Quantum chaos, dynamical systems and Schrödinger equations and quantum unique ergodicity

Dr Nalini Anantharaman is a French mathematician who is currently professor at the Institute for Advanced Mathematical Research (IRMA), as well as a member of the Institute for Advanced Study (USIAS), University of Strasbourg, France. She is the daughter of Siva and Claire Anantharaman, professors of computer science and mathematics at the University of Orléans, France. Nalini attended the Ecole Normale Supérieure (ENS) and in 2000, completed her Ph D at the Pierre and Marie Curie University in Paris, France, under the supervision of François Ledrappier. She then became a lecturer at ENS in Lyon and, subsequently at the French National Centre for Scientific Research (CNRS) and the École Polytechnique in Paris. She was Visiting Miller Professor at the University of California, Berkeley, USA, in 2008 and later became a professor at the University of Paris-Sud, France in 2009. She was at the Institute for Advanced Study in Princeton, USA from January to June 2013. Between 2014 and 2016, she held a temporary chair at USIAS, which then became permanent.

Analysis and mathematical physics are the main focus of Nalini research. She has studied quantum chaos and, more recently, harmonic analysis of large graphs. Nalini showed in a paper published by the Annals of Mathematics, which was presented in the Bourbaki seminar of June 2007, that the semiclassical measures (Wigner measures) associated to sequences of eigenfunctions of the Laplace operator on a compact Riemannian manifold with non-positive curvature have a strictly positive Kolmogorov-Sinai entropy’. This result belongs to the field of quantum chaos, which has been intensively studied by physicists and mathematicians since several decades. The problem was to derive properties of the quantum spectra from the dynamical properties of the classical Hamiltonian. In particular, her result implies that no sequence of eigenfunctions can concentrate on a closed geodesic. This last result cannot be obtained from local considerations. The proof involves two different areas of mathematics: microlocal analysis and ergodic theory of dynamical systems. Nalini knew from her Ph D work the ergodic theory of dynamical systems. Nalini knew from her Ph D work the ergodic theory of dynamical systems and she quickly learned the microlocal analysis with good intuition on what she wanted to do. The theorem of Nalini is the first progress on this problem which was proposed by several people more than 30 years ago. It is quite remarkable that this result was not even conjectured. One should also mention that this mathematical result is an answer to the question of ‘scars’, a name introduced by physicists like E. Heller. More recently, Nalini has results in two independent areas; I will only briefly describe both as follows:

1. Quantum ergodicity on large graphs: Nalini considers the Laplacian on a large regular graph and shows that if the girth is going to infinity, most of the eigenfunctions are, in some precise sense, uniformly distributed. After proving this with her student Etienne Le Masson using a pseudo-differential calculus on trees, she founded several independent proofs with the hope to extend the result to a much more general situation.

2. Controlability problems for the Schrödinger equation: In the case of the unit disk, Nalini proves an observability inequality, mentioning that the L2-norm of a solution on any open subset intersecting the boundary controls its full L2-norm. This is a difficult result using quite technical tools in microlocal analysis.

Nalini was awarded the Salem Prize in 2011 for her work on Laplace eigenvalues. She was also awarded the Jacques Herbrand Prize from the French Academy of Sciences. In 2012, she was awarded the Henri Poincaré Prize for Mathematical Physics ‘for her original contributions to the area of quantum chaos, dynamical systems and Schrödinger equations, including a remarkable advance in the problem of quantum unique ergodicity’; she shared this award with Freeman Dyson, Barry Simon and Sylvia Serfaty. Nalini received the CNRS Silver Medal in 2013. She was elected as member of the Academia Europaea in 2015, and was a plenary speaker at the 2018 International Congress of Mathematicians. She is also the winner of the 2018 Infosys Prize in Mathematics.


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