String theory and the conundrums of quantum gravity

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We give a brief survey of the attempts to understand the quantum dynamics of general relativity through the lens of quantum field theory, which has been successfully applied to the other fundamental interactions. This approach began by naively quantizing spin two massless particles but quickly ran into difficulties when considering quantum corrections to the interacting theory. We will describe how the early incarnations of string theory successfully addressed these problems. Then we go on to sketch how one has been able to go beyond this 'perturbative' picture by giving a tight microscopic description of black hole thermodynamics. This in turn has led to the much more subtle holographic description of the quantum dynamics of a large class of spacetimes which has been one of the primary engines of theoretical developments in the last couple of decades.

Keywords: General relativity, quantum dynamics, quantum field theory, string theory.

Introduction

THE quantum mechanical framework of nature is incomplete without the inclusion of the gravitational interaction. While we celebrate the centenary of Einstein's successful description of classical gravity, it is humbling to remember that quantum mechanics, in the form that emerged a mere ten years afterwards, is yet to be fully reconciled with general relativity. This is not merely unsatisfactory in some abstract, intellectual sense. It means that we do not have a full description of the initial stages of the universe despite the success of general relativity in explaining its large scale evolution and structure. It also means that we are yet to completely penetrate the enigmatic nature of black holes, which are now believed to be ubiquitous at stellar scales as well as on the much larger scales at the centre of galaxies.

In this article, we trace, from the early period to the present, the evolution of our view of gravity as a quantum interaction. We will first describe how gravity was sought to be quantized as a quantum field and some of the problems this approach ran into. We next outline how string theory in its early form addressed some of these issues which are to do with the perturbative interactions of gravitons. Then we show how an improved understanding of what string theory is, led to a first principle explanation of some of the puzzling features of black holes – their thermodynamic nature. This, in turn, has led to a new insight into the nature of quantum gravity in general – that of being holographically described by a quantum field theory (QFT) after all! This, still evolving, subject of gauge-gravity duality will also be discussed.

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Quantum gravity as quantum field theory?

Einstein's equations have many similarities to Maxwell's equations, seemingly only a more complicated, nonlinear (self-interacting) and tensorial version of the latter. In fact, the non-abelian generalization of Maxwell theories to Yang–Mills theories seems to be even closer in spirit to the Einstein theory in being nonlinear as well. All of them have the property that in a weak field limit the equations can be linearized and reduce to wave equations which have solutions propagating at the speed of light. Since the starting point of the quantization of Maxwell (and weakly coupled Yang–Mills theories) are these plane wave solutions, it was natural to mimic this procedure in the case of gravity as well.

In a weak field limit, the metric $g_{\mu\nu}(x)$ which describes the classical gravitational field has the form

$$g_{\mu\nu}(x) \approx \eta_{\mu\nu} + h_{\mu\nu}(x); \qquad |h| \ll 1, \tag{1}$$

where $\eta_{\mu\nu}$ is the usual Minkowskian spacetime metric in the absence of gravity. After imposing an appropriate gauge choice – we need to fix the linearized diffeomorphism invariance

$$h_{\mu\nu}(x) \to h_{\mu\nu}(x) + \partial_\mu \xi_\nu(x) + \partial_\nu \xi_\mu(x), \tag{2}$$

the vacuum Einstein equations reduce to the wave equation

$$\partial^2 h_{\mu\nu} = 0 \qquad (\partial^2 \equiv \partial_\mu \partial^\nu), \tag{3}$$

which has plane wave solutions of the form $\varepsilon_{\mu\nu}e^{(ik\cdot x)}$ with $k^2 = 0$, where the (constant) polarization tensor $\varepsilon_{\mu\nu}$ takes only two independent values (due to the gauge invariance).

As for electromagnetic waves, where the quantization leads to massless quanta, photons, here too one has massless quanta, now called gravitons. A crucial difference is that unlike the vector nature of electromagnetic waves which led to spin one photons, the tensorial nature of the polarization tensor $\varepsilon_{\mu\nu}$ implies that gravitons have *spin two*. The two independent polarizations correspond to states with helicity ± 2.

This was for the free theory. The complications arise on considering interactions. First, the source of gravity (the metric) is the energy-momentum tensor. Continuing to work with the weak field but allowing for (weak) matter sources, the linearized equations get modified to

$$\partial^2 h_{\mu\nu} = -16\pi G_{\mathrm{N}}\left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T^{\alpha}_{\alpha}\right), \quad (4)$$

where $T_{\mu\nu}$ is the matter energy-momentum tensor. These equations also follow from the Fierz–Pauli Lagrangian which is the unique one that is quadratic in the spin two field $h_{\mu\nu}(x)$ and invariant under the linearized diffeomorphism invariance mentioned above, together with the linear matter current coupling $h_{\mu\nu}(x)T^{\mu\nu}(x)$.

However, this is not a consistent theory in itself since the energy-momentum of the gravitational field itself has not been included in $T_{\mu\nu}(x)$. The free field part of the Fierz–Pauli Lagrangian itself gives a contribution to $T_{\mu\nu}(x)$ which is quadratic in $h_{\mu\nu}$. This in turn implies, from the coupling $h_{\mu\nu}(x)T^{\mu\nu}(x)$ that there is now a cubic term in $h_{\mu\nu}$ in the Lagrangian. This simply reflects the self-interaction of gravity. However, the process cannot truncate with this cubic interaction since this now gives an additional cubic contribution to the gravitational energy-momentum which leads to a quartic interaction and so on. One generates interaction terms with an arbitrary number of h's. Remarkably, it has been argued that this process when iterated generates the Einstein–Hilbert Lagrangian for classical gravity.

What is significant, from our present point of view of QFT, is that we have succeeded in viewing gravity as an interacting theory of spin two quanta without reference to the equivalence principle, etc. Thus it would appear as if the geometric aspect of gravity was merely a nice add-on feature but not particularly crucial to the understanding of the theory at the quantum level. Indeed Weinberg was able to show that the coupling of a massless spin two field (thus, with linearized gauge invariance) to any other field is universal and proportional to the mass/energy as expected from the equivalence principle. This followed from a very general analysis of the low energy limits of scattering amplitudes involving massless gravitons. The equivalence principle, in this sense, is a consequence of the massless spin two nature of the graviton.

Emboldened by these successes, as well as those of quantum electrodynamics (QED), theorists applied the same techniques to understanding the quantum interactions of gravity. The recipe of perturbative quantum field theory is succinctly captured by the Feynman diagram prescriptions. The building blocks of the diagrams are edges or propagators of the free graviton (and any other matter fields present), as well as vertices representing the interactions. Thus a cubic interaction corresponds to a 3-valent vertex and so on. The weights associated to these propagators and vertices depend on the precise form of the Lagrangian. In the case at hand we saw that we have interactions in $h_{\mu\nu}$ of arbitrarily high order. This means there are an infinite number of vertices, of all valencies, to consider. Each of these vertices comes with an appropriate *positive power* of the gravitational coupling constant G_{N}.

A naive treatment of quantum effects in QFT (captured by diagrams with internal loops) leads to ultra-violet or short distance divergences which do not make physical sense. These come from virtual particles of arbitrarily high energies which must, in principle, be present. Since physics at arbitrarily high energy is something we can only extrapolate to, one parametrises one's ignorance in terms of some number of independent coupling constants or parameters which characterize the theory at energy scales of interest. The quantum corrections which are naively divergent are actually unobservable corrections to these parameters which are themselves to be fixed by observation. In terms of these parameters one then extracts other finite corrections which are also observable. This is the philosophy of effective field theory.

When this approach is applied to gravity, we encounter the problem tha6t the gravitational coupling constant G_N has dimensions (in natural units where $\hbar = c = 1$) of $[M]^{-2}$ (in four spacetime dimensions). At energies comparable to the Planck scale $M_{\mathrm{P}} \propto (1/G_{\mathrm{N}})^{1/2}$ where quantum effects become significant, we have an infinite number of quantum correction terms to the Einstein Lagrangian involving more and more derivatives (e.g. higher powers of the curvature tensor) each of which would have an *a priori* independent coupling constant. Since we have parametrized our ignorance of the theory in these parameters, we can extract very few consequences of the quantum corrections. This is unlike the case of QED, where the coupling constant is dimensionless and so that at low energies, quantum effects are computable in terms of a finite number of parameters. This is the so-called problem of non-renormalizability of gravity. Explicit calculations, starting with ones by 'tHooft and Veltman in the early 70s, have confirmed the presence of additional higher derivative terms with undetermined coefficients.

In fact, the situation is reminiscent of the low energy description of pion interactions which can be captured effectively using quantum field theory with a coupling which has negative mass dimensions (f_π^{-1}, where f_π is the pion decay constant). This is a good description at energies small compared to the QCD scale which is set by $4\pi f_\pi$, just as the Einstein description is a good one at energies small compared to M_{P}. At energies of the order of the QCD scale, the pion Lagrangian contains arbitrarily many higher derivative terms with unknown coefficients and can make few robust predictions. In that case, one needs a more microscopic description like that of the

QCD Lagrangian which predicts an infinite number of other excitations (i.e. other mesons heavy compared to the pions) in the theory which need to be taken into account to make predictions at the QCD scale.

Thus the QFT approach appears to lead us to an impasse. We see that it predicts Einstein's theory as an effective description for massless spin two particles at low energies but does not show the way to go beyond that in a predictive way at energies (or distances) where quantum effects are important.

String theory – infancy and early successes

Though string theory originated in the late sixties as an attempt to understand the strong interactions, it was gradually realized by mid-seventies that it actually had the potential to provide a consistent quantum theory of gravity. In fact, at the very minimum it cured some of the problems just outlined, faced by the QFT approach to quantizing gravity. For a collection of articles on general aspects of string theory, see ref. 1.

String-like extended objects are common in nature. They arise in superconductors and also describe the flux tubes of QCD (the theory of the strong interactions). Quantizing such an object leads to a remarkable fact: there is an infinite tower of particles in the spectrum with one of the lowest excitations being a massless spin two particle. To be more precise, this particle arises from the quantization of closed strings (which like rubber bands have no endpoints, unlike open strings which are like a piece of thread and have two endpoints).

A fundamental string (i.e. without any thickness and thus truly one dimensional) is characterized by its tension T (or mass-energy per unit length). The dynamics of the relativistic string is defined by specifying the amplitude for the propagation of this string. Adopting the Feynman approach, this amplitude is given by

$$Ampl \propto \int [\mathcal{D}X^{\mu}(\sigma,\tau)]\mathrm{e}^{(\mathrm{i}S_{cl}[X^{\mu}(\sigma,\tau)])}. \tag{5}$$

Here (σ, τ) refer to the spacelike and timelike directions on the two dimensional worldsheet spanned by the time evolution of the one dimensional string. $X^{\mu}(\sigma, \tau)$ gives the spacetime location of each such point on the worldsheet in the spacetime in which it propagates. It is often referred to as the embedding or a map of the worldsheet in a target spacetime. $S_{cl}[X^{\mu}(\sigma, \tau)]$ is the classical action which is a functional of this embedding or trajectory of the string. The most natural action (largely fixed by requiring reparametrisation invariance of the functional on the arbitrary choice of (σ, τ)) is the Nambu–Goto area functional. Here

$$S_{cl} \propto T \times \text{area}, \tag{6}$$

where the area is that swept out by the string worldsheet in spacetime. This generalizes the usual proper time functional for the worldline propagation of a point particle.

The spectrum of excitations of this string is now quantized. As mentioned above there is an infinite tower

$$m_n^2 \propto nT \quad (n = 0, 1, 2, \ldots), \tag{7}$$

with exponentially increasing degeneracy at any given level n (see note 1). At $n = 0$, one can also show that the massless excitation of the closed string has spin two. For the open string, one has spin one quanta as the massless excitations.

It was the great insight of Yoneya, and independently, Scherk and Schwarz, to realize that the low energy classical interactions of these massless particles is exactly that given by Einstein gravity (for the closed string) and Yang–Mills gauge fields (for the open string). To be more precise, for energies much smaller than the string tension T, the other excitations of the string are too massive to play a significant role. The low energy spacetime Lagrangian is that of Einstein–Hilbert (plus some massless scalars as well as Yang–Mills, when one has open strings). This essentially follows from the same arguments given in the previous section once one has a massless spin two particle with the linearized gauge invariance of (2). However, the massive excitations of the string do give well defined higher derivative corrections to the Einstein–Hilbert theory which are suppressed by positive powers of (E^2/T) even at the classical level. For instance, the two graviton to two graviton scattering amplitude has a finite classical correction coming from a term proportional to R^4, which is a short form for denoting a term involving four powers of the Riemann curvature tensor.

The situation is analogous in many ways to the pion case mentioned in the previous section. When we go to energy scales comparable to the string tension scale $\sqrt{T}$, we can no longer ignore the other massive excitations. The low energy description breaks down and we need to use the full string amplitude. It must be emphasized that this is true even at the classical level, i.e. without including the effect of virtual particles. The new effects are already present at the classical level due to the additional massive particles that will contribute to any amplitude. Note that the new physics enters at the scale $\sqrt{T}$ which is not, in general, the same as M_P.

What about the quantum corrections which brought quantum gravity to grief in the conventional perturbative approach? We now need to consider virtual processes involving the propagation of strings. These string loops can be diagrammatically represented by surfaces (the two dimensional worldsheets) with holes in them (analogous to the Feynman graphs with loops). The simplest such diagram involves one hole and mathematically this is what is known as a torus (like a bicycle tube). Higher order quantum effects will have more holes and are known as surfaces of higher genus. Each additional hole is weighted by a factor of g_s^2, the string interaction parameter. Thus g_s^2 gives the amplitude for a string to split into two and rejoin (see note 2). The perturbative expansion of string theory in terms of surfaces of higher and higher genus is valid when $g_s^2 \ll 1$. The Newton constant G_N is proportional to g_s^2 times appropriate factors of T and any other parameters (e.g. compactification scales when there are additional small dimensions). Thus the Planck scale can differ from the string scale by factors of g_s.

The fundamentally different nature of string interactions (from that in a QFT) shows up when we consider these quantum corrections. Unlike the point-like vertices in Feynman graphs which implied that interactions were at arbitrarily short distances (and therefore involved virtual particles of arbitrarily high energies), in string theory, the interactions are smeared out. A worldsheet with holes is a smooth surface and there is no single point in spacetime which can be associated with a string splitting or joining – it is spread over a size set by the string tension.

Mathematically, this is seen, from the fact that the region of integration over the different shapes of the surfaces does not have any degeneration which corresponds to short distances in spacetime. The only degenerations are those coming from the infra-red or long distances, corresponding to the more familiar divergences from the long range interactions of massless quanta. The net upshot is that the quantum corrections to the Einstein action are finite and we do not have the embarrassment of an infinite number of undetermined couplings. Thus, for instance, the R^4 term mentioned earlier, that governs two to two scattering of gravitons, gets a finite calculable correction at one loop.

String theories therefore afforded the possibility, for the first time, to make computable corrections to Einstein's theory within a robust framework. It sidestepped some of the difficulties that QFT had by introducing a tower of additional particles interacting in a manner constrained by the geometry of strings.

So what was the downside? First, string theories come with a lot of additional baggage. There are, of course, all the stringy massive excitations that need to be postulated. Also, as alluded to earlier, one needs to consider supersymmetric versions of string theory to avoid problems like a runaway vacuum and/or large cosmological constant. These versions are consistent only in ten spacetime dimensions, necessitating the presence of additional curled up tiny dimensions. Both supersymmetry (even mildly broken) and additional spacetime dimensions are currently unobserved. But on the plus side these could be features rather than bugs. The Kaluza–Klein idea of additional dimensions as a way to put together gravity with other forces predates string theory and cannot be ruled out at this stage of experimental input. In fact, this may well be a feature rather than a bug since many string compactifications naturally realize popular models of grand unification with the bonus of gravity. Supersymmetry (in mildly broken form) is theoretically attractive for solving problems involving going beyond the standard model of particle physics. We might be on the threshold of knowing if this is indeed the path nature takes as we explore more of the physics beyond the standard model, at the Large Hadron Collider (LHC). The best thing is to wait and see what the verdict is.

The second (and more serious) criticism that was levelled against string theory at this stage of its development was that its approach to quantum gravity was much too tied up with the QFT one of understanding the interactions of gravitons in a perturbative expansion. One largely ignored the link to spacetime geometry. However, many of the conundrums of quantum gravity have to do with the behaviour of black holes or cosmological singularities. These cannot be addressed by studying the interaction of gravitons – they are intrinsically nonperturbative in that they involve a macroscopically large number of excitations which creates qualitatively new effects in spacetime geometry such as horizons or singularities. An analogy is with the Schwinger effect in QED where the presence of a macroscopic electric field leads to the pair production of electron–positron pairs. In fact, there is a similar effect in gravity – the Hawking radiation from black holes. Moreover, an understanding of the scattering of gravitons does not tell us about how to make sense of spacetime at the quantum level. We have been assuming spacetime to be a fixed background with small graviton fluctuations.

String theory – addressing the puzzles of black holes

Beginning in the mid-nineties, thanks to the work of Sen, Seiberg, Witten, Polchinski, Stominger, Vafa and many others, one gained a better grip of string theory, beyond the perturbation expansion in terms of surfaces with more and more holes. This expansion is limited by the requirement that the interaction strength $g_s^2 \ll 1$. The picture that was now pieced together was one where the different varieties of string theory were actually different perturbative corners of a single entity. An important ingredient in this understanding was the presence of so-called D(irichlet)-branes. These are macroscopic defect-like solutions of closed string theory. They are extended objects, ranging in extent from being point like to filling the entire spacetime. The unusual feature of these objects is that they can equally well be viewed as loci where open string endpoints can live (hence Dirichlet boundary conditions). In either way of viewing these objects they are intrinsically non-perturbative in having a tension (now mass per unit worldvolume) which is proportional to $1/g_s$ and thus invisible in an expansion in positive powers of g_s^2. In this sense they are the solitonic objects of string theory.

The understanding of these objects enabled string theorists to try and address some of the most puzzling features of black holes. Namely, the discovery of Hawking, Bekenstein and others about their behaviour as thermodynamic objects in the mid-70s. This discovery arose from the study of quantum fields in the background of black holes. The result of these studies was that black holes appeared to have a thermodynamic entropy and moreover, radiate quanta of the other fields at a fixed temperature. There is a universal formula for the entropy

$$S_{\rm BH} = \frac{A_{\rm H}}{4G_{\rm N}\hbar}, \qquad (8)$$

where $A_{\rm H}$ is the area of the horizon of the black hole. The accompanying Hawking temperature is (for a Schwarzschild black hole of mass M)

$$T_{\rm H} = \frac{\hbar}{8\pi G_{\rm N} M}. \qquad (9)$$

More generally, it is proportional to the surface gravity of the black hole. Note the presence of $\hbar$ in these formulas showing it to be intrinsically quantum. It was also seen that black holes obey the first and second laws of thermodynamics with the above assignments.

To associate such an entropy to a black hole is deeply puzzling from the point of view of Einstein's theory. Black holes solutions obey various uniqueness theorems. Thus it seems impossible to assign any microstructure to them – there is only one classical configuration characterized by a given total energy, charge and angular momentum. This is unlike the situation with respect to gas molecules in a box in which there are a huge number of internal configurations corresponding to the same total energy, charge and angular momentum. Classically, one would thus assign zero entropy to a black hole.

But there is a further puzzle: even if one were to assume that the entropy is non-zero due to some quantum microstructure, the fact that it is proportional to an area is difficult to understand. If the gravitational field were some kind of (approximately) local field like the electric or magnetic fields, then the entropy of the quanta would be extensive in the volume of the object – in this case, that enclosed by the horizon of the black hole. The fact that $S_{\rm BH}$ is proportional to the area suggests that the underlying degrees of freedom of quantum gravity are *far fewer* than one might, at first sight, imagine.

Coming back to string theory, the dual perspectives on D-branes enabled Strominger and Vafa, for the first time, to undertake a precise counting of the microstates of black holes (a special class of them which are supersymmetric). This exploited the above dual description of D-branes. On the one hand, they could be viewed as black hole (black brane) like solutions from a closed string point of view. On the other hand, they were seen to have open string excitations which could be counted microscopically. Thus in the original Strominger–Vafa case, when $S_{\rm BH}$ in eq. (8) is expressed in terms of the charges, it takes the form

$$S_{\rm SV} = 2\pi\sqrt{Q_1 Q_5 N_L}, \qquad (10)$$

where Q_1, Q_5, N_L refer to three different charges carried by the black hole. This is exactly reproduced from the statistical mechanical counting of the number of different excitations on an open string with these different charge species. In fact, this can be generalized away from the strictly supersymmetric limit by turning on an additional charge in which case we have the entropy $S'_{\rm SV} = 2\pi\sqrt{Q_1 Q_5}(\sqrt{N_L} + \sqrt{N_R})$ which is once again reproduced by a simple state counting. This was further generalized to a wide variety of cases immediately (carrying multiple charges, angular momenta, in different dimensions, etc.). In each case the agreement was striking since the functional form of A can depend quite nontrivially on the charges, etc. as we see from above.

Moreover, it was not just the entropy that was matched but also the detailed profile of Hawking radiation as well as the absorption cross section (gray body factor). Here there were seminal Indian contributions by Sumit Das, Samir Mathur, Avinash Dhar, Gautam Mandal and Spenta Wadia.

In the decade that followed, this striking microscopic agreement with the semi-classical answer was further deepened. The corrections to Einstein's equations by higher derivative terms lead to corrections to $S_{\rm BH}$. The systematic way to compute these was formulated by Wald through an explicit entropy formula. As we saw, string theory leads to definite corrections (even at the classical level) to Einstein's gravity. Considering the subleading (in the charges such as Q_1, Q_5, etc. in eq. (10)) contributions to the microscopic counting leads, in many case to a precise match of an infinite number of terms. Building on work by de Wit and collaborators, Atish Dabholkar, Ashoke Sen and collaborators, performed some of these striking checks. Ashoke Sen went on to also calculate some of the leading quantum corrections (one loop) to the entropy and matched terms which are logarithmic in the area with corresponding terms in the microscopic answer. These show how robust the matching of entropy in string theory is, to both classical and quantum corrections to Einstein's equations.

Quantum gravity as a quantum field theory!

The remarkable successes in understanding black holes gave a great deal of confidence in the ability of string theory to give a consistent description of quantum gravity beyond the perturbative regime of graviton scattering. Examining deeper the underlying reasons for the agreement between the microscopic and macroscopic computations of black hole entropy led to one of the most striking developments of contemporary theoretical physics. This was the so-called gauge–string duality or AdS/CFT correspondence proposed by the Argentinian physicist, Juan Maldacena. For an introduction to Gauge-String Duality, see ref. 2.

This duality arose from the two different descriptions of D-branes which we have already mentioned. On the one hand, they can be viewed as black hole like gravitational solutions – a closed string description involving gravity. On the other hand, they can be viewed in terms of quantum gauge field theories – the open string description. This open-closed string duality is at the origin of this remarkable connection. By taking an appropriate limit, what one has is an exact quantum equivalence between, for instance, a quantum Yang–Mills theory (with no gravity) and a closed string theory which describes quantum gravity. Moreover, the connection is 'holographic'. The QFT can be viewed as being defined on the boundary of the space-time whose quantum behaviour we are capturing. In the simplest cases, this spacetime is an asymptotically anti de Sitter (AdS) space-time. AdS spacetimes are the analogue of hyperbolic spacetimes in Euclidean signature. They can be thought of as solid cylinders with the cylindrical axis being the time direction. The QFT is then on the boundary (or surface) of the solid cylinder and is an asymptotically conformal field theory – i.e. a fixed point of the renormalization group with relativistic conformal invariance.

This relation provides a deeper insight into the puzzles of black hole entropy. It first equates the entropy with that of the QFT at the same temperature as the Hawking temperature of the black hole (in AdS). So we have a physical picture of the degrees of freedom underlying the entropy. Secondly, the fact that the microscopic description is in terms of a field theory on the boundary means that it will be extensive in one lower dimension – which is exactly what the area law indicates. Thus the idea that quantum gravity has fewer degrees of freedom compared to a QFT (in the same number of dimensions) is very well borne out. Indeed quantum gravity appears to be described by a QFT but in a more novel way than one imagined.

This duality has had consequences *both* for the physics of gauge theories as well as gravity since the equivalence is a strong–weak coupling duality. The dictionary of parameters reads as

$$\lambda \propto \ell_{\text{AdS}}^{\alpha} \Rightarrow \lambda \gg 1 \leftrightarrow \ell_{\text{Ads}} \gg 1 \text{ and}$$
$$\lambda \ll 1 \leftrightarrow \ell_{\text{AdS}} \ll 1. \tag{11}$$

Here λ is the gauge coupling while ℓ_{AdS} is the length scale of AdS and is inversely proportional to its curvature. The exponent $\alpha > 0$. Thus $\ell_{\text{AdS}} \gg 1$ corresponds to a weakly curved spacetime where Einstein's equations can be trusted. But this is exactly the regime where the field theory is strongly coupled ($\lambda \gg 1$). Thus classical Einstein equations are able to tell us about highly quantum nonperturbative correlators in a gauge theory! Conversely, the regime of weakly interacting theories ($\lambda \ll 1$) which is accessible to perturbation theory describes highly curved AdS spacetime which needs a full string theory completion. We gain complementary information from gauge theory and gravity about each other.

The two sides of the equivalence are two different pillars of theoretical physics, namely, general relativity and quantum field theory. About a century after their discovery we are unearthing new facets to them. This is what gives this particular duality a magical flavour. It is a demonstration of the theoretical robustness of string theory that it is able to encompass both QFT and quantum gravity, while at the same time providing novel insights to both fields which predate its own existence. It is not difficult to falsify many of the statements that this connection entails and yet, in a decade and a half, it has continually passed every nontrivial check. The original correspondence has been generalized to a number of situations: in diverse dimensions, away from the conformal limit, with less and even no supersymmetry. In fact, other theories of gravity like the Vasiliev theory of higher spin gauge fields (with a single tower of massless gauge fields of spin $s \geq 2$) also appear to have holographic dual QFTs. Thus the connection is very wide-ranging and indicates a universal aspect of quantum gravity. It however remains a challenge to generalize this correspondence further to time-dependent geometries (such as de Sitter, which describes our accelerating universe).

To conclude

In this brief sketch we tried to give a bird's eye view of an approach to quantum gravity that over the last 80–90 years has steadily made progress towards explicating one of the most enduring enigmas of theoretical physics – on how to reconcile general relativity with quantum mechanics. The step by step progress from gravitons to black holes has been a long and arduous one and has necessitated the development of an unexpectedly rich framework – that of string theory. The unfolding developments of gauge-gravity duality show us how surprises still lurk in gravity. The ongoing debate about firewalls at black hole horizons is another instance of our perhaps having to make radical adjustments in our thinking in order to fully grasp quantum gravity. The next century of general relativity will not lack for intellectually stimulating developments.

Notes

1. Strictly speaking this is the spectrum of the superstring which is a supersymmetric version of the string that contains fermionic excitations as well. The purely 'bosonic' string appears to be ill behaved in that its lowest state is tachyonic – a signature of instability of the vacuum.
2. Strictly speaking, g_s^2 is not a parameter, rather it is the expectation value of a scalar field and is fixed dynamically. It measures the strength of quantum interactions and always arises in combination with $\hbar$. This is unlike the string tension T which is a genuine dimensionful parameter of the theory which sets the scale of the size of the string.

1. Special Section on Superstrings. *Curr. Sci.*, 2001, **81**(12), 1547–1616.
2. Gopakumar, R., What is the AdS/CFT correspondence?, *Physics News*, January 2010.

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