Self-organized criticality and the Sikkim earthquake (2011) with comparative exemplars

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The phenomenon of self-organized criticality (SOC) has been pursued in this article in the context of the Sikkim earthquake of September–October 2011. The relatively smaller recursive fluctuations later on in macroscopic strain energy along the mountain chain are deemed as perturbations expressed by lower seismic potency. Comparative examples are analysed from the recent earthquakes (September 2010, December 2011) at Christchurch, New Zealand to unravel the systemic similarity, if any.

The key connotation of such SOC systems in nature is a complexity in their evolution. The complexity is reflected by the plurality of their manifestations carried over a wide range of scales – both spatial and temporal. SOC can occur only in large interactive dynamic systems. The complexity of the earthquake systems and their fractal growth are ascribed to the dynamic mechanism of the principles of critical self-organization. In fact, SOC constitutes a fundamental framework to explain the abstruseness and criticality in the earthquake systems and in nature as a whole.

Keywords: Complexity, earthquake systems, fractal growth, self-organized criticality.

IN California (USA) legislation, the Alquist-Priolo Earthquake Fault Zoning Act (1972) defines an active fault for the purposes of the Act as one that has ruptured the ground surface in the last 11,000 years[1]. Gaur[2] in his summary on the Himalayan earthquakes (1255–1950) has shown that the observations on the earthquakes in Sikkim and Assam are missing. He has described it as a 'historic void'. The last quake reported from Sikkim was in 1713. Since the recent (September 2011) Himalayan earthquake which devastatingly hit Sikkim (Richter scale 6.8–6.9) and the Darjeeling Himalayas had its epicentre in the East Nepal border (64 km NW from Gangtok) it is interesting to note that the last earthquake reported from East Nepal was in 1934. It hit the main Indian land mass too. However, when the present workers visited Sikkim in 2003 (May–June), they were surprised to find intense warping of a hard-court tennis court in an engineering college, south of Gangtok, around Rangpo, on its northern periphery. It gives evidence of an earthquake which must have occurred before May 2003 or in early 2003. Since information flow is non-existent, it is not on record. However, it is worthwhile to mention that the Sikkim and Darjeeling Himalayas represent a network of interconnected fault lines, fractures and slip planes. They occupy a tectonically disturbed zone. The rupture that took place both in Sikkim and Darjeeling according to rule must have followed the zones of pre-existing planes of weakness.

The September 2011 earthquake shock in Sikkim was followed by a tremor within the next 48 hours in the affected region and in the northeastern part of the country. In fact, the earthquake affected North Bengal, South Bengal, Bihar, Delhi and parts of North India. The tremor, however, was strongly felt in West Bengal and Bihar.

The consequent events in the Mediterranean–Himalayan belt have been marked by the following episodes:

16.10.2011 – Tremor in Srinagar, Jammu and Kashmir; Richter scale – 3.0.

23.10.2011 – Damaging earthquake in East Turkey; Richter scale – 7.3.

29.10.2011 – Mild shaking in Sikkim, 18 in number within 24 hours, of which six were of stronger magnitude; death in North Sikkim. On the same day, Kolkata felt a mild tremor around 6.30–7 p.m.

31.10.2011 – An earthquake in Taiwan (Circum-Pacific belt); Richter scale – 6.1.

07.11.2011 – An earthquake with tremors occurred in North India at Srinagar and other parts of Jammu and Kashmir and Jaipur, Rajasthan; Richter scale – 5.9.

10.11.2011 – Turkey in the Mediterranean–Himalayan belt was again hit by an earthquake; Richter scale – 5.7.

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11.11.2011 – India–Bhutan border felt a tremor; Richter scale – 3.8.
21.11.2011 – A tremor was felt in Northeast India, the epicentre was 130 km east of Manipur; Richter scale – 5.9.
15.12.2011 – Earthquake felt on the India–East Nepal border with epicenter near Chungthang; sixth occurrence in 2011.
19.12.2011 – Sikkim and Darjeeling experienced a tremor; Richter scale 3.7. Post tremor: Bridge collapsed in Sikkim; tremor felt in North Bengal and Kolkata.

New Zealand occupies the fulcrum of a scissor-shaped criss-cross system of faults. The mountain belt of the South Alps traverses the city of Christchurch along the NNE–SSW axis. A parallel chain of N/S–NNE/SSW faults are observed in both Christchurch and Wellington. Till 2010, Christchurch has experienced ten and a half thousand tremors. van Dissen and Berrymann[3] and van Dissen *et al.*[4] have traced surface rupture earthquakes in Wellington region too over the last 1000 years. On 25 December 2010, Christchurch was again affected by a high-intensity earthquake which killed more than 180 people. The cumulative effect of the earthquake systems to the city is reflected by the change of slope of the land mass, so much so, that about 75,000 people had to be evacuated. The said zone was declared as most vulnerable or susceptible to earthquakes (Red Alert). On 23 December 2011 around 8 p.m. local time, two earthquakes measuring 5.8 on the Richter scale hit Christchurch at an interval of about half an hour. The depth or extension of the earthquake measured was about 8 km. Wellington–Lower Hutt earthquake belt extending NNE–SSW also experienced a strong tremor. There was again an earthquake on 1 January and on 5 January 2012, measuring 5.3 on the Richter scale near Christchurch.

Earthquakes are known to occur on structures of active faults ranging from thousands of kilometres to centimetres. But there exists every degree of fault activity between active and inactive zones so that the boundary is necessarily arbitrary, but may be defined in accordance with a specific purpose. Moreover, it is difficult to argue that an active fault has stopped its episodic slipping.

The earthquakes are fundamentally scale-free in the sense that by viewing a part of the whole, their actual size cannot be deduced. The dynamics that forms the structure is characterized by a timescale no doubt, but much longer than the observed period. The observed earthquakes last only for a few seconds but the fault formations appear as static, because they have been built up over million of years.

This article is an attempt to explore the earthquake dynamics, particularly in the Himalayan belt with relevant references to the New Zealand system in the context of self-organizing criticality (SOC).

SOC, sandpile system, fractals and earthquakes

SOC is a representation of the internal interactions of large dissipative systems having many degrees of freedom to build up a critical state.

In a widely known experiment using a pile of sand, if each sand grain making up the pile is counted near the criticality of its slope, it is observed that at a critical point of value, addition of one more sand grain results in the sliding of sand off the pile. Winslow[5] confirmed that the count of added grains on an average equals the grains lost. This implies that the above sandpile has self-organized to a critical state. Moreover, the sand sliding has a periodic occurrence (recursive) similar to the observed oscillation in the earthquake system. The sliding of the sand grains off the pile is not initiated until the threshold/critical value of the pile height is reached.

According to Dhar[6], for large lattices, when we have $4N$ stable states, there are only $(3.2102\ldots..)^N$ recursive states. It shows that initiating from an arbitrary state and adding sand, the sandpile system self-organizes into a subset of $(3.21\ldots\ldots)^N$ states that form the attractor of the dynamics. It also predicates that in principle, in the elementary (2×2) example, if one adds $192 = (3.21)^4$ grains at the same site, one obtains the same state. In Bak and coworkers[7–15], examples of 2×7, 59×10^6 states are possible.

These remarkable mathematical features of the sandpile models[10] have been explored[6,16,17]. Considering that we have N sites of two-dimensional regular lattice with open boundaries, let B_g be the integers on each site, g representing the sandpile height. When a sand particle is added to a site g, the value B_g at the site is naturally increased by one unit. As the height exceeds the critical value B_{cr} by addition of individual sand grains, a toppling event occurs. In such a case, one grain of sand is transferred from the unstable site to the neighbourhood of the base of the pile and is counted to be lost. The power law suggests that the chain reaction generated, if the operation is continued, is critical. It means that the probability that the activity at the same site branches into more than one active site is balanced by the probability that the activity dies. Thus by evolving through one slide of land after another (may also be rupture along with displacement in natural ambience), the system adapts to responding critically to the next perturbation. After a local perturbation at a single site on a critical state, it spreads to the nearest block and then to the next nearest and so on in a domino effect, eventually dying out, having triggered tremors in a total time T. It may be relevant to draw a lesson from real-life phenomena involving systems whose base structure is made up by a large number of elements that interact over a time range. Such a situation probably exists in tectonically disturbed zones, as in the present case. During redistribution of stress in a dynamic milieu, a minor stress may start a chain reaction that can affect the elements in the composite system (where elements interact over a short range) showing signs of SOC.

Such systems in nature exhibit a power-law relationship as evidenced above and are clearly brought out by seismicity plots and the Gutenberg–Richter relationship. Incidentally, the Gutenberg–Richter Law exhibits fractal properties. The latter provide a dynamic framework of how nature might have produced them. The origin of fractal here is a dynamic, not a geometric problem.

Nature in similar systems appears to ignore the type of exact self-similarity in geometry in favour of different shapes. It, however, implies statistical self-similarity not only because of chance acting through random conditions, but also because of necessity as reflected by the least total energy expenditure. Thus the fractal patterns represent the interplay of local and global interactions that are organized over the farthest distances available for the developing structure. In the above backdrop the tremors felt in Turkey in the Mediterranean–Himalayan belt, in Taiwan in the Circum-Pacific belt and in New Zealand are probable reflections. In SOC, a global rule may not be enforced. When large systems operate near their equilibrium state, they may be correlated only locally. In the present scenario, however, when operating conditions are far from equilibrium, the importance of global correlation is underscored.

At a critical point, when distribution and re-distribution of stress take place, i.e. a transition of phase occurs, systems are fractal and can be correlated over all scales. Main and Naylor[18] from their applications to the Olami–Feder–Christiansen numerical model put forth the hypothesis that maximum entropy production (MEP), the driving force of SOC, occurs only when the global elastic strain is near critical. The necessity of a global rule in SOC is thus emphasized, accompanied by low seismic efficiency, power-law scaling of frequency and rupture area.

Discussion

SOC is a reference to the scientific quest for the spatially extended dynamic systems like earthquakes. These systems have both spatial and temporal degrees of freedom. Mostly landform structures in nature, inclusive of earthquakes, reflect SOC[19–21]. According to Bak and co-workers[7–15], a wide category of dissipative coupled systems is governed by SOC. The latter further interrelates the evolution of spatial structures (like earthquakes) and the temporal effects, expressed by $1/f$ noise. The temporal signatures of self-organized systems thus demonstrate a characteristic self-similar (fractal) response, where earthquakes may occur over all possible timescales. Such a behaviour is displayed in the $1/f$ power spectrum (power is inversely proportional to the frequency), as opposed to the uniform power spectrum of a purely stochastic process[14]. The systems naturally evolve towards a critical state with no intrinsic time or length scale. $1/f$ describes the power law decay. It is also considered as a fingerprint of a spatially scale-free behaviour, commonly assigned as critical. The recent Himalayan earthquakes or tremors and their recursive patterns reflect the SOC attributes; so does the one in Christchurch on 5 January 2012. The earthquake on 1 January 2012 near Christchurch suggests that the crustal domain has already reached near criticality. The stress was redistributed in the other blocks after the tremor on 25 December 2011. As redistribution of stress was carried out, self-organizingly the particular block became critical and responded with an earthquake measuring 5.3 on the Richter scale on 1 January 2012.

The evolution of large interactive systems appears to be represented in their dynamic characteristics. The ubiquity of their scaling forms and the recursive characters of the processes operating in different environmental/ecological milieu are probably related to the general dynamics of the specific belt. The earthquake could evolve in an intermittent burst-like manner, rather than in a smooth and gradual manner. The above explanation (Bak, op cit), unrelated to specific events and embedded in the mechanism of self-organization into critical states is more plausible in the present exemplars.

The Himalayan range is known to be an unstable mountain chain. The Sikkim and the Darjeeling Himalayas pose an interconnected network of fault lines and slip/fracture planes. In the Darjeeling Himalayas, in and around Ambootia tea gardens, a NE–SW fault line is traversing along the eastern fringe of the east-west Ambootia slip plane. Movement along the slip plane is continuous. This plane is advancing towards the north. The present workers are apprehensive that when the slip plane reaches the fault plane near Kurseong to the northeast, the stability of Kurseong may reach its threshold – its existence as an hill station may be endangered. A NE–SW fault line passing through the south of Darjeeling is also met with if the Ambootia E–W slip plane is followed strike-wise west.

Such a geomorphological self-organization[22] of landscape (landscape evolution in such terrains is governed by a self-organization concept arising out of a varying product mix of earthquakes, slips, climatic factors and erosion – which leads to a geomorphological threshold) does not indicate a high level of stability and should be marked as the most vulnerable (Red) zone.

In the Himalayan mountain chain, the Sikkim earthquake was a large event. It had a specific source, i.e. rupture of fault segments. It indicates that the region has attained a status of SOC. The dynamics of SOC is governed by the sites, i.e. North Sikkim with its external value of the signal (this area was totally devastated and the epicentre was closeby). It has generated a chain reaction of global size borne out by the recursive tremors at many places in the country along the Himalayas. The lower range of amplitude in the later tremors (measured on the Richter scale) is in consonance with the hypothesis[18].

New Zealand, situated on the fulcrum of a (wide-bladed) scissor-shaped criss-cross system of faults, shows similarity with the Sikkim Himalayan belt in the pattern dynamics of the earthquake system. Their attributes expectedly reflect SOC; the earthquake behaviour of the two exemplars appears to be dynamically self-similar.

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