Formation of a perfect fluid in high-energy heavy-ion collisions

Bedangadas Mohanty and Victor Roy

One of the main goals of the experiments involving high-energy heavy-ion collisions is to create a high-temperature state of matter where quarks and gluons, the basic constituents of matter, are no longer confined inside hadrons (like protons and neutrons). This state of matter is commonly referred to as the quark gluon plasma (QGP). Such a de-confined state of matter will allow us to study properties like shear viscosity, thermal conductivity and diffusion for a thermodynamic system of quarks and gluons—a study that would not have been otherwise possible using a system of commonly observed hadrons in nature. It is like properties of hydrogen and oxygen that cannot necessarily be gauged by studying the properties of water. Both hydrogen and oxygen facilitate burning in contrast to that of water. There are compelling evidences that collisions of gold and lead nuclei at the Relativistic Heavy Ion Collider (RHIC) facility at Brookhaven National Laboratory and Large Hadron Collider (LHC) facility at CERN respectively, have produced such a de-confined state of matter, thereby providing a unique opportunity to study the properties of a thermodynamic system of quarks and gluons.

For a system slightly away from equilibrium, according to the linear theory of non-equilibrium thermodynamics, the thermodynamic fluxes are proportional to the thermodynamic forces. The proportionality constants are known as the transport coefficients. Table 1 shows the thermodynamic fluxes and thermodynamic forces with their corresponding transport coefficients.

Shear viscosity arises in a fluid when a velocity gradient is present. The shear viscosity coefficient \( \eta \) is a measure of how a fluid will flow under an applied force. The inverse of \( \eta \) is called fluidity. For a dynamic system as formed in the high-energy heavy-ion collisions, collective phenomena such as the flow of produced particles due to pressure gradients created in the system have been observed. These experimental data are then confronted with results from a viscous hydrodynamic modelling of the collisions to extract a dimensionless quantity, the shear viscosity to entropy density ratio (\( \eta/s \)) (ref. 2), where the flow in experimental data is quantified in terms of asymmetry in the momentum distribution of the produced particles. The hydrodynamic model is an effective theory that describes the slow long-wavelength motion of a fluid close to equilibrium. This is the most common approach used to extract \( \eta/s \). Figure 1 shows a representative plot of a particular set (with different \( \eta/s \) values) of viscous hydrodynamic calculations compared to most recent high-statistics measurements from the experiments at RHIC and LHC.

A small value of \( \eta/s \) reflects a high degree of fluidity; hence it is logical to ask if there is any lower bound to this number. In other words, how perfect can a fluid be? A lower bound on \( \eta/s \) can be obtained from quantum mechanical considerations. For a system of quasi particles, the shear viscosity is \( \eta \sim \varepsilon \tau_{\text{mft}} \), where \( \varepsilon \) is the energy density and \( \tau_{\text{mft}} \) is the typical mean free time. The entropy density is given as \( s \sim \kappa B n \), where \( \kappa B \) is the Boltzmann constant and \( n \) is the number density. This implies \( \eta/s \sim \varepsilon /s \sim \varepsilon \tau_{\text{mft}} /\kappa B n \). Considering \( \varepsilon /n \) as the average energy per particle and using uncertainty relation between energy and time, one finds \( \eta/s \approx \hbar /2\pi \kappa B n \). In addition, using string theoretical ideas it can be shown that \( \eta/s \) for strongly interacting quantum fluids has the value \( \hbar /8\pi \kappa B \).

The current status of extraction of shear viscosity to entropy density ratio (\( \eta/s \)) from high-energy heavy-ion collision experimental data and theoretical studies is summarized in Figure 2. The references to various calculations/measurements are given in the figure itself. Diverse estimates suggest that at both RHIC and LHC energies the value of \( \eta/s \) is less than four times the conjectured quantum bound. It is lower than that for liquid helium at its critical temperature, indicating that the system formed in the high-energy heavy-ion collisions is strongly coupled having the highest degree of fluidity per entropy. Even though

<table>
<thead>
<tr>
<th>Flux</th>
<th>Force = gradient of</th>
<th>Transport coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Momentum</td>
<td>Velocity</td>
<td>Shear and bulk viscosity</td>
</tr>
<tr>
<td>Heat</td>
<td>Temperature</td>
<td>Heat conductivity</td>
</tr>
<tr>
<td>Diffusion flow</td>
<td>Number density</td>
<td>Diffusion constant</td>
</tr>
</tbody>
</table>

**Figure 1.** Comparison of experimental measurement of collective flow (termed here as elliptic flow and defined as \( \langle \cos 2\phi \rangle \), where \( \phi \) is the azimuthal angle and averaging is over all events and all particles) as a function of transverse momentum of the charged particles in heavy-ion collisions at Relativistic Heavy Ion Collider (RHIC) (Au–Au) and Large Hadron Collider (LHC) (Pb–Pb) to the corresponding simulated results from a viscous hydrodynamics modelling of the collisions with different \( \eta/s \) values, where 30–40% corresponds to a certain range of impact parameter of heavy-ion collisions.
there is a factor of 10 increase in the beam energy and hence a higher initial temperature attained for the quark–gluon system at LHC, the shear viscosity to entropy density ratio continues to have similar values as observed at RHIC. One would have naively expected \( \eta/s \) to increase as we go to higher temperature in the QGP phase, as is seen for other fluids (Figure 3).

Lattice QCD calculations indicate that for such a system to reach a weakly coupled regime would perhaps need much larger temperatures.

It is important to mention at this point about the sources of uncertainties associated with such a process of extracting the value of \( \eta/s \). The major uncertainty comes from the lack of complete knowledge of the initial conditions. Viscosity is a process that inhibits the way initial spatial asymmetry in heavy-ion collisions is effectively converted to the finally measured momentum anisotropy for the system relative to as expected for systems following ideal hydrodynamics (\( \eta = 0 \)). It is this initial spatial asymmetry that causes different pressure gradients in different directions in the system. In addition, there are uncertainties associated with how one models the viscous hydrodynamics and the freeze-out conditions (vanishing inelastic and elastic collision conditions) for high-energy heavy-ion collisions. Some of these uncertainties are reflected in the errors shown in Figure 2.

Figure 3 shows \( \eta/s \) values for the QGP system relative to those for other known fluids and calculations. The values extracted from the experimental data for high-energy heavy-ion collisions indicate that the fluid produced in such collisions have the smallest \( \eta/s \) value among any known fluids – hence the QGP is termed as the perfect fluid. This is a remarkable discovery regarding the thermodynamic properties of a system having the basic constituents of matter. That the QGP will be a strongly interacting system with the smallest observed value of \( \eta/s \) was not envisioned during the early days of research in the field. The searches for these phenomena were based on the concept of a weakly coupled gas of freely moving quarks and gluons as expected from asymptotic freedom, until the advent of the measurements at RHIC.

The value of \( \eta/s \) observed in heavy-ion collisions shows remarkable similarity with several other strongly coupled systems occurring in nature. The strongly coupled QGP then evokes a certain degree of universality among certain kinds of conventional plasmas consisting of electrically charged particles (electrons, ions or large charged mesoscopic grains), which also exhibit liquid or even solid-like behaviour. Such strongly coupled plasmas are characterized by an interparticle potential energy that dominates over the (thermal) kinetic energy of the particles. Strongly coupled plasmas occur in electrical discharges, in cryogenic traps, storage rings, semiconductors and astrophysical systems (interior of giant planets and white dwarfs).

Strongly coupled systems are characterized by the coupling parameter \( \Gamma \) that is the measure of the ratio of average potential energy to the average kinetic energy per particle. The strong coupling regime corresponds to \( \Gamma > 1 \) (Figure 4). Recent work indicates that the coupling...
parameter for the QGP formed at RHIC and LHC is expected to be in the order of one\textsuperscript{7}. The smallest value of $\eta/s$ measured for any fluid at RHIC and LHC, has led to several interesting measurements for similar strongly coupled systems in condensed matter physics – like graphene\textsuperscript{8} and dilute gases of ultracold fermions\textsuperscript{9}. It is remarkable to note that both the coldest (ultracold fermionic gas) and hottest matter produced (heavy-ion collisions at RHIC and LHC) on earth exhibit very similar flow patterns, with $\eta/s$ values close to the conjectured lower bound.

Understanding the transport properties of the QGP will remain one of the main motivations for the field of research in high-energy heavy-ion collisions in near future. These experiments will now strive towards making a more precise measurement of the transport coefficients.


Bedangadas Mohanty* and Victor Roy are in the School of Physical Sciences, National Institute of Science Education and Research, Bhubaneswar 751 005, India.
*e-mail: bedanga@niser.ac.in