# The Rs 10,000 crore club

No Indian university figures among the top 500 globally in the Shanghai Academic Ranking of World Universities. Recognizing this, the government intends to provide Rs 10,000 crores to 20 varsities, 10 private and 10 government, to make them 'world class'. This funding will span a period of five years, meaning that approximately 100 crores will be given to each university per year. This may be too little too late, but is a welcome initiative as it will create a club of universities that will be challenged to demonstrate their potential to become world class.

One worrying feature is that the selection of the 20 universities will be based on an assessment by a professional third-party agency. The present author is not aware of any agency that is equipped to deal with such a task.

The National Institutional Ranking Framework (NIRF) launched in 2015 by the Ministry of Human Resource Development (MHRD), Government of India is the country's own system of ranking Higher Educational Institutions (HEIs) using India-specific parameters. The ranking for 2017 which came out recently, considers five broad parameters for ranking: teaching, learning and resources; research and professional practices; graduation outcomes; outreach and inclusivity and perception. Within each category several sub-parameters are identified. A complex protocol is then used to arrive at a single number called the NIRF score.

NIRF 2017 (https://www.nirfindia.org/OverallRanking.html) ranks 200 institutions in engineering, university and management education according to their overall NIRF score. Here we select 22 of the top-ranked universities from the overall category and another 3 from the engineering category which did not make it to this select list. We shall find out if the research performance of these 25 institutions as well as their earnings related to innovation activities (sponsored research, consultancy, licensing of patents) are commensurate with the inputs (faculty and total expenditure) deployed by them. A simple output–input ratio becomes a measure of how the totalized input is productively (or efficiently) translated to output. The top 20 from this list are arguably the best candidates to join the Rs 10,000 crore club.

From the huge NIRF data, the two key inputs taken cognizance of are the total number of regular faculty ($F$) and the total expenditure ($S$) for three years (2013–2016). The key outputs are the total earnings ($E$) for three years (2013–2016), and the total bibliometric output ($X$) measured in units of exergy[1]. Both input and output are in incommensurable units. Here, we use space transformations

**Table 1.** Multidimensional input and output in terms of total expenditure $S$ (crores of rupees), total number of regular faculty $F$, exergy of research output $X$ and total earnings $E$ (crores of rupees), for the 25 institutions ranked in NIRF 2017

| Institution | Expenditure 2013–2016 $S$ | Regular faculty $F$ | Exergy $\sum X$ | Total earnings $E$ |
|---|---|---|---|---|
| Indian Institute of Science Bangalore (IISc) | 1287.1 | 447 | 371,508 | 812.18 |
| Indian Institute of Technology Madras (IIT M) | 2085.3 | 598 | 145,467 | 689.70 |
| Indian Institute of Technology Bombay (IIT B) | 1216.6 | 606 | 267,574 | 714.27 |
| Indian Institute of Technology Kharagpur (IIT Kgp) | 1121.1 | 679 | 223,669 | 462.28 |
| Indian Institute of Technology Delhi (IIT D) | 999.3 | 565 | 189,288 | 441.78 |
| Jawaharlal Nehru University (JNU) | 1006.7 | 673 | 55,877 | 108.60 |
| Indian Institute of Technology Kanpur (IIT Knp) | 960.5 | 447 | 144,476 | 219.09 |
| Indian Institute of Technology Guwahati (IIT G) | 945.0 | 436 | 123,284 | 131.67 |
| Indian Institute of Technology Roorkee (IIT R) | 788.6 | 444 | 170,708 | 458.80 |
| Banaras Hindu University (BHU) | 2189.9 | 1252 | 178,761 | 85.75 |
| Jawaharlal Nehru Centre for Advanced Scientific Research (JNCASR) | 176.7 | 38 | 150,313 | 142.42 |
| Jadavpur University (JU) | 538.3 | 642 | 114,179 | 154.75 |
| Anna University (Anna) | 797.9 | 909 | 70,292 | 101.07 |
| University of Hyderabad (UOH) | 387.0 | 369 | 94,085 | 137.79 |
| University of Delhi (UOD) | 1731.5 | 1015 | 270,094 | 150.04 |
| Amrita Vishwa Vidyapeetham (AVV) | 1672.3 | 1683 | 33,753 | 199.93 |
| Indian Institute of Management Ahmedabad (IIMA) | 473.7 | 118 | 1,493 | 107.57 |
| Savitribai Phule Pune University (SPPU) | 1002.9 | 366 | 55,147 | 255.18 |
| Aligarh Muslim University (AMU) | 1769.1 | 1285 | 104,401 | 35.44 |
| Jamia Millia Islamia (JMI) | 845.2 | 658 | 46,757 | 56.06 |
| Birla Institute of Technology and Science – Pilani (BITS) | 1020.0 | 664 | 51,067 | 34.82 |
| Vellore Institute of Technology (VIT) | 1296.8 | 1673 | 54,797 | 61.27 |
| Indian Institute of Technology Indore (IIT I) | 202.9 | 99 | 108,209 | 16.59 |
| Indian Institute of Technology Hyderabad (IIT H) | 334.7 | 176 | 25,936 | 63.21 |
| Institute of Chemical Technology (ICT) | 284.9 | 108 | 84,191 | 216.35 |
| Total | 25,134.0 | 15,950 | 3135,325 | 5,857 |

**Table 2.** Multidimensional output in terms of exergy of research output, *X*, and total earnings, *E* (crores of rupees), before and after fractionalizing using the conservation rule, and after recursive improvement

| Matrices and vectors | *A* | | *a* | | | | |
|---|---|---|---|---|---|---|---|
| | Exergy | Total | Exergy | Total | $au_o$ | $au_o$ normalizd | $u_{cc}$ |
| Institution | $\sum X$ | *E* | $\sum X$ | *E* | Total | Totalized output | Totalized output |
| Indian Institute of Science | 371,508 | 812.18 | 0.12 | 0.14 | 0.26 | 0.129 | 0.129 |
| Indian Institute of Technology Madras | 145,467 | 689.70 | 0.05 | 0.12 | 0.16 | 0.082 | 0.085 |
| Indian Institute of Technology Bombay | 267,574 | 714.27 | 0.09 | 0.12 | 0.21 | 0.104 | 0.105 |
| Indian Institute of Technology Kharagpur | 223,669 | 462.28 | 0.07 | 0.08 | 0.15 | 0.075 | 0.075 |
| Indian Institute of Technology Delhi | 189,288 | 441.78 | 0.06 | 0.08 | 0.14 | 0.068 | 0.068 |
| Jawaharlal Nehru University | 55,877 | 108.60 | 0.02 | 0.02 | 0.04 | 0.018 | 0.018 |
| Indian Institute of Technology Kanpur | 144,476 | 219.09 | 0.05 | 0.04 | 0.08 | 0.042 | 0.041 |
| Indian Institute of Technology Guwahati | 123,284 | 131.67 | 0.04 | 0.02 | 0.06 | 0.031 | 0.030 |
| Indian Institute of Technology Roorkee | 170,708 | 458.80 | 0.05 | 0.08 | 0.13 | 0.066 | 0.067 |
| Banaras Hindu University | 178,761 | 85.75 | 0.06 | 0.01 | 0.07 | 0.036 | 0.034 |
| Jawaharlal Nehru Centre for Advanced Scientific Research | 150,313 | 142.42 | 0.05 | 0.02 | 0.07 | 0.036 | 0.035 |
| Jadavpur University | 114,179 | 154.75 | 0.04 | 0.03 | 0.06 | 0.031 | 0.031 |
| Anna University | 70,292 | 101.07 | 0.02 | 0.02 | 0.04 | 0.020 | 0.020 |
| University of Hyderabad | 94,085 | 137.79 | 0.03 | 0.02 | 0.05 | 0.027 | 0.027 |
| University of Delhi | 270,094 | 150.04 | 0.09 | 0.03 | 0.11 | 0.056 | 0.054 |
| Amrita Vishwa Vidyapeetham | 33,753 | 199.93 | 0.01 | 0.03 | 0.04 | 0.022 | 0.023 |
| Indian Institute of Management Ahmedabad | 1,493 | 107.57 | 0.00 | 0.02 | 0.02 | 0.009 | 0.010 |
| Savitribai Phule Pune University | 55,147 | 255.18 | 0.02 | 0.04 | 0.06 | 0.031 | 0.032 |
| Aligarh Muslim University | 104,401 | 35.44 | 0.03 | 0.01 | 0.04 | 0.020 | 0.019 |
| Jamia Millia Islamia | 46,757 | 56.06 | 0.01 | 0.01 | 0.02 | 0.012 | 0.012 |
| Birla Institute of Technology & Science – Pilani | 51,067 | 34.82 | 0.02 | 0.01 | 0.02 | 0.011 | 0.011 |
| Vellore Institute of Technology | 54,797 | 61.27 | 0.02 | 0.01 | 0.03 | 0.014 | 0.014 |
| Indian Institute of Technology Indore | 108,209 | 16.59 | 0.03 | 0.00 | 0.04 | 0.019 | 0.018 |
| Indian Institute of Technology Hyderabad | 25,936 | 63.21 | 0.01 | 0.01 | 0.02 | 0.010 | 0.010 |
| Institute of Chemical Technology | 84,191 | 216.35 | 0.03 | 0.04 | 0.06 | 0.032 | 0.032 |
| Total | 3,135,325 | 5,857 | 1.00 | 1.00 | 2.00 | 1.00 | 1.00 |

with matrix multiplications that totalize the input and output, and allow productivity measures to be defined in terms of these totalized distance measures[2].

Krauze and McGinnis[2] introduced the concept of a 'scientific space', where the basic elements are scientific articles and their authors. The space is constructed on the basis of connections among these elements in the form of matrices that reflect those connections through its metric. The data are presented in the form of matrices, and matrix transformations allow various scientometric insights to be obtained. Indeed, they showed that matrix transformations are 'crucial for quantitative analyses of the structure of contemporary science'.

These insights can be extended to the multidimensional research evaluation problem here. There is now an institution space of 25 dimensions, an input space of two dimensions (*F* and *S*), and an output space of two dimensions (*E* and *X*). The primary data are given in the form of matrices, one linking the institution space to the input space and the other the institution space to the output space. Unlike Krauze and McGinnis[2], here we fractionalize the matrices into a stochastic form (i.e. columns add up to 1) so that through the multiplication process multiple counting is avoided and the conservation rule is adhered to.

For the purpose of demonstrating the procedure, we use data from NIRF 2017 for the top 22 institutions. To this, another three institutions are taken from the engineering category which did not make it to the overall list. Table 1 shows the multidimensional input and output in terms of total expenditure (*S*, crores of rupees), total number of regular faculty (*F*), exergy of research output (*X*) and total earnings (*E*, crores of rupees) for the 25 institutions chosen from NIRF 2017.

We first take up the output connections using the joint space of institutions and output. In Table 2, $A_o$ is a 25 × 2 matrix linking the 25 institutions to the output terms. The exergy of research output and total earnings are in incommensurable units, which cannot be added. We transform them into a matrix of stochastic vectors (i.e. a vector with non-negative entries that add up to 1) $a_o$. Note that this operation is a conserving one. If $u_o$ is a 2 × 1 vector of unit terms (using the terminology of Krauze and McGinnis[2], then $a_o u_o$ gives a totalized output vector (25 × 1)). We see that the terms add up to 2, as is to be expected as there are two output dimensions. We see from this sequence of matrix operations that the Indian Institute of Science (IISc) accounts for 12.9% of the totalized output of the 25 comparator institutions.

Following Krauze and McGinnis[2], we can use the $a_o a_o^T$ operation to map the network terms to the institution space. Let us call this matrix $O = a_o a_o^T$. This is now a 25 × 25 matrix where the totalized output is distributed as a symmetric matrix of terms which all add up to 2. It is a real symmetric matrix and its eigenvalues are all real. Again, if we introduce a 25 × 1 vector of unit terms called $u_c$ (i.e. each institution is given a weight of 1), then $Ou_c$ gives a totalized output vector (25 × 1) which is identical to $a_o u_o$. As

**Table 3.** Multidimensional input in terms of faculty, and expenditure before and after fractionalizing using the conservation rule, and after recursive improvement

| Matrices and vectors | A | | a | | | | |
|---|---|---|---|---|---|---|---|
| | Expenditure | Regular | Expenditure | Regular | $au_i$ | $au_i$ normalizd | $u_{cc}$ |
| Institution | S | F | S | F | Total | Totalized input | Totalized input |
| Indian Institute of Science, Bengaluru | 1,287 | 447.00 | 0.05 | 0.03 | 0.08 | 0.040 | 0.039 |
| Indian Institute of Technology Madras | 2,085 | 598.00 | 0.08 | 0.04 | 0.12 | 0.060 | 0.059 |
| Indian Institute of Technology Bombay | 1,217 | 606.00 | 0.05 | 0.04 | 0.09 | 0.043 | 0.043 |
| Indian Institute of Technology Kharagpur | 1,121 | 679.00 | 0.04 | 0.04 | 0.09 | 0.044 | 0.044 |
| Indian Institute of Technology Delhi | 999 | 565.00 | 0.04 | 0.04 | 0.08 | 0.038 | 0.038 |
| Jawaharlal Nehru University | 1,007 | 673.00 | 0.04 | 0.04 | 0.08 | 0.041 | 0.041 |
| Indian Institute of Technology Kanpur | 961 | 447.00 | 0.04 | 0.03 | 0.07 | 0.033 | 0.033 |
| Indian Institute of Technology Guwahati | 945 | 436.00 | 0.04 | 0.03 | 0.06 | 0.032 | 0.032 |
| Indian Institute of Technology Roorkee | 789 | 444.00 | 0.03 | 0.03 | 0.06 | 0.030 | 0.030 |
| Banaras Hindu University | 2,190 | 1252.00 | 0.09 | 0.08 | 0.17 | 0.083 | 0.083 |
| Jawaharlal Nehru Centre for Advanced Scientific Research | 177 | 38.00 | 0.01 | 0.00 | 0.01 | 0.005 | 0.005 |
| Jadavpur University | 538 | 642.00 | 0.02 | 0.04 | 0.06 | 0.031 | 0.031 |
| Anna University | 798 | 909.00 | 0.03 | 0.06 | 0.09 | 0.044 | 0.045 |
| University of Hyderabad | 387 | 369.00 | 0.02 | 0.02 | 0.04 | 0.019 | 0.019 |
| University of Delhi | 1,731 | 1015.00 | 0.07 | 0.06 | 0.13 | 0.066 | 0.066 |
| Amrita Vishwa Vidyapeetham | 1,672 | 1683.00 | 0.07 | 0.11 | 0.17 | 0.086 | 0.087 |
| Indian Institute of Management Ahmedabad | 474 | 118.00 | 0.02 | 0.01 | 0.03 | 0.013 | 0.013 |
| Savitribai Phule Pune University | 1,003 | 366.00 | 0.04 | 0.02 | 0.06 | 0.031 | 0.031 |
| Aligarh Muslim University | 1,769 | 1285.00 | 0.07 | 0.08 | 0.15 | 0.075 | 0.076 |
| Jamia Millia Islamia | 845 | 658.00 | 0.03 | 0.04 | 0.07 | 0.037 | 0.038 |
| Birla Institute of Technology & Science – Pilani | 1,020 | 664.00 | 0.04 | 0.04 | 0.08 | 0.041 | 0.041 |
| Vellore Institute of Technology | 1,297 | 1673.00 | 0.05 | 0.10 | 0.16 | 0.078 | 0.079 |
| Indian Institute of Technology Indore | 203 | 99.00 | 0.01 | 0.01 | 0.01 | 0.007 | 0.007 |
| Indian Institute of Technology Hyderabad | 335 | 176.00 | 0.01 | 0.01 | 0.02 | 0.012 | 0.012 |
| Institute of Chemical Technology | 285 | 108 | 0.01 | 0.01 | 0.02 | 0.009 | 0.009 |
| Total | 25,134 | 15,950 | 1 | 1 | 2 | 1 | 1.00 |

before, the terms add up to 2, as is to be expected as there are two output dimensions. This should not surprise us; the matrix transformations from institution-output space to institution space using stochastic matrices led to a transformation matrix that is orthogonal and this ensures that the conservation law is upheld.

Note that the totalized output measures (or distances) we have found so far assume that each institution is given a weight of 1. It is known from graph theoretic procedures that the fundamental eigenvector of the $O$ matrix provides a more accurate and appropriate evaluation of the output of institutions than an approach based on a simple addition of all the terms in the row. Each institution will be weighted according to its relative strength or weakness. It is possible to carry out this recursive iterated improvement using the eigenvalue relationship $Oo_{cc} = \lambda o_{cc},$ where $o_{cc}$ is the recursively weighted improvement of the totalized institution output score. We see that even after the recursive improvement, the IISc accounts for 12.9% of the totalized output of the 25 comparator institutions.

The entire sequence can be repeated with the input space, namely in terms of faculty, and expenditure before and after fractionalizing using the conservation rule, and after recursive improvement. This is seen in Table 3, where now $A_i$ is a $25 \times 2$ matrix linking the countries to the input terms. These are again in incommensurable units which cannot be added. Once again, we can transform them into a matrix of stochastic vectors. If $u_i$ is a $2 \times 1$ vector of unit terms, then $a_iu_i$ gives a totalized input vector ($25 \times 1$). We see that the terms add up to 2, as there are two input dimensions, namely expenditure and manpower. Amrita Vishwa Vidyapeetham (AVV) accounts for 8.6% of the totalized input.

Following Krauze and McGinnis[2], one can get the input network terms mapped again to an institution space. Let us call this matrix $I = a_ia_i^T$. This is now a $25 \times 25$ matrix where the totalized input is distributed as a symmetric matrix of terms which all add up to 2. Again, if we introduce a $25 \times 1$ vector called $u_c$ (i.e. each institution is given a weight of 1), then $Iu_c$ gives a totalized input vector ($25 \times 1$) which is identical to $a_iu_i$. As before, the terms add up to 2, now that there are two output dimensions. The matrix transformations from the joint institution-input space to institution space preserve the conservation law.

Note that the totalized output measures (or distances) we have found so far assume that each institution is given a weight of 1. It is possible to carry out a recursive iterated improvement using the eigenvalue relationship $Ii_{cc} = \lambda i_{cc},$ where $i_{cc}$ is the recursively weighted improvement of the totalized institution input score. We see that after repeated improvement, AVV now accounts for 8.7% of the totalized input. Table 3 is nearly self-explanatory.

Using NIRF 2017 data for 25 leading HEIs in the country, we have stated the problem in a multidimensional manner

**Table 4.** Totalized input and output measures after fractionalizing using the conservation rule and recursive improvement and ranked according to the productivity measure

| Rank | Institution | Totalized input | Totalized output | Totalized *O–I* ratio |
|---|---|---|---|---|
| 1 | Jawaharlal Nehru Centre for Advanced Scientific Research | 0.005 | 0.035 | 7.63 |
| 2 | Institute of Chemical Technology | 0.009 | 0.032 | 3.60 |
| 3 | Indian Institute of Science | 0.039 | 0.129 | 3.30 |
| 4 | Indian Institute of Technology, Indore | 0.007 | 0.018 | 2.46 |
| 5 | Indian Institute of Technology, Bombay | 0.043 | 0.105 | 2.44 |
| 6 | Indian Institute of Technology, Roorkee | 0.030 | 0.067 | 2.28 |
| 7 | Indian Institute of Technology, Delhi | 0.038 | 0.068 | 1.82 |
| 8 | Indian Institute of Technology, Kharagpur | 0.044 | 0.075 | 1.73 |
| 9 | Indian Institute of Technology, Madras | 0.059 | 0.085 | 1.42 |
| 10 | University of Hyderabad | 0.019 | 0.027 | 1.37 |
| 11 | Indian Institute of Technology, Kanpur | 0.033 | 0.041 | 1.26 |
| 12 | Savitribai Phule Pune University | 0.031 | 0.032 | 1.01 |
| 13 | Jadavpur University | 0.031 | 0.031 | 1.00 |
| 14 | Indian Institute of Technology, Guwahati | 0.032 | 0.030 | 0.94 |
| 15 | University of Delhi | 0.066 | 0.054 | 0.81 |
| 16 | Indian Institute of Technology, Hyderabad | 0.012 | 0.010 | 0.79 |
| 17 | Indian Institute of Management Ahmedabad | 0.013 | 0.010 | 0.78 |
| 18 | Jawaharlal Nehru University | 0.041 | 0.018 | 0.44 |
| 19 | Anna University | 0.045 | 0.020 | 0.44 |
| 20 | Banaras Hindu University | 0.083 | 0.034 | 0.41 |
| 21 | Jamia Millia Islamia | 0.038 | 0.012 | 0.32 |
| 22 | Amrita Vishwa Vidyapeetham | 0.087 | 0.023 | 0.27 |
| 23 | Birla Institute of Technology & Science – Pilani | 0.041 | 0.011 | 0.26 |
| 24 | Aligarh Muslim University | 0.076 | 0.019 | 0.25 |
| 25 | Vellore Institute of Technology | 0.079 | 0.014 | 0.17 |
| | Total | 1.000 | 1.000 | 1.00 |

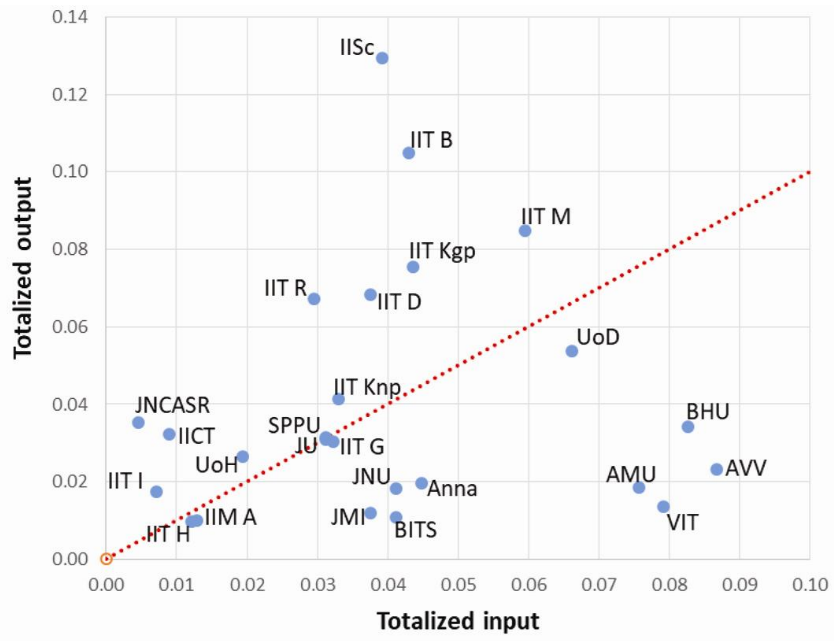


**Figure 1.** Totalized input and output after multidimensional input and output are projected to an institution space.

with input and output in terms of faculty, expenditure, bibliometric output and earnings. There are four ways in which productivity terms can be computed[3]. Using matrix transformations which project information in the institution-input and institution-output spaces to an institution space, it is possible to derive totalized input and output measures[2]. For this, fractionalizing using the conservation rule and recursive improvement using the network properties have been employed.

Table 4 and Figure 1 display the totalized input and output after the multi-dimensional input and output have been projected to an institution space.

Jawaharlal Nehru Centre for Advanced Scientific Research and the Institute of Chemical Technology are seen to be the best institutions from the productivity or efficiency point of view. They are followed very predictably by IISc and various Indian Institute of Technologies. Note that faculty size and expenditure are totalized into a single input term, and earnings and bibliometric output are totalized into a single output term for each institution. No private university finds a place in the list of top 20 here.

All the matrix operations are performed here with a cohort of 25 institutions; this restriction is due to the use of Excel spreadsheets alone. The matrix algorithms are general and if a computer algorithm is used, there need not be any restriction on the number of institutions assessed by this totalization procedure.

Research evaluation is a multidimensional problem as there are multiple input and output dimensions, and in the present case an institution space of many dimensions as well. Both research excellence (high-quality research output) and economic performance (earnings from sponsored research and consultancy) are taken into account, and this becomes a multidimensional problem with two input dimensions, two output dimensions and an institution space of 25 dimensions. The data making the connections are taken from NIRF 2017 and rearranged in matrix form. Here, we have used a protocol based on matrix normalization and multiplication so that totalized input and output measures can be obtained and comparative research evaluation can be made using NIRF 2017 data for 25 leading institutions in the country. The totalization process reveals that no private university finds a place in the top 20. The wisdom of letting 10 private universities join the Rs 10,000 crore club appears to be suspect.

1. Prathap, G., *Scientometrics*, 2011, **87**(3), 515–524.
2. Krauze, T. K. and McGinnis, R., *Scientometrics*, 1979, **1**(5–6), 419–444.
3. Basu, A., In Proceedings of the 15th International Society for Scientometrics and Informetrics Conference, Istanbul, 29 June to 3 July 2015.

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# A modification to Hirsch index allowing comparisons across different scientific fields

The Hirsch index (*h*-index) was introduced by physicist Jorge E. Hirsch in 2005, originally to determine the 'quality' of theoretical physicists by citation counts of their publications. Since then, the *h*-index has been used as a measure of the scientific proficiency of scholars in various scientific disciplines, university departments, scientific journals, etc.

However, the *h*-index also has several drawbacks. First, it does not enable comparisons across different scientific fields due to different citation habits and the number of researchers active in different fields. Secondly, the *h*-index does not account for the age of scholars, thus discriminating younger researchers. Also, the *h*-index cannot distinguish different positions in the authors' list of collaborative publications and can be biased by self-citations. Therefore, many modifications of the *h*-index were proposed in the last decade[1–6].

Here we propose a novel and simple modification of the original *h*-index, the relative Hirsch index ($h_r$-index), which assigns each researcher a value between 0 (bottom) and 1 (top), expressing his/her distance to the top in a given field of science. By this 'normalization', scientists from different disciplines can be compared.

The Hirsch index assigns each scientist a positive integer value such that a scientist with an index of *h* published *h* papers, and each of them has been cited at least *h* times[7]. The number of scholars' citations is usually acquired from main bibliographic databases such as the ISI Web of Knowledge (WoK), Scopus, Google Scholar or REPEC (for economists). However, data from these sources differ due to different coverage[8]. Moreover, according to Meho and Young[9], SCOPUS and Google Scholar have limited coverage of publications prior to 1990.

A more precise definition of the *h*-index is as follows: Let *f* be the function assigning each publication *i* its number of citations, and let *f* be in decreasing order. Then the *h*-index is given as follows

$$h = \max_i \min(f(i), i). \quad (1)$$

Table 1 provides several indices derived from the *h*-index that avoid some drawbacks mentioned above.

The $h_r$-index of a given scientist active in a scientific field *S* is defined as his/her *h*-index divided by the current maximal Hirsch index in the field *S*

$$h_r = \frac{h}{\max_S h}. \quad (2)$$

Clearly, $h_r \in [0, 1]$.

The relative Hirsch index expresses a scholar's 'distance to the top' in his/her field, as $h_r = 1$ represents the top (the case of a scientist with the highest *h*-index in his/her field) and $h_r = 0$ the bottom (the case of a scientist with no citations).