NEWTON, MATHEMATICS AND RAMANUJAN

BY

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SEPARATED by an interval of about two centuries and a half, the mathematical heritage of mankind was enriched by two remarkable men—Newton born in Britain, 25 December 1642, and Ramanujan born in India, 22 December 1887. They had just a little in common; but they differed widely in most matters—even in the force that pulled them to mathematics.

1. Dissimilarities

Newton spent his early years in fairly comfortable circumstances, in spite of his father dying before his birth and his mother remarrying before he was three; Ramanujan's parents, though both were living, could not keep him above want till he began to earn. Before entering adulthood, Newton had passed through the University of Cambridge, learned virtually all that could then be learnt in mathematics and contacted the eminent mathematicians and natural philosophers of his days, either as teachers or as friends; but till his twenty-fifth year Ramanujan could not come across any who would understand him or his language and still less teach him, nor could he finish the university course nor know the then state of mathematical knowledge. Notwithstanding his anxiety to avoid 'contention' of any kind, Newton's discoveries involved him in frequent and bitter controversies and much tension prevailed between him and his peers; it was, on the other hand, an atmosphere of uniform and spontaneous admiration, love and respect that surrounded Ramanujan after he was discovered. Newton was long-lived; Ramanujan died prematurely.

2. Similarities

And yet they had just a few things in common. Mathematics was the meeting ground of the two geniuses, though they approached it from different directions. Their life's work was done by the time they were thirty, though one lived for half a century thereafter and the other gave up his mortal coil almost immediately. Newton wrote down his results in diaries, letters and note-books, most of which are still preserved in some British libraries and had been all published in his own days though it was a job to make him release them for print; Ramanujan too wrote down his results systematically in a note-book which he left behind in England, where I was fortunate to pick it up in 1925 though it was in a tattered state; and it is now the most precious possession of the Madras University Library, still awaiting publication as a whole. The Royal Society elected both of them as its fellows before they passed thirty.

3. Integral Calculus

Newton's boyhood had been rich in hobbies of a mechanical sort like making kites, sun-dials and windmills. Kepler's *Optikas* is said to have been the first book to engage his serious thought. Consistently with this, one of the important books that Newton himself published was his *Opticks* (1704). He would grind lenses and if the telescope could not be made achromatic, he would invent a new type of telescope, the reflecting one. Consistently with this if a mathematical tool was wanting or defective, he would improvise a new tool or sharpen what existed. When his theory of gravitation led him to calculate the attraction of a sphere, which he could not do by the then known mathematical tools, he invented a new tool—the integral calculus. Particular problems of finding lengths, areas and volumes had been solved before him by special devices; but a general tool applicable to all similar problems was invented only by Newton and that because his preoccupations with his theory of gravitation needed it. The *Tractatus di quadratura curvarum* first appeared as an appendix to his *Opticks*.

4. Conics

The determination of a central orbit as a conic involved Newton in facing the problem of finding the conic when a focus and three other conditions or any other five conditions were given. These were fully investigated in the *Principia* (1687).

5. Problem of Three Bodies

The phenomenon of tides challenged Newton for an explanation on the basis of his theory of gravitation. While Proposition 66 of Book I and 24 and 26 of Book III of the *Principia* were turned on it directly, the foundations of the profound problem of three bodies were also laid incidentally.

Calculus of Variations

In considering the form of ships, Newton was obliged to determine the shape of bodies of revolution that will experience the least resistance when moved in the line of its axis. He improvised in 1686 a new tool to do this job and showed it to Fatio Duillier who published it in the pamphlet *Investigatie geometrica solidi rotundi in quod minima fiat resistantin* (1699). This tool, the calculus of variations, was lost sight of, until it was rediscovered by Lagrange in 1762.

Calculus of Finite Differences

Proposition 40 of Book III of the *Principia* is followed by Lemma 5 which gives a general solution of the problem: "To describe a geometrical curve which shall pass through any given points"—the fundamental proposition of the calculus of finite differences. *Methodus differentiates* (1711) which gives another solution of the same problem was first translated into English in the pages of the *Journal* of the Institute of Actuaries in 1918.

Cubics

The *Enumeratio linearum tertu ordinis* which appeared as an appendix to *Opticks* was the first systematic contribution to the theory of higher plane curves. 72 species of cubics were enumerated. As many propositions were without proof, this became one of the most commented of Newton's works. His method of classifying cubics appears to have inspired Waring to apply it to the quartic of which he obtained 84,551 species. To evaluate Newton's contribution to this field, we should remember that the modern tool of projective geometry had not yet been forged.

Calculus of Fluxions

The most discussed of Newton's tools from the point of view of priority of discovery is the calculus of fluxions. This again was forged to meet a definite situation in the determination of orbits. In fact he first invented the binomial theorem and then this calculus. The *Principia* had given its substance without the name in certain propositions of Book I. Evidently influenced by his dynamical interests, Newton regarded all variables as functions of time and all variation as primarily happening as time flows on. He, therefore. invented a calculus to determine the rate of change of any magnitude regarded as a function of time. These rates were called fluxions. They were with respect to time. The rate of change of a magnitude x with respect to another magnitude y he defined as the ratio of the fluxion of x to the fluxion of y. In spite of Newton's fluxional approach providing the beginner with an intuitive and familiar illustration of the concepts of calculus and in spite of time having been appropriately basic in the specific problem that led Newton to its discovery, it was soon felt that it was needless to drag in time where it was not concerned; and Leibnitz produced a calculus unobsessed by the spirit of time. But insularity kept out for long the more fertile calculus of Leibnitz from the home of Newton. It was not till 1812 that the sterile calculus of fluxions was deposited in the museum. In that year the Analytical Society of Cambridge was founded to adopt Leibnitz in place of Newton; or as Babbage, one of its founders, put it, "To promote the principles of pure *D*-ism in opposition to the *Dot*-age of the University". Nonmathematical readers can see the pun if they are told that the notation for the differential coefficient of x was $\dot{x}$ (x dot) in Newtonian calculus and Dx (Dee x) in Leibnitzian calculus.

Newton's Approach

Thus Newton's approach to mathematics was from the side of physics and the phenomena of the material universe and for the purpose of employing it as a tool. Concrete phenomena are ever so complex and there is no end to the variety of tools which they call for. Even to-day mathematics is being enriched by new tools demanded by newly discovered phenomena in physics; witness the calculus of operations, the calculus of tensors and the theory of Wave Function. Green, Kelvin and Poincare are some of the later mathematicians who enriched mathematics in this way. The Indian mathematicians of the Vedic age had enriched geometry similarly, *i.e.*, by the urge to solve a concrete problem.

Ramanujan's Approach

But Ramanujan's approach to mathematics was from another side, the side of form—its beauty and potency. Euler, Galois and Riemann had done so before. The Greek mathematicians of the classic age had enriched geometry similarly. We saw that Newton's boyhood-experience registered with his mode of approach to Mathematics. It was so with Ramanujan too. His boyhood was spent in making patterns of magic squares, playing with the formulæ enumerated in Carr's *Synopsis* and listening to *puranic* discourses on the infinite attributes of God. Two intimate friends who were with him virtually all through, except during his sojourn in England, assure me that his sensitiveness to form was unusual and that, as they could not follow mathematics, he used to entertain them till late in the night with interpretations of the *Ramayana* and the *Mahabharata* based on certain patterns of thought which were exquisitely beautiful and the mathematical correlates of which he would occasionally indulge in expounding.

Force of Form

Hardy's reminiscence about the taxi-cab No. 1729 is significant. Hardy's remark that it was 'dull' elicited a prompt reply from Ramanujan: "No, it is a very interesting number; it is the smallest number expressible as a sum of two cubes in two ways." Hardy asked for the corresponding number for fourth powers and Ramanujan replied, after a moment's thought, that the first such number must be very large. The fascination and force of form may be traced behind most of Ramanujan's work on the theory of numbers and of modular functions. We witnessed a pretty manifestation of the same in a mathematical social evening at Madras in 1914 in which Ramanujan pierced, as it were, through the traditional integer-garb of the Leibnitzian form for the differential coefficient of the nth order and utilising the Eulerian generalisation of the factorial into the gamma function entertained the audience with the beauty of fractional differentiation.

Complementary Approaches

Both the approaches are necessary to disclose the potency of mathematics. But the illumination from Ramanujan's side of approach is more subtle and visible only to a select few; while that from Newton's side is more extensive and lights up many a path in many a field of knowledge.