Analysis of the circular track experiment measuring the one-way speed of light

The invariance of the speed of light for all inertial observers is a basic postulate of the special theory of relativity[1]. However, this postulate has not been experimentally established directly. It has only been experimentally established that the two-way speed of light is independent of the velocity of the inertial frame in which it is measured[2].

One of the major challenges in performing direct measurement of the speed of light during one-way transmission is the requirement of spatially separated, synchronized clocks. A possible way of circumventing this challenge is measuring the speed of light travelling in a closed path. This will enable the use of the same clock for the initial and final measurement, thereby eliminating the need for synchronization.

The speed of light in an experiment based on this idea, conceived and performed by Unnikrishnan, was found to have an apparent dependence on the velocity of the observer[3]. We present a re-analysis of this experiment and show that the experimental results can be explained within the framework of special relativity, without requiring any violation of the principle of invariance of speed of light.

A brief description of Unnikrishnan's experiment is given below to facilitate the understanding of this paper.

The experiment is presented by Unnikrishnan[4] as an extension of the simple idea of measuring the speed of light by sending a light pulse between two synchronized clocks. According to the invariance principle, the measured speed of light must be independent of the constant velocity with which the two clocks may be moving. It was argued that, even if the path followed by the light pulse is not straight as in Figure 1 *a*, if the clocks are moving with constant velocities maintaining the length of the path, like in Figure 1 *b*, the speed of light must be invariant[4].

The path may be bent to make the initial and final clocks the same, like in Figure 2. This maintains the total path length and eliminates the concern about synchronization.

The invariance principle was to be validated by comparing the speeds of two light pulses, travelling in opposite directions, in a closed path, as schematically shown in Figure 2. The two light pulses were sent in opposite directions from an observer moving with constant velocity and the difference in their time of arrival was measured. Since the speed of the light pulses travelling in either direction must be the same with respect to the observer moving with constant velocity, it was argued in ref. 3 that the light pulses must arrive simultaneously. Note that the contribution of length contraction is the same in case of both pulses.

A schematic diagram of the actual experimental set-up used by Unnikrishnan is shown in Figure 3. The detector interferometrically measures the difference in the time of arrival with a temporal resolution exceeding 10^{-18} sec. The platform holding the detector is capable of moving with uniform velocity up to the order of 10^{-1} ms^{-1}.

The result obtained by Unnikrishnan[3] for a round trip length of the order of 2 m shows a clear linear dependence between the arrival time difference and the velocity of the observer. It was argued[3] that this observation contradicts the invariance principle.

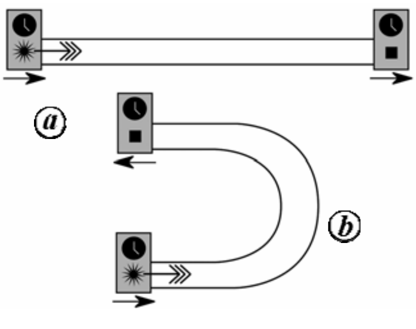

Figure 1. *a*, Simple set-up for measuring the speed of light in an inertial frame. ***b***, Modified set-up[4].

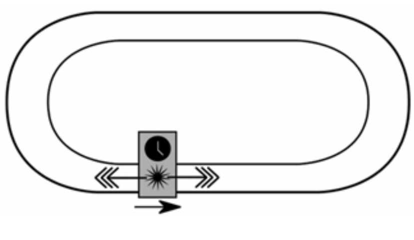

Figure 2. Schematic diagram of the concept of the experiment[4].

We will show that this apparent violation of the postulate of invariance of the speed of light arises because, according to the special theory of relativity from the frame of reference of the inertial observer, the path travelled by the two light pulses is of unequal length. This can be seen easily by examining the Minkowski space–time diagram in Figure 4.

If we consider the frame of reference of the observer, the mirrors keep moving

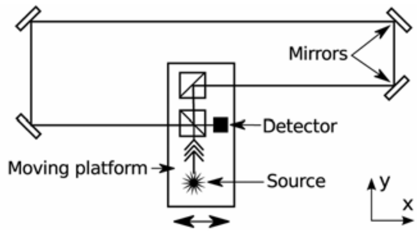

Figure 3. Schematic diagram of the experimental set-up used by Unnikrishnan[3].

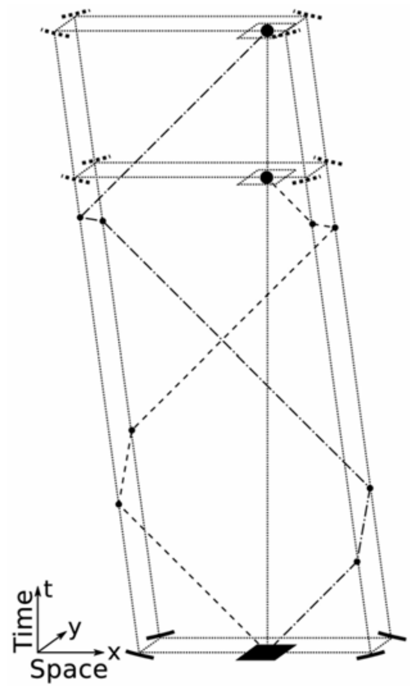

Figure 4. Minkowski space–time diagram of the experiment from the frame of reference of the observer (detector)[5]. The mirrors appear to move in the opposite direction from this frame. The inclination of the world lines followed by light to the horizontal plane does not change due to the constant speed of light. The world lines followed by the mirrors are at a greater angle due to their lower speed (i.e. they fall within the light cone).

while the light pulses travel from one mirror to another. From the start of the pulse to the detection of the pulse, this results in a change in the effective distance between the mirrors. Contrary to naive expectation, the total distance travelled by the two light pulses is seen to be unequal (Figure 4). It turns out that it was erroneous to expect Figure 1 *a* and *b* to yield the same result.

For the purpose of comparison with experimental results, the difference in the time of arrival can be calculated. This can be accomplished by writing the equations for the instantaneous positions of the mirrors from the frame of reference of the observer, calculating the time for the light pulse to travel each segment separately and summing them, to obtain the total time of travel for each pulse.

We are expecting an effect which is first order in velocity. Since the velocity of the inertial observer is much lesser than the speed of light, all contributions arising from higher orders of velocity can be ignored.

The difference between the time of arrival of the two light pulses at the detector is found to be

$$\Delta t \approx \frac{2l}{c^2}v,$$

where l is the length of the path, v the speed of the detector with respect to the rest frame of the mirrors and c is the speed of light.

Substituting $l = 1.5$ m (since the total path length was of the order of 2 m) in the above expression, we obtain

$$\Delta t = (3.33 \times 10^{-17}\ \mathrm{s}^2\ \mathrm{m}^{-1})v,$$

The slope calculated above agrees well with the slope of the experimental plot in ref. 3.

Figure 4 shows the basic difference between our analysis and the analysis in ref. 3. In our analysis, with respect to the observer (detector), the world lines of the mirrors are inclined in the same direction, since all the mirrors have the same relative velocity with respect to the observer. In ref. 3, the world lines of the mirrors are shown to be inclined in different directions.

We have reanalysed the experiment in ref. 3 and have shown that the difference between the time of arrival of the two light pulses and its dependence on the velocity of the observer does not violate the principle of invariance of the speed of light. As explained above, the time of traversals corresponds to that of two different distances, and so they cannot be expected to be the same. The velocity dependence of the difference in the time of arrival naturally arises as the difference in the distance of traversal depends on the velocity of the observer. The experimental results of ref. 3 appear to be numerically in good agreement with the calculations performed using the special theory of relativity, without requiring any violation of the principle of invariance of the speed of light.

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ACKNOWLEDGEMENTS. I thank Prof. S. C. Phatak, Centre for Excellence in Basic Sciences, Mumbai and Prof. A. M. Srivastava, Institute of Physics, Bhubaneswar for their encouragement and critical comments. I also thank Ankit Pandey, National Institute of Science Education and Research, Bhubaneswar for inspiring me to tackle the problem.

Received 24 September 2012; accepted 2 January 2013

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Estimation of Indian coastal areas inundated into the sea due to sea-level rise during the 20th century

The sea-level rise caused by climate change leads to inundation of coastal areas and may be a threat to the low-lying coastal regions[1,2]. Bindoff *et al.*[3] reported that the average rate of sea-level rise was 1.7 mm/year in the twentieth century and this rate was not uniform over decadal periods throughout the century[4–6]. The sea-level rise also varies in a regional scale and accordingly causes inundation along the coast. Unnikrishnan *et al.*[7] estimated the sea-level rise along the Indian coast. Their result indicates that the rate of sea-level rise at Mumbai is 0.78 mm/year during 1878–1994, at Kochi it is 1.14 mm/year during 1939–1997 and at Visakhapatnam it is 0.75 mm/year during 1939–1994; whereas the rate of decrease of sea level at Chennai is 0.65 mm/year during 1955–1994. In 2007 Unnikrishnan and Shankar[8] extended the above work and reported that sea-level rise occurs between 1.06 and 1.75 mm/year, with a regional average of 1.29 mm/year[8]. Nandy and Bandyopadhyay[9] show that the trend of annual sea-level rise along Hugli, West Bengal coast is 1.09 mm/year[9]. This indicates that some areas along the Indian coast might have gone under the sea during the above-mentioned period and this inundation might be different in the regional scale depending on the topography of the region. A report by the Geological Survey of India, Hyderabad shows that the coast between Azhikkod and Edavilangu has suffered a net loss of 2.093 sq. km area during the last century[10]. Similarly, the coast between Munambam and Nayarambalam has suffered a net loss of 4.675 sq. km during the last hundred years[10]. Kumaravel *et al.*[11] reported that erosion of 3.21 sq. km shoreline occurred in Cuddalore district in the east coast of Tamil Nadu. Kumar and Jayappa[12] studied a coastal stretch of 18 km from the New Mangalore Port in the north to Talapadi in the south of