figure 2. ***a***, The impact of p_1 and p_2 on total revenue of the seller. ***b***, Relationship between p_2, T and total revenue of the seller. ***c***, Relationship between p_1, T and the total revenue of the seller.

revenue is maximized to 40.9346, when the markdown time T^* is 0.999.

In contrast, Figure 2*c* shows different results. When $p_2 = 0.2353$, the seller's profit increases as p_1 increases continually, that is, the objective function of the seller is increasing strictly monotone. Moreover, the profit of the seller decreases with delay of the markdown timing, when the original price p_1 has an arbitrary fixed value. We calculate that the seller revenue is maximized to 40.9349, when the markdown time T^* is 1.

Conclusion

In this study we mainly examine the marketing strategy of perishable products considering both dynamic pricing and price reduction. Consumers arrive randomly during the selling period, while short-sighted and strategic consumers exist simultaneously. The markdown timing can be divided into two cases, namely confirmation and uncertainty. The example shows that with increase in the proportion of strategic customers in the market, the perineum price, the discounted level in the second period and the profits of the seller are significantly reduced. Irrespective of the proportion of strategic consumers, the profit will first increase and then decrease with the delay in the discount timing. If the proportion of strategic customers is far greater than myopic customers in the market, the more delayed the lower price is, the more advantageous it is to the seller.

This article assumes that the seller's inventory is unrestricted, but in general the initial inventory is determined by customer demand and firm profits. Our model takes into account the uncertainty of the customer's arrival and heterogeneity of basic valuation. However, analysis of heterogeneity of the base valuation is ignored in the numerical example. These need to be further studied. In addition, future research should also consider the asymmetry of information between the enterprise and the customer.

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