Experimental tests of macrorealism: an assessment

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The macrorealistic inequality of Leggett and Garg is reviewed, along with a recent experiment that aims to test it.

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Introduction

IF pressed to express the difference between the quantum and classical philosophies of physics in one pithy phrase or sentence, I would say that it lies in the way the two approaches conceive of the state of the world, or what it means for something *to be*. In the classical view, a system has definite properties at all times, and in principle, these properties are all knowable to us with perfect precision. In quantum mechanics, this is not so, and there is no system for which *all* the properties are definitely knowable, or even definite to begin with. The detailed development of the theory in terms of the quantum mechanical amplitude provides a complete prescription for calculating and predicting just which properties are indefinite in a given situation and in precisely what way. There is however, one knotty problem, in that one is compelled to make a dichotomy between the observer and the observed for which there is no satisfactory basis. It is easy to see why such a dichotomy must be made. It is necessitated by the underlying idea that the state of a system is indefinite. Therefore, its properties cannot be said to have any independent existence of their own. They can only be brought into existence, or actualized, by the act of measurement. This act lies outside the processes by which the world goes about its business, and all efforts to bring it into the ambit of the latter have so far failed. Capping it all is the fact that all experiments to date are consistent with the predictions of quantum theory, which have been found accurate to better than a part per billion in some cases.

Given this state of affairs – of a theory at once so successful and so bewildering – it is natural to ask if one can enlarge the theory so as to make it compatible with our classical world view, and yet retain its quantitative predictive content. One such suggestion dates to the earliest days of quantum mechanics, namely that the properties of systems are in fact definite, but are not definitely knowable. In other words, that a system has *hidden variables*, in terms of which its properties are definite, but that these variables are inaccessible to us for some reason. It is difficult to imagine how such variables could be viable in light of phenomena such as two-slit interference, but that is not the same as a proof of their nonexistence. This gap is filled by the Bell inequality (by which I mean not only Bell's original inequality[1], but all subsequent derivatives based on the same logic, such as that of Clauser *et al.*[2]). This inequality is a consequence of any *local* hidden variables theory satisfying a small number of very reasonable requirements. Most importantly, the inequality is testable, and as is well-known today, it is violated by the experimental data. A large class of theories is thus ruled out, and quantum mechanics survives.

In the same way, the macrorealistic inequality is a consequence of any macrorealistic theory satisfying a small number of classical-minded requirements, chief among which is that the state of a macroscopic system really be definite[3]. This inequality is also testable, and although experiments have been performed to this end, it is unclear how significant the tests done to date are. The purpose of this article is to review this inequality and one of the main experiments[4]. In addition to their direct implications for hidden variables theories, the tests of Bell's inequality have spurred a great deal of experimental innovation and research into quantum communication and information theory. Perhaps the same will happen with the macrorealistic inequality.

The plan of the article is as follows. The next section contains a brief exposition of macrorealism. The inequality is discussed in the following section, and one of the early experiments involving a dc SQUID-based device[4] is discussed in the text. The implications of the experiment are briefly discussed in the conclusion.

What is macrorealism?

Phenomena such as kaon oscillations, neutron interferometry, and Rabi oscillations make it difficult to sustain the idea that the state of systems at the *microscopic* scale can be definite. At the *macroscopic* scale, however, our everyday experience is that the very opposite is true. We

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formalize our classical macroscopic world view, which we call macrorealism, in terms of the following three assumptions. (A1) *Macroscopic realism*: A macroscopic system with two or more macroscopically distinct states available to it will at all times be in a definite one of those states. (A2) *Noninvasive measurability at the macroscopic level*: It is possible, in principle, to determine the state of a macroscopic system with arbitrarily small effect on its subsequent dynamics. (A3) *Induction*: If a measurement is made on a macroscopic system, it is meaningful to talk about what the result of an earlier measurement would have been, had such a measurement been made. All three assumptions are implicitly made by us without conscious thought countless times every day. However, each of them explicitly contravenes quantum mechanical precepts.

The assumption A1 lies behind our understanding of the functioning of every experimental apparatus. Indeed, if it did not, we would not trust that our apparatus was doing what we wanted it to do. We stress that A1 does not require us to actually know the state of the system. Thus, we can assert that a coin lying on the floor *is* either heads or tails, even if we do not know which. A strict extrapolation of quantum mechanics to the macroscopic realm would allow the coin to be in a superposition of heads and tails.

The assumption A2 is invalid in quantum mechanics, but is natural in classical mechanics. For example, any thermometer used to measure the temperature of a glass of water necessarily draws some heat from the water and thus perturbs it, but we believe that we can make this perturbation as small as we like. It would in fact be very odd to accept A1 but reject A2, because there would then be no way to give operational meaning to the notion of being in a definite state at all times. Still, we adopt A2 explicitly and not as a logical corollary of A1.

The assumption A3 is equally natural. Imagine that we have 100 identically manufactured balloons, 50 of which are released and allowed to float away. The other 50 are punctured with a pin. Would this second set of balloons also have floated away if they had been released without being punctured? Classically, this is an entirely meaningful question, with the clear answer of yes. Quantum mechanically, the question is illegitimate. This is the central point of Bohr's reply[5] to Einstein *et al.*[6], and is perfectly captured by Peres's[7] dictum: 'Unperformed experiments have no results.' We can assign a value only to those physical quantities that the apparatus is set to measure.

The macrorealistic inequality

To derive the macrorealistic inequality, let us suppose that we can unambiguously divide into two sets the states in which our macrosystem can be, and distinguish these two sets by assigning to the first a value +1, and to the second a value −1 of a variable Q. The value of Q at time t is denoted by $Q(t)$. For specificity, we may imagine an rf SQUID threaded by an external flux of half a superconducting flux quantum. At low temperatures, the low energy states correspond to a current in the SQUID ring of about 1 nA, flowing either clockwise ($Q = +1$) or anticlockwise ($Q = -1$). (We assume that the probability of observation of any other state is negligibly small, but we could include a nonzero value for it without any change in our conclusions by extending the analysis in analogy with Garg and Mermin[8], for example.) The dynamics of the variable Q can be arbitrary and unknown, even stochastic.

We now imagine a large ensemble of such systems, all identically prepared at an initial time t_0. Next we consider three subsequent times $t_1 < t_2 < t_3$. It is an elementary algebraic identity that

$$Q(t_1)Q(t_2) + Q(t_2)Q(t_3) - Q(t_1)Q(t_3) \le 1, \tag{1}$$

where the quantity on the left in guaranteed to exist by assumption A1. The values of $Q(t_1)$, $Q(t_2)$ and $Q(t_3)$ may vary from one member of the ensemble to the next, but the identity holds for every member of the ensemble. It must therefore hold when averaged over the entire ensemble. Denoting this average by angular brackets, we have

$$\langle Q(t_1)Q(t_2)\rangle + \langle Q(t_2)Q(t_3)\rangle - \langle Q(t_1)Q(t_3)\rangle \le 1. \tag{2}$$

For future reference, we define

$$\langle Q(t_1)Q(t_2)\rangle + \langle Q(t_2)Q(t_3)\rangle - \langle Q(t_1)Q(t_3)\rangle = K, \tag{3}$$

so the inequality (2) is

$$K \le 1. \tag{4}$$

Next, we divide the full ensemble into three subensembles S1, S2 and S3, each assumed to also be very large. We measure Q at t_1 and t_2 for members of S1, at t_2 and t_3 for S2, and t_1 and t_3 for S3 (Figure 1). We then find the average of the measured values of the product $Q(t_1)Q(t_2)$ for subensemble S1, and denote it by $\langle Q(t_1)Q(t_2)\rangle_{S1}$. The averages $\langle Q(t_2)Q(t_3)\rangle_{S2}$ and $\langle Q(t_1)Q(t_3)\rangle_{S3}$ are defined in the same way. The existence of these averages is assured by the assumption A2. For example, for a member of S1 the value of $Q(t_2)$ is unaffected by the measurement of Q at t_1 and so what we measure for the product $Q(t_1)Q(t_2)$ is the underlying value. We now define

$$K_{\text{expt}} = \langle Q(t_1)Q(t_2)\rangle_{S1} + \langle Q(t_2)Q(t_3)\rangle_{S2} - \langle Q(t_1)Q(t_3)\rangle_{S3}. \tag{5}$$

As the notation suggests, the quantity K_{expt} is experimentally measurable.

The final step in the argument is to invoke A3, the assumption of induction. Consider the average $\langle Q(t_1)Q(t_3)\rangle_{S3}$.

If instead of measuring Q at t_1, we had measured it at t_2, A3 allows us to conclude that the values we would have obtained would have exactly the same characteristics as those of subensemble S2, or that if we had made the second measurement at t_2 instead of t_3, we would have got exactly the same result as we did from S1. Thus, the subensemble averages are identical to those over the entire ensemble, and we may write

$$\langle Q(t_1)Q(t_2)\rangle_{S1} = \langle Q(t_1)Q(t_2)\rangle, \tag{6}$$

etc. Hence,

$$K_{\text{expt}} \equiv K, \tag{7}$$

and the inequality (4) becomes

$$K_{\text{expt}} \leq 1. \tag{8}$$

This is a macrorealistic inequality. Similar inequalities may be derived by considering measurements at four times, for example[3].

Quantum mechanically, it makes no sense to talk of the quantity K, but K_{expt} is entirely meaningful. In any two-state situation where we have perfect flip-flop oscillations between the two states at a frequency Ω, quantum theory predicts that

$$\langle Q(t_1)Q(t_2)\rangle = \cos \Omega(t_2 - t_1). \tag{9}$$

Choosing

$$t_3 - t_2 = t_2 - t_1 = \pi/3\Omega, \tag{10}$$

we obtain

$$K_{\text{expt}} = 3/2, \tag{11}$$

and

$$K_{\text{expt}} > 1, \tag{12}$$

in violation of the macrorealistic inequality.

We thus see that quantum theory is at odds with macrorealism. This is hardly surprising, but just as with Bell's inequality, the significance of the inequality, i.e. eq. (8) is that it is, in principle, experimentally testable. Since the inequality is a necessary condition for macrorealism to hold, any experimental observation that it is violated would make macrorealism that much less tenable. In this connection it is important that the inequality is violated by a fairly large margin in the most extreme case, i.e. eq. (12). This increases the chances for observing a violation when the flip-flop is less than perfect. Naturally, the systems which are most likely to reveal a violation are those which display coherent oscillations of a macrovariable, also known as macroscopic quantum coherence (MQC). However, as stressed by Leggett[9], observation of MQC by itself is not sufficient to rule out macrorealism.

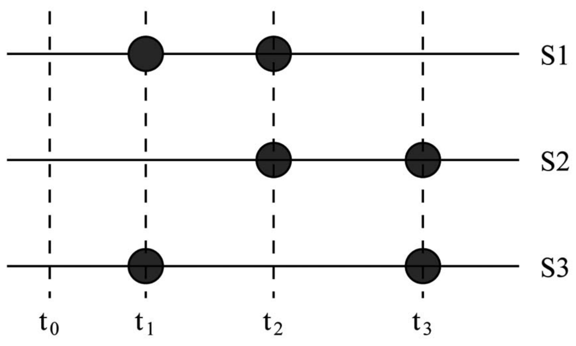

Figure 1. Illustration of the subensembles S1, S2, and S3. All members of all subensembles are identically prepared at t_0. The heavy circles indicate subsequent measurements.

It is worth making two additional remarks. First, we could add to A2 the demand that the measurements be of the *ideal negative result* type. We imagine that the measuring apparatus is set up to respond only if the system is in the $Q = +1$ state, say. Then we are allowed to conclude that $Q = -1$, if the apparatus shows no response. Quantum mechanically, a measurement is a measurement is a measurement, whether of the positive or negative type. Macrorealistically, however, it is difficult to argue that the state of a system has been altered by a non-observation, and the notion of a nonivasive measurement becomes much more credible.

Second, many authors refer to macrorealistic inequalities as Bell inequalities in time. While this usage is fine in so far as the structure of the mathematical argument is similar in the two cases, it is likely to obscure the import of the entire exercise. Bell's inequality derives its force from the requirement that the two subsystems be widely separated, and would hardly be regarded as relevant to a mu-mesic He atom in which we measured the correlations of the muon and electron spins (assuming this were even possible). In the same way, the macrorealistic inequality derives its force from the requirement that the states in question be macroscopically distinct. If we find that the inequality is violated for a microscopic system, that would not be front-page news today. (At least it should not be.)

(I conclude this section by noting that the assumption of induction did not appear in the original paper[3], as indeed it did not in Bell's paper[1]. By the time of writing the original paper[3], however, the importance of this assumption was well and widely established. In addition, the three-time distribution $\rho(Q_1, Q_2, Q_3)$ that was introduced in this paper[3] was described in a way that left scope for it to be confused *for the distribution of three actual sequential measurements*, as opposed to a *postulated distribution which exists only under the truth of the assumptions of macrorealism*. The first interpretation makes nonsense of the entire argument, and so while a careful reader might not have fallen into the trap of making this interpretation, it should not have entered into the writing in the first place. I am certain that neither mistake would have occurred if my co-author had not generously let me write the parts of the paper in question, and there is no adequate apology that I can make for letting him down so completely.)

The transmon experiment

In this experiment[4], the system is what is now known as a transmon[10,11], of which we now give a brief discussion. Readers interested in a more accurate and detailed one must consult the original references (see also ref. 12). Very broadly speaking, the transmon consists of a two-Josephson-junction dc SQUID capacitatively coupled to a electromagnetic cavity resonator. The back and forth tunnneling of Cooper pairs across the Josephson junctions (also described as a Josephson plasma oscillation) is coupled to the oscillations of the electric field inside the cavity, and both the plasma oscillations and the cavity field oscillations must be regarded as quantum mechanical dynamical degrees of freedom in the quantum mechanist's view of the world. The remarkable thing is that a large number of experiments on this and related systems are strongly in accord with the quantum mechanical description (where they would not have been 30 years ago), so this patently macroscopic device, made by lithographic deposition of bits of metals and oxides on insulating substrates, is (at least a priori) a viable candidate for a test of the macrorealistic inequality.

In what follows, we give a highly condensed discussion of the relevant degrees of freedom and their quantum mechanical description.

The dc SQUID is formed by connecting two superconducting islands by two Josephson junctions (Figure 2). Cooper pairs can tunnel through the junctions, and if we denote the state where the number of pairs on one of the islands is N by $|N\rangle$, the Hamiltonian of the junction can be written as[10,12]

$$\mathcal{H}_{\mathrm{J}} = \sum_N \Big[4E_{\mathrm{C}}(N - N_{\mathrm{g}})^2 |N\rangle\langle N| - \frac{E_{\mathrm{J}}}{2}(|N\rangle\langle N+1| + \mathrm{h.c.}) \Big]. \quad (13)$$

Here, E_{C} is a charging energy reflecting the fact that because of the charge on the superconducting islands,

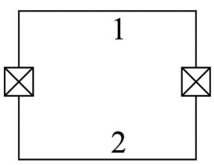

Figure 2. Schematic of the dc SQUID. The two line segments labelled 1 and 2 represent the two superconducting elements, and the boxes with X's represent the Josephson junctions.

there are electric fields between the islands and the voltage gate. The energy E_{C} is determined by the various capacitances between these elements. Likewise, E_{J} is the Josephson energy reflecting the propensity of Cooper pairs to tunnel between the islands. This quantity is determined by the intrinsic junction properties (or the Josephson plasma frequency, equivalently) and the external flux, Φ, through the SQUID loop: $E_{\mathrm{J}} = E_{\mathrm{J0}}\cos(\Phi/\Phi_0)$, where $\Phi_0 = eh/2c$ is the superconducting flux quantum. Finally, $N_{\mathrm{g}} \in (0, 1)$ is a charge offset determined by the gate voltage and stray environmental charges. The value of $E_{\mathrm{J}}/E_{\mathrm{C}}$ chosen for transmons is 50–100, which is critical to the difference with a Cooper pair box, where $E_{\mathrm{C}} \gg E_{\mathrm{J}}$. For the phase qubit: $E_{\mathrm{J}}/E_{\mathrm{C}} \sim 10^4$, although this is a different circuit from the transmon and Cooper pair box. There are no dc connections to the SQUID.

As shown in Figure 3, the above device is placed inside a one-dimensional (1D) transmission line resonator in the form of a long section of a coplanar waveguide (a 2D analog of a coaxial cable). The charge distribution on the superconducting islands of the SQUID couples to the electric field of the resonator, $\mathcal{E}$. To a first approximation, we may take $\mathcal{E}$ as transverse to the long axis of the resonator, $\hat{\mathbf{x}}$, and write it as a sum of normal modes akin to 1D standing waves with antinodes at the ends, $x = \pm L/2$. For the nth mode, the wavenumber $k_n = n\pi/L$, the mode function $f_n(x)$ is $\sin k_n x$ (n odd), or $\cos k_n x$ (n even), and the frequency $\omega_n = vk_n$, where v is the speed of a travelling wave. Denoting the inductance and capacitance of the waveguide per unit length along $\hat{\mathbf{x}}$ by ℓ_{W} and c_{W}, we have $v = 1/\sqrt{\ell_{\mathrm{W}} c_{\mathrm{W}}}$. The voltage, $V(x, t)$, between the central and outer conductors of the cavity at a position x along the long axis can then be written as a linear superposition of different modes, and since each mode can be regarded as an independent harmonic oscillator, we can obtain the quantum mechanical operator corresponding to $V(x, t)$ by promoting the mode expansion coefficients to

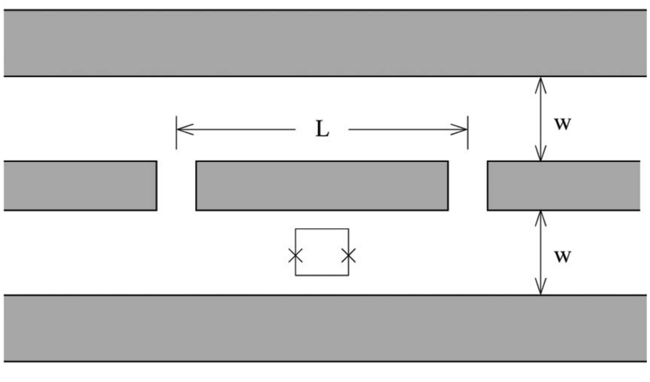

Figure 3. Schematic top view showing the 2D cavity resonator, the waveguide in which it sits, and placement of the dc SQUID. The shaded regions are superconducting. The sketch is not to scale; in reality the L/w ratio is larger, and the SQUID loop is much smaller in relation.

harmonic oscillator raising and lowering operators. In this way we obtain

$$V(x,t)=\sum_n\left(\frac{\hbar\omega_n}{Lc_{\mathrm{W}}}\right)^{1/2}f_n(x)(a_n(t)+a_n^\dagger(t)). \qquad (14)$$

(One way of seeing that this is correct is to consider the potential energy of the charges on the cavity walls. This energy is given by integrating $c_{\mathrm{W}}V^2(x,\ t)/2$ over the length of the cavity. If the nth mode is in its ground state, the potential energy in that mode should be half the zero point energy, i.e. $\hbar\omega_n/4$, and this is precisely what eq. (14) gives.)

Next we discuss the coupling of the SQUID to the resonator. Since the SQUID is placed at $x = 0$, the lowest cavity mode with a nonzero coupling is the $n = 2$ mode, whose rms voltage at $x = 0$ is $V_{\mathrm{rms}} = (\hbar\omega_2/Lc_{\mathrm{W}})^{1/2}$, and the corresponding rms electric field is $\mathcal{E}_{\mathrm{rms}} = V_{\mathrm{rms}}/w$, where w is the distance between the central and outer cavity conductors. If we denote the dipole moment of the charge distribution on the islands by d, then the coupling strength may be written as

$$\hbar g\simeq d\mathcal{E}_{\mathrm{rms}}. \qquad (15)$$

Let us now imagine diagonalizing, $\mathcal{H}_{\mathrm{J}}$, the SQUID Hamiltonian (eq. (13)), and thus obtaining eigenstates $|0\rangle$, $|1\rangle$, etc. with energies E_0, E_1, etc. The lower the quantum numbers of the states involved, the further away from the correspondence limit and classicality one is. The device will behave most nonclassically if we can stay just within the manifold of the $|0\rangle$ and $|1\rangle$ states, when we will able to treat it as a two-state system. Writing $E_{ji} = E_j - E_i$, it is found[10] that for the low-lying levels with $E_{\mathrm{J}}/E_{\mathrm{C}} \geq 20$

$$E_{j,j-1}\approx\sqrt{8E_{\mathrm{J}}E_{\mathrm{C}}}-jE_{\mathrm{C}}. \qquad (16)$$

That is, the levels are approximately those of an anharmonic oscillator with a spring that is not quite Hookean but softens upon stretching. Strictly speaking, we should write $E_{j,j-1}$ as a function of N_{g}, the offset charge, and give this dependence. For $E_{\mathrm{J}}/E_{\mathrm{C}} \geq 20$, however, this dependence contains an exponential factor $\exp(-\sqrt{8E_{\mathrm{J}}/E_{\mathrm{C}}})$, and is thus essentially negligible. This means that the SQUID is highly insensitive to charge noise, or fluctuations in N_{g}. At the same time, the anharmonicity of the energies (eq. (16)) is enough that if $n = 2$ mode of the resonator is tuned to the E_{10} transition energy, the $|1\rangle \leftrightarrow |2\rangle$ transition is essentially never excited. Not only that, but the anharmonicity is enough that if we wish to apply $\pi/2$ or π pulses to the SQUID, we can make the pulses broad enough in time (and thus narrow enough in frequency) as to also almost never excite the $|1\rangle \leftrightarrow |2\rangle$ transition and yet be of durations small in comparison to T_1 and T_2 decoherence times. (Greater anharmonicity would allow us to work with even shorter pulses, but there is not much point in doing so as microwave pulses much shorter than 10 ns are difficult to attain.) It is this combination of charge noise insensitivity and sufficient anharmonicity that allows us to treat the SQUID in the transmon as an artificial atom with just two states, with furthermore, a high degree of coherence. It is these two states that will correspond to the distinct states of our purported macro-variable, and to which we will assign the values $Q = \pm 1$.

We arrive in this way at a simplified but quite accurate description of a transmon as a two-level atom coupled to just one mode of the cavity, or harmonic oscillator. If we represent the two-level atom as a spin-1/2 system, and make the rotating wave approximation, we obtain the classic Jaynes–Cummings Hamiltonian

$$\mathcal{H}=-\frac{1}{2}\hbar\omega_{10}\sigma^z+\hbar\omega_r a^\dagger a+\frac{1}{2}\hbar g(a^\dagger\sigma^-+a\sigma^+). \qquad (17)$$

Here, $\omega_{10} = E_{10}/\hbar$, ω_r is the resonator frequency, and $\sigma^{\pm} = \sigma^x \pm \mathrm{i}\sigma^y$. We have not shown the couplings of the cavity and the SQUID to the rest of the universe, which give rise to spontaneous decay of excited states of either system. Estimates of the various frequencies may be made as follows. For the $n = 2$ mode, $\omega_r/2\pi = v/L$. Taking $L = 1$ cm, and v as the speed of light in order of magnitude, we get $\omega_r/2\pi = 30$ GHz. In the experiment[4], the large dielectric constant of the sapphire substrate reduces v, and measured frequencies are $\omega_r/2\pi = 5.8$ GHz, and $\omega_{10}/2\pi = 5.3$ GHz. The coupling $\hbar g$ may be estimated via eq. (15). As is typical of similar coplanar waveguide resonators[13], the capacitance $Lc_{\mathrm{W}} \simeq 1$ pF, $V_{\mathrm{rms}} \simeq 4$ μV, and $\mathcal{E}_{\mathrm{rms}} = V_{\mathrm{rms}}/w \simeq 0.4$ V/m. The dipole moment d is estimated as $(2 \times 10^4)ea_0$ (with a_0 the Bohr radius), corresponding to a displacement of the Cooper pair by 0.5 μm. This leads to $g/2\pi \sim 50$ MHz. The significant point here is that this coupling is substantially greater than the 'atomic' linewidth $\gamma \geq \kappa/2\pi \sim 3$ MHz or the cavity linewidth $\kappa/2\pi \simeq 30$ MHz (corresponding to a cavity Q-factor of about 200, not to be confused with our dynamic variable, Q). It is thus possible to drive many cycles of Rabi oscillation between $|0\rangle$ and $|1\rangle$ states of the SQUID.

While the coupling is large in the sense just discussed, it is also quite a bit smaller in magnitude than the detuning, $\Delta = \omega_{10} - \omega_r$. We can then eliminate the atom–cavity interaction in eq. (17) to first order in g by an obvious unitary transformation U, obtaining

$$U\mathcal{H}U^\dagger=-\frac{1}{2}\hbar\omega_{10}'\sigma^z+\hbar(\omega_r'+\chi\sigma^z)a^\dagger a, \qquad (18)$$

where

$$\chi=g^2/\Delta, \qquad (19)$$

is a dispersive frequency shift, and the primes on ω_{10} and ω_r indicate frequency renormalization or shifts of the same order. (In what follows, we shall not explicitly show the primes, and use unprimed quantities for the renormalized frequencies. Second, a more accurate expression for χ includes effects from the $|2\rangle$ state of the SQUID.) The last term in eq. (18) has the interpretation that depending on whether the SQUID is in its ground or first excited state, the frequency of the resonator is shifted up or down by an amount $\chi/2\pi$, which equals 1.75 MHz in the experiment. This is the means by which the value of Q is determined, as we now discuss.

The experiment consists of applying a continuous microwave signal (the *drive tone*) at ω_{10}, and another continuous signal (the *measurement tone*) at ω_r. Because the resonance frequency of the cavity is shifted up or down by χ, the cavity oscillator is being driven slightly off-resonance, and the reflected signal at the frequency ω_r acquires a phase shift that depends on the state of our two-level atom, i.e. on the value of Q. This reflected signal is homodyned, and both quadratures $X(t)$ and $Y(t)$ are measured. The key point is that if the measurement tone is weak, both X and Y provide *weak*, *continuous* and *nondestructive* measurements of $Q(t)$. We can write

$$X(t) = X_{\text{avg}} + X'Q(t) + \xi(t), \tag{20}$$

where X_{avg} is some average or dc offset, X' is a scaling factor that depends on the details of the analysing circuitry, and $\xi(t)$ is noise. A similar expression applies to $Y(t)$. It thus follows that a measurement of the $X-X$ correlation $\langle X(t)X(t')\rangle$ will contain information on $\langle Q(t)Q(t')\rangle$.

In fact, the experiment does not measure $\langle X(t)X(t')\rangle$ directly. Rather it measures its Fourier transform or the power spectrum $S_{XX}(\omega)$ of the X signal. The inverse Fourier transform of this power spectrum is then taken to obtain K_{expt}. The result quoted by the authors is that for $t_3 - t_2 = t_2 - t_1 \simeq 17$ ns, the quantity in eq. (5) is found to be

$$K_{\text{expt}} = 1.44 \pm 0.12, \tag{21}$$

in clear violation of eq. (8). We tentatively conclude that the observed behaviour of the SQUID system excludes a macrorealistic description. To make this conclusion more firm, however, we must discuss the assumptions inherent in the measurement and analysis scheme further.

We do not give the details[14–16] of the signal processing that must be performed in order to obtain K_{expt}, and let it suffice to say that this proccessing is plausible, if heavy. Thus, from the raw measurement of the power spectrum, one must subtract the noise power spectrum of the output amplifier, and scale by the frequency response curve of the measuring line (which contains filters, amplifiers, digitizers and whatnot). While many of these effects are independently measurable, it is clearly desirable to be able to minimize the number of such subsidiary transformations of the data. Nevertheless, it is clear that if $Q(t)$ is indeed undergoing underdamped oscillations at a frequency Ω, then $S(\omega)$ will display a narrow peak near Ω with a large area under it. The experimenters find a relatively narrow peak in $S(\omega)$ at $\omega/2\pi \simeq 10$ MHz with a width of about 2 MHz. (Note that this is the Rabi frequency, and not $\omega_{10}/2\pi$.) It is not surprising that under certain conditions, it is possible[16] to transcribe the inequality (8) directly into a condition on $S(\omega)$.

Next, let us discuss how well the conditions under which the experiment is done permit a true test of the macrorealistic inequality. First, the measurements are of the weak continuous type, and not the classic projective type. They are certianly not of the ideal negative result type, but are they nevertheless noninvasive? It seems to me that the answer is no, and this seems to be confirmed by Palacios-Laloy *et al.*[4]. It is reported that as the strength of the measurement signal is increased, the Rabi oscillations are washed out. Second, it seems to be assumed that measurements of $X(t)$ and $Y(t)$ do not affect the noise power spectrum, and that the measured quantity $\langle X(t)X(t+\tau)\rangle$ depends only on the delay τ. Both assumptions are plausible and testable via subsidiary experiments on the ensemble. More troublesome is a tacit assumption that the signal $Q(t)$ is unaffected because it is not time correlated with the noise, and it is not clear how to test this assumption.

But the biggest question hanging over the experiment is whether the two states in which the system lives are macroscopically distinct. As noted above, the dipole moment change is 10^4 times ea_0, which is indeed large compared to atomic and molecular moments. At the same time, the change in the number of Cooper pairs, N, as we go from the ground to the first excited state of the SQUID, is minor[10]. In both states $|0\rangle$ and $|1\rangle$, $\langle N\rangle = 0$ for $n_g = 1/2$. But even the uncertainty ΔN is of order 1: for $E_J/E_C = 50$, ΔN is 1.1 and 1.9 for the states $|0\rangle$ and $|1\rangle$ respectively. It is hard to unambiguously identify a macroscopic difference in the character of the states themselves. Still, the system studied is quite remarkable, and I cannot help but wonder if the founders of the subject conceived that quantum mechanics would one day be applied to such a strange degree of freedom.

Conclusion

In summary, we have reviewed the macrorealistic inequality and one of the leading experiments that purport to test it. One major improvement would be to perform a test using negative result measurements, since otherwise it is not plausible that a system would show both a violation of macrorealism and also that measurement does not perturb the dynamics of the system. (The two possibilities are almost irreconcilable, since it is, so to speak, the 'interference' caused by the observation at t_2 that makes the inequality fail for quantal systems.) Precisely such an test has recently been claimed in an experiment involving Cs atoms[17]. An atom is prepared in a superposition of hyperfine states in the ground state manifold,

$$\frac{1}{\sqrt{2}}(|F = 4,\ m_F = 4\rangle + |F = 3,\ m_F = 3\rangle), \tag{22}$$

and placed in two overlapping optical lattices, one that is sensitive to atoms in an $F = 4$ state, and the other to the $F = 3$ state. The two lattices coincide to begin with, so the atom will seek one of the coincident (deep) minima of the potential wells of the optical lattices. The two lattices are then dragged with respect to each other, and correlations of the position of the atom can be measured by means of operations on the spin states. The key points are that the measurements are indeed of a negative character, i.e. the atom is deduced to be in position *B* only if it is not found in *A* (if it is found in *A* that particular run is discarded), and that the correlations are measured for position differences up to 2 μm. While this is indeed a large distance on the atomic scale, the object whose position is indefinite on this scale is itself decidedly microscopic. Thus, while the experiment is undoubtedly a feat of skill, whether it entails a macroscopic object is open to question. Indeed, *its disconnectivity*[18] is less than that of an earlier experiment[19] showing diffraction effects with C_{60} (buckminsterfullerene) molecules, and neutron interferometry has been done with 'slits' 10 cm apart.

All this once again raises the question of when the states participating in a superposition can be regarded as *macroscopically* distinct. This is a vexing question that has resisted easy answers. It is clear that the state originally envisioned by Schrödinger of his hapless cat qualifies (Griffiths[20] refers to it as a 'grotesque' state), but is there some numerical measure that one can use to quantify grotesqueness? Despite their drawbacks, two of the best are still those proposed by Leggett: the disconnectivity[18], already mentioned, and the 'extensive difference'[9]. We refer the reader to the cited papers for details.

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