In this issue

Mathematical finance

Application of mathematics has contributed immensely in the genesis of mathematical finance. A special section on mathematical finance in this issue of *Current Science* puts together various mathematical models that have been applied in various financial sectors.

Madan and Eberlein (**page 647**) explain the role, nature and purpose of mathematical modelling in financial analysis which are elucidated with a view to clarifying many misconceptions about such activities. It is emphasized that models must and do support important financial decision making while at the same time recognizing their simultaneous inability to coincide with or capture reality. Decision making requires modelling even when it is clear that this is done at best with error and open to inconsistencies ignored or unperceived. As a result, models supporting decisions are theoretically and of necessity, varied, possibly mutually inconsistent, dynamically evolving in a process of continuous improvement leveraging advances occurring in information processing abilities. These very dynamic calls into question the concept of a true model as the immediate adequacy of currently used models inevitably fades into future irrelevance. The objective of mathematical modelling is then not that of discovering some nonexistent true model but one of enhancing the quality of decision making in line with technological capabilities. As a consequence, financial modelling is as exciting as it is unsettled. Though humans interact in markets, as they evolve in their information processing abilities so do the markets that respect their interactions. Hence the extent to which the risk of changes in volatility matter in pricing today is far advanced over its status in years gone by and in all probability the future will bring yet other dimensions into play. Hopefully having explained the evolutionary nature of financial modelling from a fundamental, practical and economically necessary viewpoint, the authors invite mathematicians to participate in this development. They can help not only in improving the use of current markets to design trades and manage risks but also assist in developing new markets that follow on the footsteps of mathematical advances.

Kumar and Pal (**page 650**) introduce a new continuous time portfolio model consisting of money market account, a stock and a defaultable asset. They consider a financial market with a basket of defaultable assets, one stock and a money market account. At any given time the basket of defaultable assets contain M-type of assets with ith type being identified by a pair of parameters $(b(i)\,\bar{\sigma}(i))$ where $b(i)$ denotes the expected rate of return and $\bar{\sigma}(i)$ denotes the volatility vector, $i = 1, 2, \ldots, M$. The portfolio model at any given time consists of a single defaultable asset chosen from the basket, the stock and the money market account. The evolution of the portfolio over time is as follows. Upon default of the defaultable asset in the portfolio, investors choose another defaultable asset from the basket as a replacement and the procedure continues up to the terminal time. The default time and the replacement of defaulted asset in the portfolio is according to a continuous time nite state Markov chain. The authors study two types of portfolio optimization problems using the above portfolio model.

In the first they look at portfolio optimization using power utility and in the second, they study benchmark optimization problem. Using dynamic programming approach, they prove the existence of optimal portfolios for the associated power utility maximization problem and the benchmarked optimization problem. They also characterize the value function as the unique positive solution to the corresponding Hamilton–Jacobi–Bellman equation for power utility maximization problem and for the benchmarked optimization problem. Finally, the authors provide a numerical method for computing optimal control for power utility maximization problem.

Credit risk refers to the potential losses that can arise due to the changes in the credit quality of financial instruments. These changes could be due to changes in the ratings, market price (spread) or default on contractual obligations. Credit derivatives are financial instruments designed to mitigate the adverse impact that may arise due to credit risks. However, they also allow the investors to take up purely speculative positions. The financial crisis set off by the default of Lehman Brothers in 2008 leading to disastrous consequences for the global economy has focused attention on regulation and pricing issues related to credit derivatives. Banerjee *et al.* (**page 657**) provide a brief introduction to the notions of credit risk, the credit derivatives market and describe some of the important credit derivative products. There are two approaches to pricing credit derivatives, namely the structural and the reduced form or intensity-based models. In the structural approach explicit assumptions are made about the dynamics of a firm's assets, its capital structure, debt and shareholders. A firm defaults when its asset value reaches a certain lower threshold, defined endogenously within the model. In the intensity-based approach the firm's asset values and its capital structure are not modelled at all. Instead the dynamics of default are exogenously given through a default rate or intensity. The challenge in pricing credit derivative instruments ranges from developing realistic yet elegant mathematical models to calibrating and implementing such models of importance as well is an understanding of the limitations of the models and developing efficient hedges to cover risk.

The Markowitz model of risk–return tradeoff has been a staple for financial decision-making over the past six decades. The measure of return has always been the mean of the distribution and the measure of risk has been either the variance of the distribution or at times semi-variance or some such measure. In another strand of the literature of the practice of measuring risk, the value at risk (VaR) has not only become the standard tool in the industry but also a requirement by the regulators. When the rates of return are Gaussian (normal), these two measures are interchangeable. In other words, it does not matter if we use the mean-variance framework or the mean-VaR framework. Unfortunately, most financial markets do not have rates of return that are Gaussian. The distributions are asymmetric and with fatter tails. Therefore, decisions taken using the Markowitz mean-variance model produce outcomes that are not optimal. A measure is provided by Karandikar and Sinha (**page 666**) in this article to improve upon the Markowitz portfolio using VaR and median as the decision-making criteria. A method is then proposed for calculating this measure with distributions that are far from Gaussian. It is then shown that there are computational limitations for solving the most general problem. An algorithm is provided which circumvents the computational problem by providing 'nearly optimal' solutions. The algorithm is tested out with an example to demonstrate how it would work in practice.

In recent days, financial mathematics is catching up in India and it is going hand-in-hand with the setting up of back offices of major investment banks from all over the world in India. While investment banking industry may be influenced by financial mathematicians but policymakers of the central banks and governments are usually dominated by economists. As, there is hardly any common forum for these two groups to interact, they hardly talk or understand each other's language. This may push our country to a crisis as we have seen in other parts of the world where, before the crisis, policymakers were hardly aware what investment bankers were doing, and after the crisis, they are very eager to contain/limit the activities of the investment bankers. In this issue Basak *et al.* (**page 673**) address problem pertaining to foreign exchange crisis that concerns both policy makers and bankers, emanating from international borrowing and exchange rate dynamics without assuming any statistical model of exchange rate determination.

Production of venom and antivenom in India

Reliable snakebite statistics are only now available, thanks to the Million Death Study, an initiative of the Registrar-General of India and the Centre for Global Health Research at St Michael's Hospital and the University of Toronto, Canada. Based on this study, the upper estimate for snakebite deaths in India is a staggering 50,000 per year, approximately 1 for every 2 HIV/AIDS deaths! In the face of this new data, hospital returns to the Government of India from 31 of 35 states and Union Territories between 2004 and 2009 state that there was an average of only 1,350 snakebite deaths each year.

Whitaker and Whitaker (**page 635**) review the production of venom and antivenom in India and suggest areas of improvement using a combination of current herpetological knowledge, a sampling of medical statistics and interviews with venom and antivenom producers in India to collect and assess their data set. They show that several factors complicate the treatment of snakebite in India and that changes in venom/antivenom production (based on World Health Organization protocol), treatment protocols, as well as Government policy are urgently needed to improve the current scenario.

The primary victims of snakebite are farmers, rural labourers and their families. Most bites are on the feet and legs and occur after sunset. There are four snakes that are held responsible for most of the serious snakebites: the cobra, krait, Russell's viper and saw-scaled viper, popularly known as the 'big four'. Complicating the issue is the fact that India has four species of cobras, eight species of kraits and one subspecies of saw-scaled viper, besides a number of other venomous species.

Two other specific factors are not adequately addressed by the currently available antivenom. The first is geographic, intra-species variation in venoms, considering that available antivenom contains antibodies raised primarily to venom collected by the Irula Snake-Catchers Cooperative from only two adjacent districts in Tamil Nadu state. And importantly, the potency of Indian made antivenoms is woefully low and prone to causing allergic reactions.