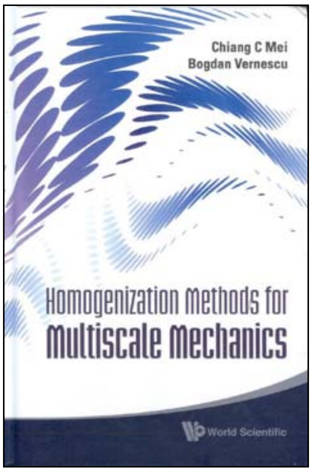

Homogenization Methods for Multiscale Mechanics. Chiang C. Mei and Bogdan Vernescu. World Scientific Publishing Co. Ptd Ltd, 5 Toh Tuck Link, Singapore 596224. 2010. xvii + 330 pp. Price: £48.00/US$ 72.00.

Homogenization is a matured subject in applied mathematics and there are quite a few nicely written textbooks on it. One may therefore ask why this new book is needed. What are its contents? The subject of homogenization has seen enormous progress over the past few decades and therefore a critical review will place the book in the context of these overall developments.

As the title indicates, this book deals with multiscale models of continuum mechanics. Sources of multiscales are usually inhomogeneities and complexity of the mechanical processes. In concrete terms, the dataset in a partial differential equations (PDE) model consists of coefficients which model properties of the medium in the bulk or on the boundary, external bulk forcing or external boundary forcing, the initial state of the system, the domain which is occupied by the mechanical body, etc. These data may contain multiscale structure. The question then is to know how this influences the structure of the solution. It is pertinent to recall here that in classical PDE theory, one is interested in good properties (e.g. regularity) of the solution when the data possess them. In contrast, the aim in homogenization theory is rather unconventional and one is interested in analysing how bad properties (e.g. oscillations, singularities, concentration, etc.) get transferred from the data to the solution.

This aspect is admirably illustrated in the book with many examples. The book, for most part, deals with problems with multiscales arising from inhomogeneities in the medium. Moreover, inhomogeneities are assumed to define what is known as periodic microstructure, e.g. in composite materials. Under this hypothesis, one sees a description of oscillatory structure of the solution when the coefficients are oscillatory. In section 2.3, where boundary layer analysis is performed, one sees a mixed structure: concentration effects with respect to normal variable to the boundary along with oscillating pattern in the tangential variables.

Apart from the dataset, there is another source for multiscale structure. It concerns the nonlinear nature of the model which represents the complexity of the dynamic processes. There can be explicit nonlinear terms present in the equation or there can be nontrivial constraints in an optimization problem. In such cases, solution may exhibit multiscale structure while the data have none. In these cases, nonlinearities create oscillations and concentration spontaneously and this aspect is not covered in the book.

On the mathematical side, one can treat arbitrary microstructures which are not necessarily periodic, but this is not pursued in the book. Of course, it is clear at the heuristic level that arbitrary microstructures can be approximated by periodic ones in a certain sense and this is usefully exploited in the literature in tackling some questions. However, developing homogenization theory for arbitrary microstructures is difficult and this is not touched upon here.

In the theory of composites, we are interested in their behaviour on scales much larger than the period. How to describe this behaviour through a model which eliminates small scales but incorporates their effects on large scales? This simple question has given rise to enormous literature in applied mathematics, Applied mechanics and related areas. This analytical scheme of upscaling is called homogenization. An analogous task in wave propagation problems is to seek the evolution of wave envelopes of large wavelength and the procedure is known as envelope theory.

In real problems, there may be several scales interacting strongly with each other. A classical example of such a situation is fluid turbulence. Such problems are difficult and this book does not deal with them. It essentially focuses on one method: multiscale asymptotic expansion method (and some of its variants) in the presence of finitely many scales weakly coupled to each other. In such situations, the canonical procedure is to proceed iteratively: first, one eliminates the smallest scale by adding their effects in the next scale and this process is repeated. Thus the problem is reduced to studying the case of two weakly coupled scales. We also say that the scales are separated and the situation is recognized by the presence of a small coupling parameter in the problem. The asymptotic expansion is then based on this small parameter and adapted to the presence of two scales.

Multiscale asymptotic expansion method is limited to microstructures which are periodic and it is not applicable to other types of microstructures. That is why various other methods are devised in the literature: oscillating test function method, G-convergence method, gamma convergence method, etc. Since these demand knowledge of functional analysis, they are outside the scope of this book.

In general, there are two aspects in the homogenization theory. Large-scale properties of periodically composite media depend on the properties of individual constituents/phases which make up the composite, their volume proportions, their sizes, their contrast and most importantly on the microstructure inside the periodic cell defined by the different phases. The subtle aspect of the homogenization theory is this: relate the overall properties of the composite to the microstructure. This is the so-called direct aspect of the problem. The inverse problem is to design suitable microstructure with possible contrast which gives rise to a desired prefixed macro property of the composite on homogenization.

Aims of this book are more modest. It exploits asymptotic expansion method to bring out the dependence of macro property on the microstructure by actually deducing the macro model from the micro one. Microstructure contribution is seen through the so-called cell solutions and their use in the definition of macro coefficients.

One of the high points of the theory of homogenization is the characterization of all possible macro behaviours that one can obtain by varying all possible microstructures in a scalar heat/electrical conduction problem. This result is known as

Murat–Tartar–Lurie–Cherkaev (MTLC) theorem stated in sections 2.6 and 2.7. In particular, given a macroscopic behaviour, it is easy to see whether microstructure can be manipulated to achieve it. This is indeed an exceptional result and there are not many such results in the literature. This constitutes what is known as the G-closure problem of homogenization. The MTLC theorem is the only point at which the book makes contact with inverse problems of homogenization, a topic which has seen tremendous developments in physics (optics) and engineering (materials science) in recent years. Some of the keywords in this topic of research are metamaterials, transformation optics, cloaking, superlenses, negative refractive index materials, etc.

There are often surprises at the end of the homogenization process. The resulting model may possess properties very different from the original one. For instance, it may be nonlocal, although the initial model is a local one. In other words, nonlocality may arise because of small scales. Anisotropy results when isotropic phases are arranged in laminate microstructure. Other surprises include: a composite with negative Poisson ratio while individual phases have positive ratios and a composite with negative thermal conducting coefficients made up of individual phases with positive coefficients. Also bubbly microstructure in a fluid attenuates the acoustic waves in it. All these surprising behaviours are of course due to the microstructure inside the composite. Similarly, this book describes its share of surprises. One of them is the so-called Taylor dispersion in a pipe flow: it represents the enhanced diffusion of a passive scalar in the longitudinal direction which is due to the shear microstructure in the velocity field in the transverse directions.

The MTCL theorem is presented in sections 2.6 and 2.7 by following a procedure and a language accessible to many scientists. In fact, this is the strategy followed by the authors throughout the book. Mathematical theory of homogenization is dominated by various sophisticated concepts of convergence, which are totally avoided by the authors. One of the aims of the book is to make some subtle ideas of the theory accessible to a larger community of scientists and the aim is largely fulfilled.

Quite often, the unknown field in the problem is characterized by an optimization principle in mechanics. Constraints may also be present in the optimization process. Such problems are of wide interest in mechanics. But unfortunately, such models are not treated in this book. Equations which are not in divergence form may not be of much relevance in continuum mechanics because its equations are based on conservation laws. But homogenization theory of non divergence form equations is a fascinating story in applied mathematics. Models containing weak randomness, weak nonlinearity and Bloch wave method are merely touched upon in the text. Maybe, each one of these topics deserves separate texts for more elaboration.

The book is written in a style useful and accessible to engineers and scientists. One finds various physical and mechanical interpretations useful to them. Abstraction is absent, but concrete examples are found in plenty. Language of appropriate convergence concept (a dominant theme in applied mathematics) is totally avoided. One original aspect of the book: in each example, the authors perform nondimensionalization on the model to find the relative importance of various terms of an equation under the considered physical situation. This is an important step usually avoided in other texts. This step is important to see how and where small parameter appears in the equation. In applied mathematics texts, this step is usually taken for granted and one starts directly with the equation with small parameter.

Described above are some of the highlights of the book as well as some of its drawbacks. In the existing literature on homogenization, one sees texts of completely mathematical nature or completely meant for engineering use. There are not many which lie in between and the present book is one of them. Consequently, it is a welcome addition which may be fruitfully used by all communities of scientists: applied mathematicians, physicists and engineers.

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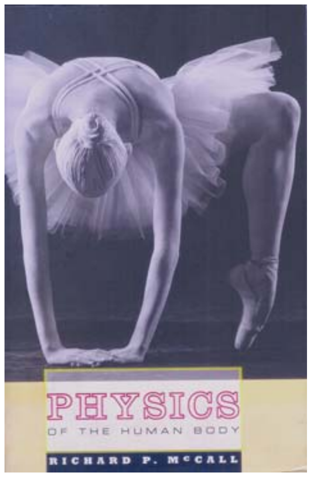

Physics of the Human Body. Richard P. McCall. The Johns Hopkins University Press, 27/5 North Charles Street, Baltimore, Maryland 21218, USA. 2010. xvi + 153 pp. Price: US$ 25.00. Paper back; US$ 75.00 hard cover.

Understanding the human body in terms of physical processes is as old as ideas about modern medicine. And yet, it is only recently that the old problems have begun to be re-examined, in the light of material science, biomechanics, rheology and bioelectricity. In that regard the book under review is a timely one. The author, Richard McCall, attempts to cover a huge amount of ground – from classical and fluid mechanics, through energetics, sound, light and electricity and even radiation, ending with pharmaco-kinetics. The author states at the outset that he has not written a textbook, although the book does attempt to cover a wide variety of topics. Each chapter is structured to begin with an introduction to the basic physics concepts, assuming only high-school physics, followed by a section on the human physiology associated and ending with associated medical conditions and diseases. The emphasis, as stated by the author, is on concepts and not so much on mathematical treatment of a topic.

On the topic of classical mechanics, the author attempts to connect everyday mechanical processes by the body to illustrate concepts. Blood, the heart and circulation form the bulk of the chapter on fluid mechanics, while briefly touching on other systems – the flow of air through the lungs, humors of the eye and fluids in the bladder and brain. The production and dissipation of energy in terms of heat begins with ideas from