

From hunches to surprises – discovering macro-scale quantum phenomena in charged particle dynamics

Ram K. Varma*

Physical Research Laboratory, Navrangpura, Ahmedabad 380 009, India

This article describes how, an intuitive guess – a hunch – in relation to a system of charged particles in a magnetic field pursued over four decades, has led to the discovery of an entirely new set of phenomena, which could not have been conceived in view of the prevailing conceptions. They pertain to the existence of quantized residence times in an adiabatic magnetic trap, and more surprisingly, the existence of macro-scale matter wave interference effects, with an \hbar independent matter wavelength. They even include the observation of a curl-free vector potential on the macro-scale as against its micro-scale detection *à la* Aharonov–Bohm. Though on the macro-scale, these results cannot be understood in terms of the Lorentz equation, which is known to govern the dynamics on the macro-scale. They have, in fact, been shown to be of quantum origin and are found to be attributed to the quantum modulation of the de Broglie wave, and hence could not have been covered by the Lorentz equation. All these phenomena are seen to run counter to the well-entrenched canonical perception that matter wave interference effects and the vector potential observation – the Aharonov–Bohm effect – pertain only to the micro-scale. The unusual phenomena so discovered constitute a complete surprise as they are entirely unexpected under the canonical view and appear to upturn the latter.

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HUNCHES have many a time played a role in the germination of ideas, which may or may not always fructify into a tangible outcome. But whenever they did, they have resulted in interesting developments and unforeseen discoveries. On the role of intuition and ‘hunches’ on the germination of ideas, it is pertinent to quote Harish-Chandra:

‘I have often wondered over the role of knowledge or experience on the one hand, and imagination and intuition, in the process of discovery. I believe that there is a

certain fundamental conflict between the two, and knowledge by advocating caution, tends to inhibit the flight of imagination. Therefore, a certain naivete, unburdened by conventional wisdom, can sometimes be a positive asset. I regard Dirac’s discovery of relativistic equation of the electron as a shining example of such a case.’¹ If he were not speaking on the occasion of Dirac’s 80th birthday celebration to say these words, he could well have included the discoveries of radiation law by Planck, the light-quantum by Einstein and that of quantum mechanics by Heisenberg as the other examples.

The above-mentioned fundamental discoveries have indeed come out through an intuitive jump, which provided a break from the earlier conceptual order into a new paradigm and a new conceptual order. However, some of the concepts of the old order which may not be consistent with the new conceptual order may well continue into the present so that it becomes a part of our conceptual perception even when it may not be entirely correct, and may thus become quite restrictive to free flow of ideas. A corrective perceptual change is then required. But such a change requires overcoming the ‘prejudice of the prevalent usage’. A prime example of such a corrective perceptual change is the familiar case of the Aharonov–Bohm (A–B) effect. Earlier, before the observability of a curl-free vector potential was pointed out by Aharonov and Bohm² and established experimentally sometimes thereafter^{3,4}, it was rather firmly believed – a perception, to be sure, a carry forward from the deemed classical concept – that only the fields, the electromagnetic fields, were the observables, and not the potentials. It was a rather entrenched perception. But the observation of the A–B effect changed all that, though the final acceptance was not an easy passage.

An intuition about the possible nature of phenomena pertaining to a physical system may be triggered by an analogy with a phenomenon in another system in a different setting. Analogies usually have no formal logic. But the play of intuition comes in recognizing some basic structural similarity between the two situations, even as the existing point of view may not support a behaviour suggested by the analogy. Following a lead suggested by an analogy is then what constitutes an intuitive ‘jump’.

*e-mail: ramkvarma@gmail.com

A new paradigm in charged particle dynamics in a magnetic field

I wish to share with the readers my own experience in pursuing a ‘hunch’ – an intuitive guess inspired by some analogies. Following this hunch relating to charged particle dynamics in a magnetic field, resulted in the evolution and development of a whole new paradigm^{5–11}, and the prediction of some associated new effects, which were far from obvious a priori, and could not have been conceived with the restraining prejudice of the earlier perception. In fact, they appeared to be so radically at odds with the prevailing conception that, not surprisingly, they attracted considerable amount of disbelief. Eventually, Nature – the ultimate arbiter – as revealed through experiments^{12–19} helped establish the physical reality of the predicted phenomena, and an understanding and an appreciation of the nature thereof.

In the present case that I shall discuss, the perceptual change involved is actually a little more non-trivial than the one relating to the Aharonov–Bohm effect. It relates to the division of physical phenomena strictly into ‘macro-classical and micro-quantum’ classes and the consequent widely and deeply held perception that all macro-scale dynamical phenomena necessarily belong to the classical domain and therefore classical mechanics ought to be adequate to describe them, barring the low-temperature phenomena of superconductivity and superfluidity. Likewise, it is believed that quantum phenomena belong essentially to the micro-scale. It will be pointed out, on the basis of the observed facts predicted by the theoretical formalism, that it need not always be the case: certain phenomena in charged particle dynamics have been identified that arise on the macro-scale, but are in fact of quantum origin. As such, they have been found to be inexplicable in terms of the classical Lorentz dynamics.

The next section introduces the physical problem relating to charged particle dynamics in a magnetic field, in the context of which the intuitive idea originated.

Residence times against leakage from the adiabatic traps – a heuristic treatment

I describe here the interesting and important problem of the theoretical determination of residence times against nonadiabatic leakage of particles from magnetic mirror traps. It is towards the solution of this problem that the intuitive guess was invoked leading to a rather unconventional formulation.

However, the nature of the intuitive guess can be appreciated only after the problem has been properly introduced to the reader. This is what is presented in the following by first explaining what a magnetic trap is, and what constitutes a nonadiabatic leakage from it.

A charged particle moving along an inhomogeneous magnetic field is known to possess an adiabatic invariant corresponding to its motion perpendicular to the magnetic field – the gyromotion. This is in accordance with a theorem – the adiabatic theorem – of classical mechanics, which states that a bounded periodic or quasi-periodic motion in a degree of freedom of a system, admits of an ‘adiabatic invariant’. Such an invariant – the gyro-action invariant – for charged particles in a magnetic field is given by

$$\mu = \mathcal{E}_\perp / \Omega, \quad \Omega = eB/mc, \quad (1)$$

where \mathcal{E}_\perp is the energy residing in the perpendicular motion of the particle, and $\Omega = eB/mc$ is its gyro-frequency in the magnetic field B . Making use of this invariant it is possible to define an approximate reduced motion along the magnetic field, which is a potential motion with the potential $V = \mu\Omega$ and the equation of motion

$$m \frac{dv_\parallel}{dt} = -\nabla_\parallel(\mu\Omega), \quad (2)$$

with v_\parallel being the velocity component along the field. With μ being an (adiabatic) invariant of motion, the variation of the potential along the field is essentially determined by the magnetic field variation. This equation is referred to as the ‘adiabatic equation of motion’, and is discussed in detail by Northrop²⁰, along with the question of the existence of the gyro-action adiabatic invariant.

A ‘magnetic trap’ is now realized by having a magnetic field configuration (axisymmetric, for simplicity) which has a minimum of the field bounded on either side by two maxima. Such a configuration then provides a one-dimensional potential well corresponding to $V = \mu\Omega = \mu(e/mc)B$, which has a minimum at the position of the field minimum. A charged particle can then be trapped in this effective potential well, provided that the energy \mathcal{E} of the particle is less than the height of the potential maximum $V_{\max} = \mu(e/mc)B_{\max}$, taking for simplicity, the two maxima to be of the same strength. That is, one must have $\mathcal{E} < \mu(e/mc)B_{\max}$. Recalling that $\mu = \mathcal{E}_\perp / \Omega = (1/2)mv^2 \times \sin^2 \delta (mc/eB)$ (δ being the pitch angle), the condition $\mathcal{E} < \mu(e/mc)B_{\max}$ translates to $\sin \delta > (B/B_{\max})^{1/2}$, where δ is the angle which the velocity vector of the particle makes with the magnetic field locally. If B here corresponds to the field minimum, then $\delta_c = \sin^{-1}(B_{\min}/B_{\max})^{1/2}$ defines the ‘loss cone angle’. A pitch angle $\delta > \delta_c$ corresponds to a trapped particle in the ‘adiabatic trap’, while $\delta < \delta_c$ corresponds to an untrapped particle. The boundary value $\delta = \delta_c$ defines the loss cone angle for the trap.

However, recalling that the effective adiabatic potential is generated because of the (adiabatic) invariance of the gyro-action μ , the trapping in this potential is as good as the invariance is. Therefore, if there is a departure from

the invariance – which is referred to as nonadiabaticity – then the adiabatic trapping will get leaky. Particles will leak out of the trap with time. But the actual nature of the decay of this population from the trap – referred to as nonadiabatic loss – is a rather tricky and nontrivial problem, because the question of the breakdown of an adiabatic invariance and departure from the adiabaticity is a rather complex mathematical problem, belonging as it does to the asymptotic theory and singular perturbations.

In the conventional viewpoint, the problem mentioned above being on the macro-scale, is supposed to belong to the classical mechanical domain in accordance with the prevailing tenet, and its solution must therefore be sought in the framework of the Lorentz equation – the governing equation for the dynamics. It turned out that the problem was much too complex to be amenable to an easily trackable solution to the posed question of residence time determination. The essential issue relating to the solution was to be able to extract out the nonadiabaticity from the formalism in a consistent manner so as to relate it to the nonadiabatic loss. Since the non-adiabatic effects are of a nonanalytic, non-expandible kind in the small parameter of the form $\sim \exp[-1/\varepsilon]$, a perturbation theory will fail to extract such effects. In view of this difficulty and consequent absence of a mathematically consistent procedure to extract the nonadiabaticity which is directly related to the loss, some approximate ‘patchwork’ solutions have been attempted. This attempt pioneered essentially by Chirikov²¹ and reviewed in ref. 22 is the best that could be done under the circumstances. Unfortunately, this approach did not achieve the desired objective of explaining the characteristics of the experimentally determined residence times²³.

However, in a radical departure from this conventional approach, several years before these attempts by Chirikov were published, the present author had developed a heuristic point of view whereby the nonadiabatic loss of particles from the adiabatic trap across adiabatic potential maxima was likened to ‘quantum tunnelling’ across classical potential barriers due to quantum effects. The non-adiabatic effects were then likened to quantum effects. The analogy turned out to be quite apt, since the adiabatic invariance requires that fields be slowly varying as defined through the smallness of a parameter – the adiabaticity parameter, $\varepsilon = \rho_g/L$ – which specifies how small the gyro-radius ρ_g is in comparison with a length scale L , characterizing the spatial variation of the magnetic field. The nonadiabatic effects arise when this adiabaticity parameter ε fails to be small enough. Likewise, the quantum effects become noticeable when the characteristic length of the potential variation is not sufficiently large compared to the de Broglie wavelength λ_{dB} for the given momenta involved in the problem. In the opposite case, classical equation of motion provides a good description.

Following this intuitive idea, it was thus conjectured that if nonadiabatic effects are to be likened to quantum

effects, then the former should be describable by a Schrödinger-form probability amplitude equation which ought to yield the (one-dimensional) adiabatic equation of motion in the appropriately defined adiabatic limit, $\varepsilon \rightarrow 0$. It was, to be sure, a heretical proposition on at least one count, namely that a probability amplitude description for particles is not supposed to exist for particles on the macro-scale as the present system belongs to. Such a (probability amplitude) description is known to be necessary and relevant only for the micro-scale of quantum phenomena. All electro-dynamic phenomena on the macro-scale are supposed to belong to the classical domain and ought therefore to be necessarily and sufficiently describable by the Lorentz equation.

However, if such a (probability amplitude) equation were to exist, setting aside the reservations against the proposition for the moment, then the adiabatic potential $V = \mu\Omega$ would take the place of ‘potential’ in the quantum Schrödinger equation. What then should be the analogue of the Planck quantum in it? The guess was that it ought to be the gyro-action μ which is an action on the macro-scale. This entire conjecture could, however, be summarily dismissed as being bizarre and absurd, because of the above-mentioned reason. But this turned out to be, precisely the defining input of the formulation. There was a strong intuitive reason to explore this conjecture, if one observes that the Schrödinger wave equation led to tunnelling effects, not contained in the classical equation of motion, and moreover that these effects are of a nonanalytic type of the form $\sim \exp[-1/\eta]$, non-expandible in the small parameter $\eta \sim \hbar$. Likewise, the nonadiabatic effects that one wishes to extract, which too are of the nonanalytic type, as noted above, could be well described by an equation of the Schrödinger type. Such a Schrödinger-form equation would be to the ‘adiabatic equation of motion’ as the Schrödinger wave equation is to classical equation of motion.

If one were to look for a starting point for a derivation of such an equation, neither of the two known formalisms – classical and quantum – would appear to serve the purpose as none of them would support a probability amplitude description for particle dynamics on the macro-scale. An appeal to the Feynman path integral methodology, however, provided a way to translate these intuitive ideas into a heuristic derivation⁵, which came after a bit of struggle. But it did yield something quite interesting and unusual namely a set of Schrödinger-form equations – not just one, like the Schrödinger equation – but an infinite set of them as given below:

$$\frac{i\mu}{n} \frac{\partial \psi(n)}{\partial t} = - \left(\frac{\mu}{n} \right)^2 \frac{1}{2m} \frac{\partial^2 \psi(n)}{\partial x^2} + \mu\Omega \psi(n), \quad n = 1, 2, 3, \dots, \quad (3)$$

where now $\psi(n)$ represents by construction a set of probability amplitudes corresponding to different n values,

$n = 1, 2, 3, \dots$ and where the total probability density \mathcal{P} over all the n modes is given by the sum, in consonance with Born's probability prescription,

$$\mathcal{P}(x,t) = \sum_n \psi^*(n) \psi(n), \quad (4)$$

but now it includes all the n modes and is a part of the construction itself, and not as an external prescription.

It may be remarked that in the limit $\mu \rightarrow 0$ taken via $B \rightarrow \infty$ (implying $\varepsilon \rightarrow 0$), each one of the above equations yields a Hamilton–Jacobi equation corresponding to the ‘adiabatic equation of motion’, i.e. eq. (2). This is in accordance with the requirement that the adiabatic equation of motion should be recovered in the adiabatic limit defined by $\varepsilon \rightarrow 0$. Departures from adiabatic limit – referred to as nonadiabatic effects – should then be describable by the above set of equations in the same spirit as the departures from the classical dynamics are describable by the Schrödinger equation.

This formulation – even as it was essentially intuitive-heuristic – appeared to be quite appealing. Its probability amplitude character is on the macro-scale by virtue of the presence of a macro-scale action μ in lieu of \hbar . But it was manifestly contrary to the prevailing conception that such a probability description holds only for the micro-scale domain of quantum phenomena. It defied the current norms.

The real test of the validity of the above formulation would rest on its ability to describe real experimental situations. This set of equations enabled residence times in the trap to be calculated in the same manner as in quantum tunnelling problems. Fortunately, experimental results pertaining to such residence times in certain mirror trap configurations and determined for varying magnetic field strengths, had been reported around that time (1969–70)²³. This enabled a ready check to be carried out for the correctness of its description and its predictive ability. The experimentally determined residence times for the various magnetic field strengths as reported in ref. 23 were compared with the theoretically calculated ones, using the equation corresponding to $n = 1$ and a magnetic field variation defining the adiabatic potential, approximating that used in the experiment.

The experimental residence times, which were found to increase exponentially with the magnetic field strength were found to be in surprisingly good agreement with the calculated magnetic field dependence⁵. But there were now additional predictions that these equations afforded: the existence of other discrete residence times corresponding to the other equations of the above set for $n = 2, 3, 4, \dots$. These predictions made in ref. 5 would now serve as an acid test of the validity of these set of equations. It is worth remarking that the prediction of the existence of such quantized residence times is quite unexpected from

the conventional point of view which uses nonlinear dynamical methods based on the Lorentz equation²¹.

Experiments were next carried out to check these predictions, which have been reported by Bora and co-workers^{17–19}. The experimental results have indeed revealed the existence of these additional quantized residence times as well, with all the characteristics as predicted by the above set of equations. We recount here briefly the various characteristics as deduced from the above set of equations for the given experimental situation.

Assuming the magnetic field variation in the region of the ‘mirrors’ – which are the regions of magnetic field maximum – to be approximately described by the following form

$$B = B_0 + [B_{\max} - B_0][\cosh \alpha x]^{-2}, \quad (5)$$

along a certain field line with x as the coordinate, then the probability of transmission per unit time across the potential hill, eq. (5), is given by (as obtained in Varma⁵)

$$P = \frac{1}{T} \sum_n C(n) e^{-\beta_n B}, \quad (6)$$

with β_n given by

$$\begin{aligned} \beta_n &= (2m)^{1/2} \frac{2\pi n}{\alpha \mu B} \{(\mu \Omega_{\max} - \mu \Omega_0)^{1/2} - (\mathcal{E} - \mu \Omega_0)^{1/2}\} \\ &= n \left(\frac{2}{m}\right)^{1/2} \frac{2\pi L e}{c\sqrt{\mathcal{E}}} f(\delta), \\ f(\delta) &= \left\{ \left(\frac{B_{\max} - B_0}{B}\right)^{1/2} \sin \delta - \left(1 - \sin^2 \delta \frac{B_0}{B}\right)^{1/2} \right\}, \quad (7) \end{aligned}$$

where B is the value of the magnetic field at the point of injection, and δ is the pitch angle of injection so that $\mu = \mathcal{E} \sin^2 \delta / B$; B_0 is the magnetic field in the straight middle section of the magnetic mirror trap. It is seen that the probabilities of transmission corresponding to the various modes n are exponentially smaller for successively larger values of n , as we see that $\beta_n = n\beta_1$. The corresponding residence times τ_n , which would be given by

$$\tau_n = T e^{\beta_n B} = T e^{n\beta_1 B}, \quad (8)$$

are exponentially longer. Note that in the case of slight nonadiabaticity (quasi-adiabatic approximation), T may be taken as the bounce period between adiabatic turning points. $C(n)$ in eq. (6) represents relative magnitudes of the transmission probabilities for the various n , which the

model does not determine. It may be conjectured that they fall off as $\sim \varepsilon^n$, where ε denotes the adiabaticity parameter measuring the magnitude of the gyro-radius relative to the magnetic field scale length.

We next give a summary of the experimental results as obtained in the literature¹⁷⁻¹⁹, while for the details of the experiments the cited references may be referred to. Note that there are essentially three control parameters in the experiment – the energy \mathcal{E} of the particles (electrons), the scale L of the magnetic field variation expressed through α as $L = 1/\alpha$, and the pitch angle δ at the point of injection. We only summarize here results for the various energies \mathcal{E} and scale lengths L used in the experiment. Results with respect to pitch angle variation are available in ref. 19.

To measure the residence times in the trap, the leakage current collected from the end of the trap is monitored as a function of time from the instant of injection and trapping and is then analysed in terms of two or more exponential decay terms of the following form

$$I = A_1 e^{-t/\tau_1} + A_2 e^{-t/\tau_2} + A_3 e^{-t/\tau_3}, \quad (9)$$

where the three residence times τ_1, τ_2, τ_3 are taken to correspond to the three different modes $n = 1, 2, 3$. The leakage current in most of the cases pertaining to different energies and scale lengths was found to fit with two exponential terms. In some cases, however, one needed three exponential terms to get the best fit with the data, identifying three residence times τ_1, τ_2, τ_3 . With experiments carried out with different magnetic field strengths, these residence times could then be determined as a function of the field strengths.

Figure 1 exhibits two sets of $\ln \tau$ versus B plots for the two residence times τ_1, τ_2 identified from the analysis of the time series for the leakage current. One set pertains to four different energies $\mathcal{E} = 2.2, 2.9, 3.7$ and 4.5 keV for the same magnetic field scale length $L = 8$ cm, while the other set corresponds to the same energy $\mathcal{E} = 2.9$ keV, but three different scale lengths $L = 8, 11$ and 13 cm. All these plots are seen to exhibit linear dependence of $\ln \tau$ with respect to the magnetic field. This is thus seen to be in accordance with the expectation as expressed by eq. (6), which gives an exponential dependence for the residence times with the magnetic field strength B , with the exponent β_n given by eq. (7). Table 1 summarizes the various characteristics of the observed results for the residence times and their dependences with respect to the magnetic field strength and the index n . The third and fourth columns of Table 1 give the experimentally determined values of the quantities β_1, β_2 corresponding to the two residence times τ_1, τ_2 identified for the set of energy \mathcal{E} and scale length L values as indicated respectively, in first and second columns of Table 1. The last column gives the ratio β_2/β_1 for each of the cases.

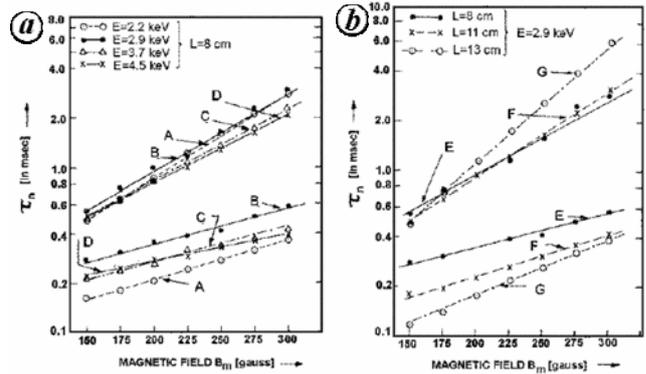


Figure 1. *a*, $\ln \tau$ versus B plots depicting two different residence times corresponding to $n = 1$ and $n = 2$, for four different energy values $\mathcal{E} = 2.2, 2.9, 3.7$ and 4.5 keV, but the same field variation scale length $L = 8$ cm. The pairs of plots AA, BB, CC and DD correspond to $n = 1$ and $n = 2$ for a given set (\mathcal{E}, L) . *b*, Plots for the same energy $\mathcal{E} = 2.9$ keV, but three different field variation scale lengths $L = 8, 11$ and 13 cm. The pair of plots EE, FF and GG correspond to $n = 1$ and $n = 2$.

Table 1. Slopes $\beta_{1,2}$ of the $\ln \tau$ versus B plots corresponding to $n = 1, 2$ for the various indicated energies \mathcal{E} and magnetic field scale lengths L . The last column gives the ratio β_2/β_1 for the various cases, which are close to the theoretically expected value of 2

\mathcal{E} (keV)	L (cm)	β_1	β_2	β_2/β_1
2.2	8	$(5.56 \pm 0.18)10^{-3}$	$(11.9 \pm 0.5)10^{-3}$	2.14 ± 0.03
2.9	8	$(4.9 \pm 0.24)10^{-3}$	$(10.46 \pm 0.7)10^{-3}$	2.12 ± 0.04
3.7	8	$(4.54 \pm 0.3)10^{-3}$	$(9.95 \pm 0.68)10^{-3}$	2.15 ± 0.06
4.5	8	$(3.96 \pm 0.42)10^{-3}$	$(9.15 \pm 0.7)10^{-3}$	2.31 ± 0.01
2.9	11	$(6.16 \pm 0.18)10^{-3}$	$(12.62 \pm 0.62)10^{-3}$	2.05 ± 0.03
2.9	13	$(8.24 \pm 0.3)10^{-3}$	$(17.2 \pm 0.53)10^{-3}$	2.09 ± 0.038

Apart from the observation of the two distinct residence times as revealed in these results, the most striking result, as given in the last column, is that the ratio β_2/β_1 for the various sets of parameters clusters around the value 2, which is the value expected in accordance with the exponential dependence of the residence times on the index n , as expressed by eqs (7) and (8).

Therefore, not only did we observe distinct residence times, but their crucial dependence on the index n and other parameters was found to be fully validated. In fact, as pointed out already, the time series for the leakage current for some of the cases has been found to give a ‘least square fit’ with three distinct residence times τ_1, τ_2, τ_3 , particularly for the smaller scale length case $L = 8$ cm. The corresponding $\ln \tau$ versus B plots are also found to yield the values of β_n , which were found to be in the required ratio $\beta_3 \sim 3\beta_1 \sim 3\beta_2/2$. We do not present these results, but refer the reader to ref. 19.

Some comments on the implications of these results

We now comment on the interesting implications of these results, and evaluate their significance with respect to their unusual nature.

The following points stand out:

(i) The model – described as a ‘wave-mechanical’ model in the original paper⁵ – and as represented by the Schrödinger-form equations, has been found to be surprisingly successful in not only describing the already observed residence times, which were seen to correspond to the $n = 1$ equation, but also predicting successfully additional quantized ones corresponding to $n = 2, 3, 4, \dots$, which were subsequently observed.

(ii) Such additional residence times were entirely unexpected and were therefore seen to be quite surprising, because they could not have been even suspected from the conventional viewpoint of using nonlinear dynamical methods in the classical mechanical framework with the Lorentz equation. In fact, the only such serious attempt made by Chirikov²¹ could not properly explain even the existing ones, let alone describing the other quantized ones observed subsequently. It is worth pointing out that the paper⁵ describing the above ‘wave-mechanical’ model pre-dated by about seven years the work by Chirikov²¹, which is based on the classical Lorentz dynamics.

(iii) An examination of the expression for the probability of transmission as given by eqs (6) and (7) across the potential hump eq. (5) shows that it is of the form $\sim \exp[-1/\varepsilon]$. This essentially has the structure of a non-adiabatic change of the gyro-action as calculated, for example, in ref. 24. It is thus clear that the above model – with the Schrödinger-form equations – is able to correctly associate the leakage probability from the trap with nonadiabaticity, for which the leakage from the trap is considered to be accountable. This formulation thus describes in a natural way the characteristic nonanalytic, non-expandible behaviour associated with nonadiabatic effects.

It is pertinent to recall here the form of the expression for the probability of transmission across classical potential humps due to quantum effects. This has the form $\exp[-2\int|p|dx/\hbar]$, where the limits of integration are the ‘turning points’. This is easily seen to be of the form $\exp[-2L/\lambda_{dB}]$. This form bears a close similarity with the probability of transmission across the adiabatic potential hump due to nonadiabatic effects, since the parameter of smallness in the latter, namely ε is analogous to the parameter of smallness in the quantum case $\eta = \lambda_{dB}/L$. This shows the existence of a close formal similarity between the two – the nonadiabatic effects and quantum effects.

What is noteworthy about this formulation, even though it is heuristic, is that it covers both adiabaticity and nonadiabaticity in a self-consistent manner. The adiabatic equation of motion is recovered in the limit $\varepsilon \rightarrow 0$ in the form of the corresponding Hamilton–Jacobi equation, while the nonadiabaticity is contained in it in a natural way as its integral part.

In view of the observations noted above, and the remarkable success of the formulation in describing the residence times in the trap, one is left with little doubt that this formulation is indeed a correct description of the physical processes involved in the problem under consideration. However, one may argue that there is a serious shortcoming afflicting the model formulation; it lacks formal legitimacy. The success of this formulation is pleasantly surprising, but it still requires the revelation of the real nature of its description. Which of the two dynamical theories – the classical and the quantum – can it be related to and in what manner?

The search for a proper connection, which was carried out over the next two decades did eventually lead to a rather fascinating relationship with the known formalisms, and a consequent unravelling of a set of surprising new physical phenomena relating to charged particle dynamics on the macro-scale. These phenomena which could not have been anticipated beforehand, include as one of them, the existence of the predicted and observed ‘quantized’ residence times. The word ‘quantized’ is used here in the sense of their discreteness labelled by the integers $n = 1, 2, 3, \dots$, and not quantized in the sense of quantum mechanics. Not yet in any case!

In the following pages, I relate the rather interesting journey which led to these revelations. I would like to emphasize the crucial role that the experimentation played in the success of this long journey, without which the rather heterodox ideas that this formulation represented would have been regarded just as a fancy. Predictions made at the various stages of development of the ideas were subject to experimental scrutiny.

The search for a relationship with known formalisms

The probability amplitude character of the above set of equations suggested a relationship of these equations with quantum mechanics. An attempt in that direction was made already in ref. 6 soon after the publication of the first paper⁵. But it was not very fruitful and far from a natural one. Such a connection was, however, realized later⁸ in a much more interesting and meaningful way. However, the macro-scale character of these equations suggested that a connection with the classical dynamical formalism ought to exist.

Relationship through the classical Liouville equation

A way forward was suggested by the observation that the Liouville equation of classical mechanics, which represents a Hamiltonian flow in the phase space, a linear partial differential equation in the phase space density, shares the property of linearity with the Schrödinger-form

equations, i.e. eq. (3). Given that the classical Liouville equation describes macro-scale phenomena, it was surmised that it could somehow lead to a derivation of the Schrödinger-form equations. However there is a fundamental issue in this proposition. The Liouville equation is a first-order partial differential equation for the phase space density describing a Hamiltonian flow for the probability, while the eq. (3) is a set of second-order (in space) partial differential wave equations for the probability amplitudes on the configuration space.

This issue appeared to pose a serious problem of mathematical nature. However, a solution was found through a somewhat unorthodox procedure which finally led to the derivation of the above set of Schrödinger-form equations (eq. (3)) complete with the generalized probability connection eq. (4). The procedure amounts to constructing a Hilbert space representation of the classical Liouville equation for the problem under discussion. The important point to emphasize here is that the ensemble that was chosen to be described by the Liouville equation was not any arbitrary one, but was chosen to represent for this purpose what Dirac²⁵ has termed as a ‘family’, and Sygne²⁶ a ‘coherent system of trajectories’ and is specified through a δ -function distribution in the initial momenta. This corresponds essentially to the ensemble of particles injected into the trap in the actual experiment. This derivation was reported in ref. 7, and it aroused considerable curiosity in terms of its structure and implications.

Predictions of the matter wave interference effects on the macro-scale

The derivation of the set of the Schrödinger-form equations (eq. (3)) through a formal procedure, provided the desired formal legitimacy to the above set of equations. Based on this, a further prediction was now ventured into about the existence of one-dimensional matter wave interference effects that these equations were interpreted to predict. This prediction was made by Varma⁷.

It was tempting to check whether such interference effects as predicted therein could be observed experimentally. The experiments were planned to check out these unusual predictions. I describe below the experiments carried out over a number of years and the results obtained to detect the possible existence of such effects.

Experiments to check the existence of macro-scale matter wave interference effects

The experiment to be carried out was designed such that the one-dimensional motion along the magnetic field was through a periodically varying magnetic field, which would appear to be analogous to a periodic ‘crystal lattice’, with the separation between the two maxima/minima of the field being analogous to the lattice period.

The experiment was quite simple and consisted of studying the electron current collected by a plate detector as an electron beam (of a very low current $\sim \mu\text{A}$) propagates along a magnetic field from one end of a vacuum chamber $\sim 5 \times 10^{-7}$ torr. The experiment was conducted in three different modes: (i) Keeping the energy of the electron beam fixed, and sweeping the magnetic field from a small value to a certain large value, and recording the plate current at the other end of the chamber during the sweep. The detector could simply be a grounded plate, or a Faraday cup with a negatively biased central grid to screen out secondary electrons. The results of this mode were first reported by Varma²⁷. (ii) The other mode was to keep the magnetic field fixed at some appropriate value, switch on the electron beam at a certain electron energy, and sweep the negative bias on the Faraday cup grid from a large value to zero. The results from these experiments are reported by Varma and Punithavelu^{12,13}. (iii) The third mode consisted in sweeping the electron energy from zero to a certain large value (typically 1 keV) with the magnetic field kept fixed at a certain value. I will discuss the results from the third mode later.

These experimental results were compared with the expectations from the theoretical formalism, which were worked out in ref. 27. Using the Schrödinger-form equation (eq. (3)) for $n = 1$, the probability density for the electrons arriving at the plate from the gun was evaluated for a given μ . Later this expression for the probability density was averaged over a small spread in μ , and in the energy of injection \mathcal{E} , to take care of an inevitable spread in both these quantities that can exist in the injected beam. The averaged probability density is given by

$$|\overline{\psi^* \psi}|(x_p) \approx A + B \sin[K(x_p - x_o) + \phi], \quad (10)$$

where $K = \Omega/v_{\parallel}$ is seen to be a wave vector which characterizes the oscillating term above, and where ϕ is a possible phase term. According to the expression above, this now indicates the existence of maxima/minima in the detector plate current which corresponds to a wavelength $\lambda_M = 2\pi v_{\parallel}/\Omega$. These are thus interpreted as interference maxima/minima with a matter wave on the macro-scale with the above wavelength – which is obviously \hbar -independent. The results of all the related experiments were thus interpreted in the light of the above expression.

First, about the qualitative nature of the obtained experimental results against the expectations of the standard view. With the rather simple system and manner of carrying out the experiments with it, as indicated above, it would appear that in each of these cases, the response of the plate current could not be more than a monotonic one, as viewed in the conventional picture with the Lorentz equation as the governing equation. However, each of these cases presented a surprise, since the plate current exhibited the presence of sharp maxima/minima. More-

over, these maxima were found to be in agreement with eq. (10) for the probability density which would be reflected in the probability current measured by the plate detector. The condition for the existence of maxima following from the above expression is given by

$$KL \equiv \frac{\Omega}{v_{\parallel}} L = 2\pi\ell - \phi, \quad \ell = 1, 2, 3, \dots, \quad (11)$$

where ℓ denotes the order of the interference maximum. If one were to look for interpeak separation between the maxima, then (assuming the phase ϕ to be just a constant term), it results in the following relation

$$\Delta\Omega L = 2\pi v_{\parallel} \Delta\ell, \quad \Delta\ell = 1, 2, 3, \dots \quad (12)$$

where $\Delta\Omega \equiv (e/mc)\Delta B$ represents the magnetic field interval corresponding to the order interval $\Delta\ell$. For a given electron energy and a given distance $L = x_p - x_o$, this condition describes equally spaced peaks with respect to a linear magnetic field sweep. This is precisely what has been reported²⁷. On the other hand, with the sweep of the retarding potential on the grid, with a fixed magnetic field, as carried out in the experiment¹², the peaks of the detector current are seen to dilate with the linear potential sweep (which translates to the energy sweep) in the manner $\delta\mathcal{E} \sim \mathcal{E}^{3/2}/\Omega L$, where $\delta\mathcal{E}$ denotes an interpeak separation at energy \mathcal{E} . The plots relating to these results can be looked up in the appropriate refs 12 and 27.

It is not the purpose of the present article to go into the details of the various experiments carried out, which can be looked up in the above references. It is essentially to point out that in each of the three different modes of experimentation, the observed maxima/minima in the plate current are found to be in sharp contrast to the expectation according to the Lorentz dynamics, which can at best lead to a monotonic response. In fact, the monotonic response seems to be recovered when the magnetic field is either not strong enough or the gun-plate distance not large enough, for a given energy \mathcal{E} . The minimum condition to be satisfied for the occurrence of the undulations is that $\Omega L > 2\pi v_{\parallel}$.

These observations aroused a lot of curiosity, and they were attempted to be repeated by two groups of workers^{28,29}. Both these groups successfully repeated the results, with the latter ones being more definitive. But, not surprisingly, they tried to give different explanations for their occurrence – using essentially the classical Lorentz dynamics as the basis. In particular, both used the well-known property of periodic ‘focusing’ of an electron beam with a small angular spread at every focusing length distance ℓ_f , given by $\ell_f = 2\pi v_{\parallel}/\Omega$, with Ω as the gyro-frequency and v_{\parallel} as the parallel electron velocity of the particle.

Confrontation with the standard canonical view

The models advanced by these two groups of workers represent a confrontation of this obviously unorthodox proposition of the existence of matter wave interference effects on the macro-scale with the standard classical mechanical paradigm as represented by the Lorentz equation. These models take the obvious orthodox view that the observed phenomena reported^{12,27} belong to the macro-scale and ought to be explainable in terms of the classical Lorentz dynamics.

It so happens that there is a fortuitous coincidence between the expression for the focusing length $\ell_f = 2\pi v_{\parallel}/\Omega$ and the wavelength $\lambda_M = 2\pi v_{\parallel}/\Omega$. Thus the two groups tried to give a model which makes use of precisely this property of multiple focusing of the beam at every focusing length along the field. While both the groups were able to reproduce correctly the ‘location’ of the maxima in the parameter space, their models applied only to the case reported in ref. 12 where there was a sweep of the retarding potential, and not to the case reported in ref. 27 involving the magnetic field sweep, or in the case where the electron energy is swept with both the plate and grid kept grounded, reported later¹⁴. This was because their models required for the operation of their mechanism, the presence of a negatively biased grid. A detailed critique of the models of these groups of workers in relation to our interpretation of the results is given in the Appendix of ref. 12.

I shall return later to another phenomenon in relation to the third mode of experimentation; that is, sweeping the electron energy while keeping the magnetic field fixed. This phenomenon will be seen to provide further confirmatory evidence identifying these phenomena with matter wave interference effects on the macro-scale.

The next crucial phase of development – relationship with quantum mechanics

Notwithstanding the above-mentioned observations of macro-scale matter wave interference effects in accordance with the predictions of the formalism⁷, there remained a disquiet that the true nature of these effects was still eluding. If they were some unfamiliar manifestation of classical dynamics, then what precise underlying structure of the latter could these be attributed to?. Could they be related to some topological aspects of classical dynamics discussed by Synge²⁶? This particular question was explored in ref. 30.

However, motivated by the parobability amplitude character of these equations, their relationship with quantum mechanics was addressed again. This led to the revelation of a rather fascinating relationship with the quantum formalism, which was reported by Varma⁸. According to the picture so obtained, a scattering of the

particle against any fixed centre in the path of the electron beam, or against any sharp inhomogeneity of the magnetic field, would cause transition across one or more Landau levels, resulting in a corresponding energy deficit in the parallel degree of freedom. However, concomitantly, the pre-scattering plane wave state of the particle along the magnetic field gets modulated. The derivation given in Varma⁸ essentially determines the equation of evolution for this modulation. It is shown there that this equation of evolution is essentially the same as the Schrödinger-form equation (eq. (3)) obtained heuristically⁵ or from the Liouville equation⁷. Furthermore, this leads to an interesting meaning to the index n which labels the various equations of the set. Accordingly, now n denotes the Landau level interval, the transition across which leads to the generation of the corresponding modulation of the de Broglie wave along the field. There is a modulation corresponding to any value of n , and the corresponding equation in the set (eq. (3)) describes the evolution of that modulation. As will be described later, each of these modulations characterized by the index n exhibits its own interference effects, which show up as harmonics of the fundamental wave number $K = \Omega/\nu$, as will be seen in the Fourier plots of the plate current.

This manner of derivation of these equations from the quantum formalism, enabled an important generalization to be effected, namely to include a curl-free vector potential. The set of equations so obtained⁸ are then

$$\frac{i\mu}{n} \frac{\partial \psi(n)}{\partial t} = \frac{1}{2m} \left(\frac{\mu}{in} \frac{\partial}{\partial x} - \frac{e}{c} \hat{A}_x \right)^2 \psi(n) + \mu\Omega\psi(n), \quad (13)$$

which include a component of the curl-free vector potential along the direction x of the magnetic field locally.

The implications of this generalization (to include a curl-free vector potential) are rather extraordinary: these equations thus predict the observability of a curl-free vector potential on the macro-scale. Such a prediction was made by Varma⁸. But such an observation could be considered rather ‘unthinkable’ as it posits against the well-known Aharonov–Bohm effect, which is a quantum effect – regarded now as of topological origin. Furthermore, according to the canonical view, a curl-free vector potential is an ‘observable’ only quantum mechanically and therefore only on the micro-scale. On the macro-scale, one would again invoke the Lorentz equation to describe the phenomena. And since the Lorentz equation disregards a curl-free vector potential, there could be no meaning to the observability of a curl-free vector potential on the macro-scale – so would the argument go.

Again, there could be no better way to counter the above argument, than to just demonstrate the above-mentioned effect experimentally; this was eventually done.

However, before I take up discussion of the experimental demonstration of this effect, I describe here a phenomenon

associated with the macro-scale matter wave interference, which provides evidence for the matter wave nature of the observed effects on the macro-scale. This refers to the observation of ‘matter wave beats’ which display the wave property that the beat frequency equals the difference between the two beating frequencies. These observations are described here. It is the observation of these beats which constitutes the right qualifying evidence in favour of the identification of the observed phenomena with wave phenomena involving the matter waves on the macro-scale.

Observation of matter wave beats

To be able to describe the phenomenon of beats mentioned above, it is necessary to first explain what is meant by the term ‘frequency’ in the experiments described in the section on checking the existence of macro-scale matter wave interference effects. To do so, we refer to the term in eq. (10) which describes a sinusoidally oscillating contribution to the plate current with the argument $K(x_p - x_0) \equiv KL$. If in an experiment, one keeps the distance L between the gun and plate fixed, and sweeps in whatever manner the quantity K , then L will act as a ‘frequency’ with respect to the variation of K . Since $K = \Omega/\nu \equiv (eB/c)(2m\mathcal{E})^{-1/2}$, a sweep of K can be effected either by a sweep of the magnetic field B , keeping the energy fixed, or that of the electron energy \mathcal{E} keeping the magnetic field fixed. Here I describe briefly the results of experiments carried out with the sweep of the energy.

Figure 2 represents the plate current plots for the electron energy sweep for the parameters indicated in the figure. Figure 2a gives both the plate and grid current response as a function of the electron energy. Figure 2b depicts the same plate current response, but now replotted as a function of $\mathcal{E}^{-1/2}$. One would notice the presence of equidistant peaks on the $\mathcal{E}^{-1/2}$ scale, whereas the interpeak separation exhibits dilation with the energy in the top frame. Such an equidistant peak response is in accordance with the expectation of the oscillating sine term in eq. (10) since $K \sim \mathcal{E}^{-1/2}$, so that equi-intervals of K correspond to equi-intervals of $\mathcal{E}^{-1/2}$. It has also been found, in agreement with the above expression, that the frequency of undulations increases with the distance in the required manner.

We should also point out the presence of two subdominant frequency peaks, besides the dominant frequency peak in Figure 2c, which represents the Fourier plot of the plate current. The dominant peak corresponds to interference effects relating to $n = 1$ modulational wave, while the sub-dominant ones correspond respectively, to $n = 2$ and $n = 3$ modulational waves. These are clearly seen to be second and third harmonics with frequencies twice and thrice that of the fundamental.

If one now introduces two distances in the system $L_p = (x_p - x_0)$, the gun–plate distance (L above, but now

redesignated as L_p), and a gun-grid distance $L_g = (x_g - x_0)$ with the grid now situated at a finite distance D from the plate, so that $D = L_p - L_g$, then one has two frequencies in the system corresponding to these two distances. Figure 3 shows the plot for the response of the plate current with the above grid position. Figure 3a gives the response of both the plate and grid, found to anti-correlate with each other. Figure 3b gives the plate current response when re-plotted as a function of $\mathcal{E}^{-1/2}$ as before.

These plots exhibit some remarkable features. One clearly observes the presence of beats riding over a high

frequency undulation. In Figure 3b, one finds both the high frequency and beat oscillations exhibiting equidistant peaks. A Fourier decomposition of the curve in Figure 3b, which is presented in Figure 3c, depicts the

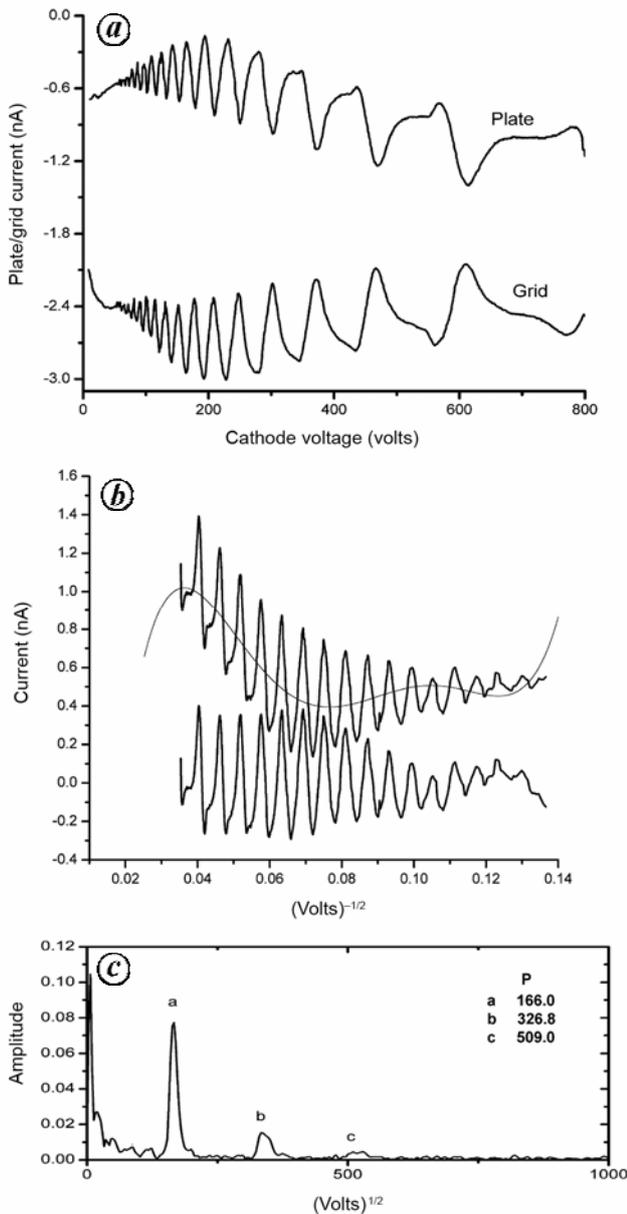


Figure 2. Plate/grid current variation as a function of electron energy. *a*, Observed plate and grid current responses. *b*, Plate current response of (*a*) replotted as a function of $\mathcal{E}^{-1/2}$. This yields equally spaced peaks. *c*, Fourier plot of curves (*b*) showing a dominant frequency peak and two non-dominant peaks corresponding to second and third harmonics.

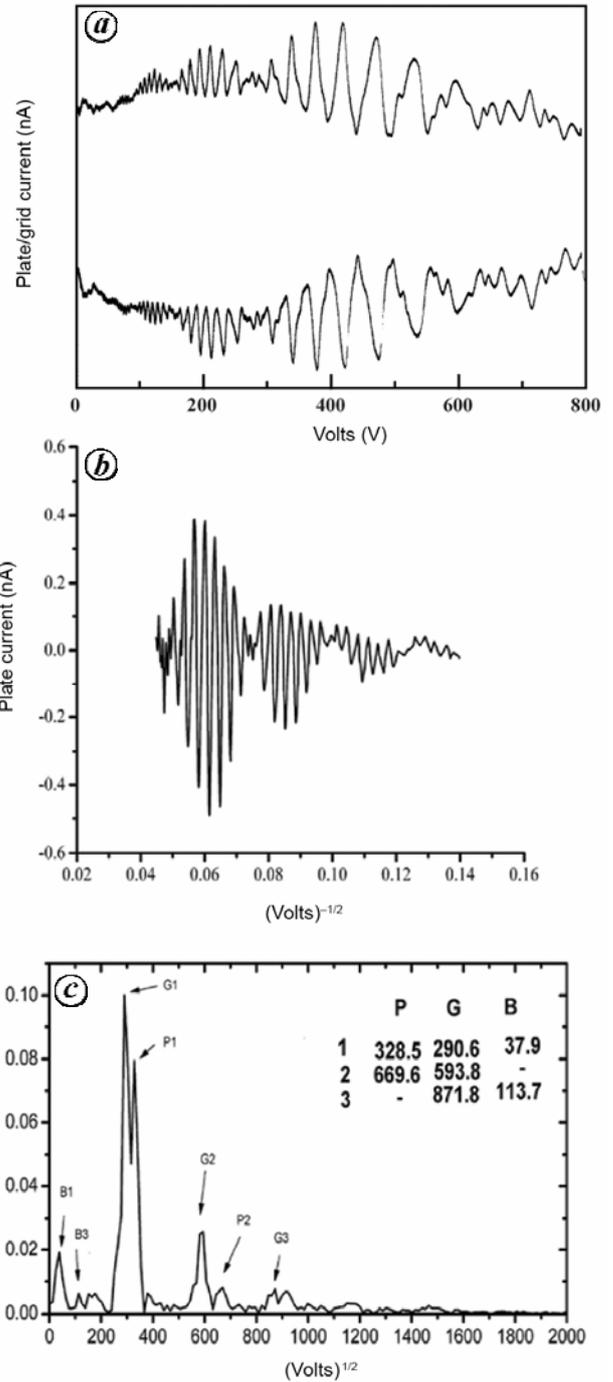


Figure 3. Variation of plate/grid current as a function of electron energy with grid-plate distance $D = (L_p - L_g) = 6$ cm. *a*, Plate/grid (upper curve plate, lower curve grid) current as a function of energy \mathcal{E} . *b*, Plate current of (*a*) transformed in terms of $\mathcal{E}^{-1/2}$. *c*, Fourier plot for the curve of (*b*). Two close frequency peaks $P_1(328.5)$ and $G_1(290.6)$ are clearly visible which beat together to produce the beat frequency peak $B_1(37.9)$.

presence of two close frequencies. But more importantly, there also exists a frequency peak corresponding to the difference between the two frequencies. This provides a critical evidence that the phenomenon represented here is indeed a wave phenomenon, because it is only in that case the beat frequency equals the difference between the two beating frequencies.

Confrontation with the classical Lorentz dynamical picture

Our claim was attempted to be refuted by an author³¹, who again invoked the fortuitous identity between our expression for the macro-scale matter wavelength $\lambda_M = 2\pi v/\Omega$ and the expression for the focusing length which is a well known classically understood effect. However, the model constructed by him on this basis failed to reproduce the beat structure observed by us, contrary to his claim. Our rebuttal of his claim is presented in ref. 32.

Mechanism of the generation of the modulational wave

An important question that had remained unattended during the course of the development so far, is the one related to the mechanism of generation of the ‘modulation’ in an actual experiment. It is already implicit in the derivation of the Schrödinger-form equations in ref. 8, that the modulation of the de Broglie wave along the field arises in consequence of a transition across Landau levels. It is important to examine the actual mechanism of generation and how the above expression for the wavelength of the modulation follows.

This question has been addressed in two papers: One, in ref. 9 in a more general context, where a general formulation has been presented based on the Feynman path integral formalism and then applied to a model problem involving a system with two degrees of freedom, in one of which there is a harmonic oscillator with a free motion in the other. The second, a more recent paper¹¹, is specifically devoted to the current problem of charged particles in a magnetic field. It has been shown there that the grid in the experiment provides a scattering centre for the particles moving past it, whereby the scattering can lead to transitions across a few Landau levels. In the process, it is shown in the framework of the standard inelastic scattering theory, that the de Broglie wave gets modulated such that the wavelength of the modulation is found to be $\lambda_M^{(n)} = 2\pi v/n\Omega$.

This is an important link which now completes the picture relating to the predicted existence, the generation, and the experimental demonstration of the existence of the modulational matter waves. According to this picture now, any scattering object in the path of the electron

beam such as the grid, for example, would serve as a source point of generation of the modulational wave from where the path length for these waves would be reckoned. Using this crucial fact, one can now explain the various experimental results in a given situation. In fact, experiments have been reported¹⁵, where the plate current responses have been recorded for the runs with different grid positions all along the length of the chamber between the gun and the plate. All such plate current responses have been duly explained¹⁵, in terms of the above algorithm involving the grid-scattering generated modulational waves.

Furthermore, and more importantly, this algorithm can now be used to plan other experiments and predict their outcomes.

Observation of a curl-free vector potential on the macro-scale

I now relate the observation of a phenomenon, which appears to come into the most severe conflict with the standard view – the observation of a curl-free vector potential on the macro-scale.

A curl-free vector potential is not known to be an observable classically, because the Lorentz equation which involves only the magnetic field, will not allow any curl-free vector potential to affect the dynamics of a charged particle. And if it is decreed that a macro-scale belongs necessarily to classical dynamics, then the observation of a curl-free vector potential on the macro-scale just cannot be possible, as it would be in violation of our current understanding.

The other point of conflict relates to the fact that the observability of a curl-free vector potential is so far known to be true only quantum mechanically, and therefore on the microscale – as the celebrated Aharonov–Bohm effect. Therefore, its observation on the macro-scale goes against all the understanding that has been developed so far in relation to it – as the argument may go. However, the very fact of its observation, as I shall relate here, should negate all the arguments objecting to its possible existence. The first point that needs to be recognized is that while the Aharonov–Bohm effect is a micro-scale effect relating to the de Broglie wave, the present macro-scale effect is attributed to the modulational wave which is on the macro-scale. There is, therefore, no conflict between the two. They both exist independent of each other, though their nature is different, as also the manner of their observation.

The observation of the above-mentioned phenomenon was first reported by Varma *et al.*³³. The apparatus to conduct the experiment was essentially the same as that for the observation of the interference effects – a vacuum glass chamber immersed in an external axial magnetic field, with an electron gun at one end of the chamber and

a grounded collector plate at the other. For the current experiment, a toroidally wound solenoid was placed in the middle of the chamber and external to it. The passage of a current in the solenoid produces a curl-free vector potential field in the space around, by virtue of the magnetic flux trapped completely inside it except for a possible small leakage field, just in its vicinity.

The above reference provides the details of the experimentation and the algorithm behind it, as also a more recent one¹⁶. Basically, the experiment is carried out first by tuning the external magnetic field such that it corresponds to one of the interference maxima (typically corresponding to $\ell = 1$ below) as defined by the relation

$$\frac{\Omega L}{v} = 2\pi\ell, \quad \ell = 1, 2, 3, \dots \quad (14)$$

where L is the gun–plate distance in the experiment, with the electron beam with a certain value of the energy \mathcal{E} turned on, with v being its parallel velocity.

The current in the toroidal solenoid is then swept keeping the system in the tuned state. This would lead to a variation in the curl-free vector potential field in the space around, which the electrons sense.

In simple terms, if the electrons were to remain unaffected by the curl-free vector potential field so produced, then the plate current response during the sweep of the toroidal solenoid current would be flat. This would be in accordance with the expectations of the Lorentz equation. On the other hand, if it exhibits undulations, then clearly the curl-free vector potential is deemed to have been detected.

Figure 4 depicts the plate current response as a function of the above-mentioned sweep, for various electron energies $\mathcal{E} = 600, 800, 1000, 1100$ and 1200 eV as reported in ref. 16. The plate current response clearly shows the presence of current undulations – a sequence of equally spaced maxima/minima, as against the expectations of the Lorentz dynamics. The presence of these undulations signals the detection of a curl-free vector potential. This would come as a complete surprise to anyone who firmly believes in the inviolability of the Lorentz dynamics in its own macro-scale domain. However, these results turn out to be in accordance with the predictions of the formalism developed in ref. 8. A detailed analysis of the expectations according to the above predictions has been presented in ref. 10, where various characteristic features of the expected detection have been identified, and an algorithm for the experimentation has been worked out. This analysis then enables a quantitative check to be carried out through appropriate experimentation.

Such a detailed quantitative demonstration of the vector potential observation has been recently reported¹⁶. Without going into the details of the analysis reported in ref. 10, I provide below a relation which gives the condi-

tion for the maxima of the plate current undulation in terms of the change ΔI of the current I in the toroidal solenoid in terms of the formalism of Varma⁸, as worked out in ref. 10.

$$\Delta I = \frac{c}{2e} \frac{mvL_p}{\Gamma G} \frac{B_p}{B_0} \sin \delta \tan \delta, \quad \Delta \ell = 1, 2, 3, \dots \quad (15)$$

This expression shows that the interpeak separation of the maxima varies directly as the speed v of the electrons, which translates to $v \sim \mathcal{E}^{1/2}$. Figure 5a shows the variation of the interpeak separation $\Delta^{(1)}I$ with $\mathcal{E}^{1/2}$ as determined from the curves in Figure 4 corresponding to the various energies given above. It clearly exhibits a linear dependence on $\mathcal{E}^{1/2}$. This linear dependence is in accordance with eq. (15). In this expression, B_0 is the magnetic field at the point of injection of the electron beam, and B_p is the magnetic field which satisfies eq. (14) for a given energy. The ratio B_p/B_0 is thus independent of energy.

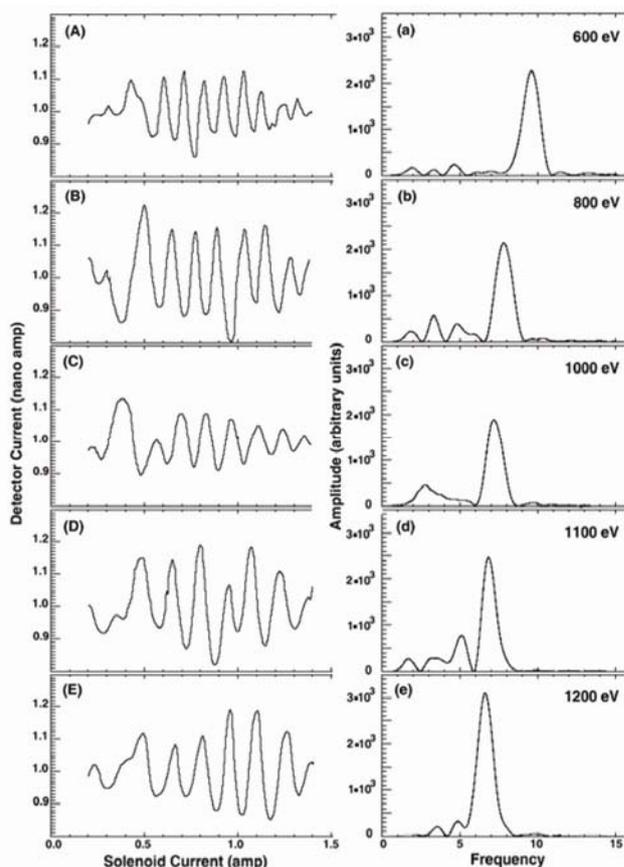


Figure 4. Detector plate current against the current in the toroidal solenoid: The plots (A)–(E) (left panel) denote detector current in nA against the solenoidal current in amperes (A), for the electron energies \mathcal{E} (eV) = 600, 800, 1000, 1100 and 1200 respectively. These have been corrected for the saturating core which leads to dilation in the interpeak separation in the original plots with increasing solenoidal current. The plots (a)–(e) (right panel) denote the Fourier plots corresponding to (A)–(E).

The other important dependence is with respect to the geometrical factor G , which involves the position of the grid in the various runs of the experiment. As mentioned earlier, the grid plays a crucial role as the generator of the modulational wave. The path difference for the modulational waves for the interference effects at the plate is determined by the position of the grid as one of its generators, and it is given by the gun–grid distance $L_g = (x_g - x_0)$. As the grid is moved towards the gun, the position of the source of the modulational wave, namely the grid, moves towards the gun, this path difference decreases, and with it the distance, the line integral over which contributes to the phase shift. This phase shift is then responsible for the ‘fringe shift’. The quantity G contains this information.

The continuous curves in Figure 5 *b* depict the plot of R – the ratio of the interpeak separation $\Delta_D I$ for a given D and its value $\Delta_{11} I$ for $D = 11$ cm – as a function of the

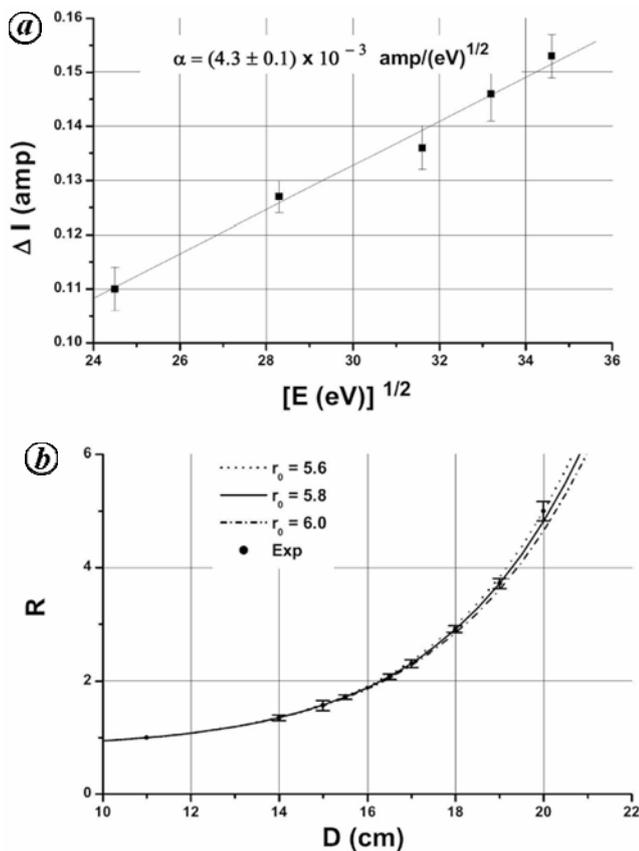


Figure 5. *a*, Interpeak solenoidal current interval ΔI against the different electron energies. The interpeak separations ΔI depicted here is expected to be equally spaced with respect to the toroidal current. The error bars in the plots are obtained using several different runs for a given electron energy with external magnetic field values differing by 0.1 G on either side of the best value. The figure gives the plot of ΔI against the square root of electron energy $\mathcal{E}^{1/2}$, confirming the theoretically expected linear dependence. *b*, The ratio $R = \Delta_D I / \Delta_{11} I$ for the various distances D (cm) for the same energy $\mathcal{E} = 830$ eV. The various curves (continuous, dotted, dashed-dotted) are the theoretical ones, with the ‘dots’ as experimental points.

gun–plate distance $D = L_p - L_g$, as calculated using the expression for G as given in Varma¹⁰. As is seen from the figure, the ratio $R = \Delta_D I / \Delta_{11} I$ increases rather sharply with the increase of L_g . This is basically because the factor G decreases with L_g . It should be noted that the ratio R is essentially independent of all parameters in eq. (15), except the factor G . The three curves correspond to three different values of the ‘effective’ radii of the core: $r_0 = 5.6, 5.8$ and 6.0 cm. The ‘dots’ on the plot with appropriate error bars give the experimentally determined points. The agreement of the experimental points with the calculated curve for $r_0 = 5.6$ cm appears to be remarkable.

The plots in Figure 5, which together represent a quantitative agreement with the predicted characteristics of the vector potential observation, thus provide conclusive evidence for the validity of the formalism of Varma⁸, which had predicted the observation of a curl-free vector potential. However, one crucial aspect of the formalism, relating to the generation of the modulational wave as a result of scattering was addressed only later⁹ in a more general context, and more specifically with respect to the charged particle dynamics¹¹. Figure 5 *b* presents a strong support for this mechanism of generation.

The results presented above on the observation of (curl-free) vector potential would appear to overturn some of the closely held beliefs, namely that (i) the curl-free vector potential observation can only be a micro-scale \hbar related quantum effect, and (ii) that it is of a topological origin, in that it requires the two interfering paths to go around the spatial domain which encloses the flux. Consequently, such an observation requires a minimum of two spatial dimensions to be able to provide an appropriate multiply connected space. The confrontation of these results with the canonical view is presented here with respect to these two beliefs. Both beliefs which were based on the micro-scale Aharonov–Bohm effect, however, stand overturned by the results presented here, which pertain to the modulational matter wave. With respect to the first point, our results have clearly demonstrated the observation on the macro-scale without the involvement of any \hbar . Moreover, these results lie beyond the purview of the Lorentz dynamics.

The second point is a little more subtle. Our detection of the vector potential has been effected essentially in the one-dimensional space along the magnetic field line. This has a trivial topology, and would not permit a detection in the manner of Aharonov–Bohm, as there are no two paths which can enclose the flux topologically since the toroid containing the flux (the source of the vector potential) exists outside the glass chamber – about 10 cm away from the axis. All our paths are one-dimensional and ‘open’. The crucial role in this detection is played by the very mechanism of the generation of the modulational wave to which this detection is attributed. The essential point in the detection is the generation of a path difference between the two modulational waves generated at

two different points along the field line, and the corresponding vector potential-induced phase shift which is eventually responsible for the detection. Readers can refer to the original report¹⁶ to see how this comes about.

Epilogue

The most striking fact about the new macro-scale phenomena discovered, as described above, is that while they have been shown to have a quantum origin, none of them involves \hbar explicitly as a signature for their quantum nature. In fact, they do not possess a $\hbar \rightarrow 0$ classical limit. For example, the expression for the macro-matter wavelength $\lambda_M = 2\pi v/\Omega$, characterizing the interference phenomena is \hbar -independent, and survives through this limit. Most quantum effects are known either to disappear in the $\hbar \rightarrow 0$ limit, going over into their classical limits, or lead to divergences, such as the expression for Stefan's constant as determined from the Planck radiation law, mirroring the ultraviolet catastrophe. These phenomena as quantum phenomena are thus rather unique and may be referred to as non-Planckian quantum phenomena.

Recent reports¹¹, reveal the rather subtle manner in which quantum dynamics 'sneaks' into the macro-scale, by involving the property of quantum entanglement leading to the quantum modulation of the parallel dynamics, and demonstrating thereby how the above-mentioned \hbar -independent macro-scale matter wave for the modulation arises.

The manner of the discovery of these phenomena has also been rather unusual. The entire formalism had its origin in the problem of the residence times determination in a magnetic trap, which was ostensibly a problem pertaining to classical dynamics by virtue of its being on the macro-scale. But its solution was approached through an intuitive guess, rather than using the standard approach of classical dynamical methods with the governing Lorentz equation. Being essentially a strongly nonlinear problem because of the inhomogeneity of the magnetic field, it appeared to be a rather formidable task to extract from the Lorentz dynamics the nonanalytic, nonexpandable part which would describe the nonadiabatic leakage from the adiabatic trap. The intuitive-heuristic approach, which was formalized later, not only provided the right solution to the problem, but led to an entirely different and an unexpected new territory of macro-scale modulatory matter waves which could not have been foreseen a priori. A whole new set of phenomena on the macro-scale were thus uncovered which are certainly not classical, but they are also not quantum either in the conventional sense. They are now recognized to be attributed to the quantum modulation, rather than to the 'particle' itself. All such phenomena have thus caused 'surprises', because they constitute radical departures from the prevalent conceptual framework.

The surprises include: (i) the observation of the predicted existence of the unexpected quantized residence times in the adiabatic trap; (ii) the observation of macro-scale matter wave interference effects, again not expected in the canonical framework, and finally, the most surprising of them all, (iii) the observation of a curl-free vector potential on the macroscale, and in one dimension.

However, all these apparent heterodoxical departures need not cause any disquiet in terms of the validity of the various existing formalisms in the given situation, if one were to realize that these new effects are attributed to quantum modulation of the de Broglie wave. The disquiet comes only if one were to be in the 'either-or' exclusivity mode of thought. Thus classical Lorentz equation, as already argued¹⁰, remains essentially unscathed through these modulations; the particle retains its Lorentz trajectory even as the modulation is on. On the other hand, it cannot describe the matter wave effects, that are attributed to the modulation. We here have a situation similar to the 'wave-particle duality'. After a scattering episode of the electron with a fixed centre, it is still a particle described by the Lorentz trajectory, but it also carries a matter wave with it because of the quantum modulation generated in it. All the new effects are now to be understood in terms of this modulation. A simple physical picture of the generation of modulation has been presented in ref. 34.

I conclude this discussion with a final comment which may further justify the title of this article – 'From hunches to surprises'. Perhaps, the most interesting and surprising general outcome of the studies reported here is the revelation of the existence of a new class of phenomena which though on the macro-scale do not belong to classical dynamics, but have their origin in quantum dynamics and yet do not involve \hbar explicitly which characterizes the quantum phenomena. In other words, a phenomenon existing on the macro-scale need not be automatically categorized as classical, and will need to be properly scrutinized. It is clear that this revelation came through the evolution and development of the 'hunch', which led to this entire gamut of investigations.

It may be noticed that the process of generation of the quantum modulation is not specific to the particular problem investigated, but is indeed quite generic, as it essentially involves transition across bound quantum levels in consequence of scattering. Any free degree of freedom associated with this bound system could then be quantum modulated. The generation of macro-scale matter wave associated with quantum modulation in consequence of such a scattering for the system of atoms and molecules, has in fact been pointed out by Varma³⁵. The current investigation on charged particle dynamics, which has involved experimental verifications of the various predicted effects, has led to the establishment of the importance of this quantum entity. It would be interesting to carry out similar experiments with atoms and molecules.

1. Tandon, R., Harish-Chandra. *Resonance*, 1996, **1**, 1–4.
2. Aharonov, Y. and Bohm, D., Significance of electromagnetic potentials in quantum theory. *Phys. Rev.*, 1959, **115**, 485–491.
3. Chambers, R. G., Shift of electron interference pattern by enclosed flux. *Phys. Rev. Lett.*, 1960, **5**, 3–5.
4. Tonomura, A. *et al.*, Evidence for Aharonov–Bohm effect with magnetic field completely shielded from electron waves. *Phys. Rev. Lett.*, 1986, **56**, 792–795.
5. Varma, R. K., Wave-mechanical model for the nonadiabatic loss of particles from magnetic mirror traps. *Phys. Rev. Lett.*, 1971, **26**, 417–420.
6. Varma, R. K. and Horton Jr, C. W., Schrödinger-like equations for nonadiabatic escape of charged particles. *Phys. Fluids*, 1972, **15**, 1469–1473.
7. Varma, R. K., Quantum like theory for the nonadiabatic behavior of charged particles in inhomogeneous magnetic fields. *Phys. Rev. A*, 1985, **31**, 3951–3959.
8. Varma, R. K., Probability amplitude description of the dynamics of charged particles in a magnetic field in the macrodomain. *Phys. Rev. E*, 2001, **64**, 036608 (1–10); Erratum, *Phys. Rev. E*, 2002, **65**, 019904.
9. Varma, R. K., Quantum manifestation of systems on the macro-scale – the concept of transition state and transition amplitude wave. *Pramana – J. Phys.*, 2007, **69**, 901–911.
10. Varma, R. K., Observability of the effects of curl-free magnetic vector potential on the macroscale and the nature of the transition amplitude wave. *Pramana – J. Phys.*, 2010, **74**, 491–511.
11. Varma, R. K., Macro-scale matter wave generation in charged particle dynamics in a magnetic field, a consequence of quantum entanglement. *Eur. Phys. J. D*, 2012, **66**, 39 (1–12).
12. Varma, R. K. and Punithavelu, A. M., Behavior of charged particle motion along a magnetic field and a retarding potential-evidence for the existence of discrete energy states in classical mechanical domain. *Mod. Phys. Lett. A*, 1993, **8**, 167–176.
13. Varma, R. K. and Punithavelu, A. M., Behavior of charged particle motion in a magnetic field and a retarding potential II – observations of the existence of ‘forbidden states’ and Aharonov–Bohm analog in classical mechanical domain. *Mod. Phys. Lett.*, 1993, **8**, 3823–3834.
14. Varma, R. K., Punithavelu, A. M. and Banerjee, S. B., Observation of matter wave beat phenomena in the macrodomain for electrons moving along a magnetic field. *Phys. Rev. E*, 2002, **65**, 026503 (1–9).
15. Varma, R. K. and Banerjee, S. B., Observation of matter wave interference on the macro-scale for charged particles in a magnetic field and the manifestation of Landau levels. *Phys. Scr.*, 2007, **75**, 19–28.
16. Varma, R. K., Banerjee, S. B. and Ambastha, A., Macro-scale observation of curl-free vector potential: A manifestation of quantum modulation of the de Broglie wave. *Eur. Phys. J. D*, 2012, **66**, 38 (1–11).
17. Bora, D., John, P. I., Saxena, Y. C. and Varma, R. K., Experimental results on multiple life times for nonadiabatic leakage of charged particles from magnetic mirror traps. *Phys. Lett. A*, 1979, **75**, 60–62.
18. Bora, D., John, P. I., Saxena, Y. C. and Varma, R. K., Multiple life times in the nonadiabatic leakage of particles from a magnetic mirror trap. *Plasma Phys.*, 1980, **22**, 652–662.
19. Bora, D., John, P. I., Saxena, Y. C. and Varma, R. K., Behavior of charged particles confined in a magnetic mirror. *Phys. Fluids*, 1982, **25**, 2284–2288.
20. Northrop, T. G., *Adiabatic Motion of Charged Particles*, Interscience Publishers, New York, 1963.
21. Chirikov, B. V., *Fiz. Plazmy*, 1978, **4**, 521–541 [Translation: Stability of the motion of a charged particle in a magnetic confinement system. *Sov. J. Plasma Phys.*, 1978, **4**, 289–300]; A universal instability of many dimensional oscillator systems. *Phys. Rep.*, 1979, **52**, 264–379; Particle dynamics in magnetic traps. In *Problems in Plasma Theory* (ed. Kadomtsev, B. B.), Consultants Bureau, 1987, vol. 13, pp. 3–73.
22. Varma, R. K., Classical and macroquantum dynamics of charged particles in a magnetic field physics. *Phys. Rep.*, 2003, **378**, 301–434.
23. Dubinina, A. N., Krasitskaya, L. S. and Yudin, Yu N., Motion of charged particles in a magnetic trap with a mirror configuration. *Plasma Phys.*, 1969, **11**, 551–564.
24. Hastie, R. J., Hobbs, G. E. and Taylor, J. B., Non-adiabatic behaviour of particles in inhomogeneous magnetic fields. In Proceedings of the 3rd International Conference on Plasma Physics and Controlled Nuclear Fusion Research, International Atomic Energy Agency, Vienna, 1969, vol. 1, pp. 389–402.
25. Dirac, P. A. M., The Hamiltonian form of field dynamics. *Can. J. Math.*, 1951, **3**, 1–23.
26. Synge, J. L., Classical dynamics. In *Handbuch der Physik: Classical Mechanics and Field Theory*, Springer-Verlag, Berlin, 1960, vol. III/1, pp. 1–225.
27. Varma, R. K., A variety of plasmas. In Proceedings of the International Conference on Plasma Physics, Delhi, India (eds Sen, A. and Kaw, P. K.), Indian Academy of Sciences, Bangalore, 1989, pp. 235–255.
28. Unnikrishnan, C. S., Do quantum-like theories imply non-classical observable effects? *Curr. Sci.*, 1999, **76**, 413–422; Unnikrishnan, C. S. and Safvan, C. P., Experimental test of a quantum-like theory – Motion of electrons in a uniform magnetic field in a variable potential well. *Mod. Phys. Lett.*, 1999, **14**, 479–490.
29. Ito, A. and Yoshida, Z., Resonant production of secondary electrons generating a discrete structure in a magnetized electron beam system. *Phys. Rev. E*, 2001, **63**, 026503 (1–5).
30. Varma, R. K., Topological manifestations in classical mechanics: discrete allowed and forbidden states of motion. *Mod. Phys. Lett. A*, 1994, **9**, 3653–3661.
31. Unnikrishnan, C. S., Comment on ‘Observation of matter wave beat phenomena in the macrodomain for electrons moving along a magnetic field’. *Phys. Rev. E*, 2004, **70**, 028501 (1–5).
32. Varma, R. K., Punithavelu, A. M. and Banerjee, S. B., Reply to ‘Observation of matter wave beat phenomena in the macrodomain for electrons moving along a magnetic field’. *Phys. Rev. E*, 2004, **70**, 028502 (1–7).
33. Varma, R. K., Punithavelu, A. M. and Banerjee, S. B., Observation of the magnetic vector potential in the classical macrodomain. *Phys. Lett. A*, 2002, **303**, 114–120.
34. Varma, R. K., Macroscopic and mesoscopic matter waves. *Eur. Phys. J. D*, 2002, **20**, 211–218.
35. Varma, R. K., Some surprising manifestations of charged particle dynamics in a magnetic field. *J. Plasma Phys.*, 2010, **76**, 355–367.

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