Analysis of travel-time variation over multiple sections of Hanshin expressway in Japan

Chalumuri Ravi Sekhar*, Takamasa Iryo and Yasuo Asakura

Travel-time variation frequently occurs in urban arterial road networks as a result of various uncertainties in the transportation system. Sources of uncertainty can be an element of demand-side factors, supply-side factors and other external factors of the road network system. Study of travel-time variation is useful for measuring travel-time reliability studies. The objective of this study was to analyse the factors influencing travel-time variation among multiple sections of the Kobe route of the Hanshin Expressway in Japan. In this study, the Seemingly Unrelated Regression Equation (SURE) model was adopted to analyse the travel-time variation due to traffic volume, traffic accidents and rainfall as factors from the supply side, demand side and external factors of the transportation system respectively. This study identified that there was contemporaneous error correlation among multiple sections of the Kobe route. This error causes significant difference in estimated model parameters between the SURE model and Ordinary Least Square (OLS) model. Particularly, it has been observed that SURE model coefficients obtained for the rainfall parameter were 42% lower than OLS model coefficients. The results of this study emphasize that the SURE model is good for analysing the influence of various factors on travel-time variation among multiple sections of the Hanshin Expressway.

Keywords: Multiple sections, SURE model, transportation system, urban expressway.
factors such as road closure due to accidents and external factors such as adverse weather effects and natural disasters. Li et al. examined travel-time variability under the influence of time of the day, day of the week, weather effect and traffic accident using the MLR model. In another study, the Florida Department of Transportation developed empirical travel-time variability models as a function of frequency of incidents, work zones and weather conditions. For this, it had considered regression analysis for different scenarios of uncertainty sources and weather conditions. For this, it had considered regression function of frequency of incidents, work zones and developed empirical travel-time variability models as a function of travel time. All these models are based on separate estimate equations. The models are unable to estimate the correlation across various time intervals, known as contemporaneous error correlation. To identify this error, the SURE model has been adopted in this study. The approach of the SURE model for travel-time variation is discussed in the following section.

Methodology for modelling travel-time variation

This section describes the econometric model structure for travel-time variation analysis among multiple sections. Figure 1 shows the typical schematic representation of a freeway into various sections. The main route is classified into various sections based on on-ramp and off-ramp criteria. Travel time mainly varies due to demand-side, supply-side and external factors from the system of transportation. In this study a linear relationship between the travel time and the factors affecting the travel-time variation has been assumed. The mathematical relationship between the factors affecting the travel time is explained in eq. (1).

\[ y_{it} = \beta_0 + \beta_1 X_{1it} + \beta_2 X_{2it} + \beta_3 X_{3it} + \epsilon_{it}, \]  

where \( y_{it} \) is the travel time at section \( i \) for time interval \( t \), \( X_{1it} \) the demand-side factor at section \( i \) for time interval \( t \), \( X_{2it} \) the supply-side factor at section \( i \) for time interval \( t \), \( X_{3it} \) the external factor at section \( i \) for time interval \( t \) and \( \epsilon_{it} \) the covariance error of the joint disturbance.

Demand-side factor such as traffic volume (no. of vehicles per hour), supply-side factor such as number of accidents which occur within the section and external factors such as rainfall intensity were considered in this study. The coefficients in eq. (1) \( (\beta_0, \beta_1, \beta_2, \beta_3) \) can be estimated in two ways – by considering sections either as independent or dependent. If a section is considered as independent, this means that the error variation across the equation is assumed to be zero. On the other hand, if the section is considered as dependent, the error covariance \( (\text{cov}) \) across the equation is not equal to zero; this is expressed in eq. (2) below.

\[ \text{cov}(\epsilon_i, \epsilon_j) = 0 \text{ (} i \neq j \text{)} \text{ if the section is independent,} \]

\[ \text{cov}(\epsilon_i, \epsilon_j) \neq 0 \text{ for all } i, j \text{ if the section is dependent.} \]

The model specification described in eq. (1) can be estimated in each section independently, but in reality there could be several unobserved characteristics of the uncertainties among various sections that will affect the travel-time variation. Therefore, the error terms can be correlated across sections. Estimating equations separately ignores this correlation. Furthermore, the model estimation would not be as efficient as it could be. This problem can be addressed by estimating the regression equations jointly as a set of seemingly unrelated regression equations. For the case where the number of sections is 3, there exist three systems of equation corresponding to each section, called the system of seemingly unrelated regression equations. This is explained as a stacked model and presented in eq. (3)

\[
\begin{bmatrix}
Y_1 \\
Y_2 \\
Y_3
\end{bmatrix} =
\begin{bmatrix}
X_{11} & 0 & 0 \\
0 & X_{22} & 0 \\
0 & 0 & X_{33}
\end{bmatrix}
\begin{bmatrix}
\beta_1 \\
\beta_2 \\
\beta_3
\end{bmatrix}
+ 
\begin{bmatrix}
\epsilon_1 \\
\epsilon_2 \\
\epsilon_3
\end{bmatrix},
\]

where \( Y_1 \) is the travel time at section 1 for various time intervals, \( Y_2 \) the travel time at section 2 for various time intervals, \( Y_3 \) the travel time at section 3 for various time intervals, \( X_1 \) the factors which influence travel time at section 1 for various time intervals, \( X_2 \) the factors which influence travel time at section 2 for various time intervals and \( X_3 \) the factors which influence travel time at section 3 for various time intervals.

\[
Y_i =
\begin{bmatrix}
y_{i1} \\
\vdots \\
y_{iT}
\end{bmatrix}, \quad Y_2 =
\begin{bmatrix}
y_{21} \\
\vdots \\
y_{2T}
\end{bmatrix}, \quad Y_3 =
\begin{bmatrix}
y_{31} \\
\vdots \\
y_{3T}
\end{bmatrix},
\]

where \( y_{iT} \) is the travel time at section 1 for time interval \( t \), \( y_{2T} \) the travel time at section 2 for time interval \( t \) and \( y_{3T} \) the travel time at section 3 for time interval \( t \).
where $x_{1T}$ is the traffic volume at section 1 for time interval $t$, $x_{2T}$ the number of traffic accidents at section 1 for time interval $t$, $x_{3T}$ the intensity of rainfall at section 1 for time interval $t$, $x_{1T}$ the traffic volume at section 2 for time interval $t$, $x_{2T}$ the number of traffic accidents at section 2 for time interval $t$, $x_{3T}$ the intensity of rainfall at section 2 for time interval $t$, $x_{1T}$ the traffic volume at section 3 for time interval $t$, $x_{2T}$ the number of traffic accidents at section 3 for time interval $t$, and $x_{3T}$ is the intensity of rainfall at section 3 for time interval $t$.

Using the obvious notation, we can rewrite eq. (3) as

$$Y = X^\beta + \varepsilon. \tag{4}$$

The coefficients from all the three model equations can be estimated using the Generalized Least Square (GLS) estimator. This has the best linear unbiased estimator for $\beta$ and has lower variance than the least square estimator because it takes into account the contemporaneous correlation between the disturbances in different equations.

$$\beta = (X'W^{-1}X)^{-1}X'W^{-1}Y,$$

$$=[X'(\Sigma^{-1} \odot I_M)X]^{-1}X (\Sigma^{-1} \odot I_M)Y, \tag{5}$$

where

$$W^{-1} = \begin{bmatrix}
\sigma_{11} & \sigma_{12} & \sigma_{13} \\
\sigma_{21} & \sigma_{22} & \sigma_{23} \\
\sigma_{31} & \sigma_{32} & \sigma_{33}
\end{bmatrix}^{-1} = \Sigma^{-1} \odot I_M, \tag{6}$$

and

$$I_M = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}.$$
hourly rainfall data in Nishinomiya for January and February, the Kobe rainfall data were considered for travel-time variation analysis.

**Travel-time estimation for the study area**

From the collected vehicle detector data, spot speed was estimated for every 500 m interval and corresponding travel times for the same sections were calibrated by transforming the spot-speed data. Furthermore, path travel time for the three sections of study area was estimated using the time slice method, which considers the variation of speed over time by constructing the vehicle trajectory. Travel time obtained from this method is sufficiently close to the actual travel time as discussed below.

Conventionally the travel time of an entire route is calculated simply by accumulating the travel times of each section at a given time. It is expressed as

$$T(s) = \sum_{i=1}^{N} t_i(s),$$

where $t_i(s)$ denotes the travel-time of section $i$ at a given time $s$. Small sections (500 m interval) in a route are numbered sequentially in the downstream direction. This method generates an instantaneous travel time based on the assumption that vehicles instantaneously traverse the route. When traffic condition is stable and travel speed is constant, the travel time can be calculated correctly using this method. However, the estimated travel time may not be correct when traffic flows are not stable. The alternative method of calculating the route travel time is the time slice method, with which the travel times of each section are accumulated successively with the delay of the section travel time. The route travel time is represented as:

$$T(s) = \sum_{i=1}^{N} t_i(s + \tau_i(s)),$$

where $\tau_i(s)$ denotes the travel time from section 1 to section $i - 1$ and can be written as

$$\tau_i(s) = \sum_{i=1}^{i-1} t_i(s + \tau_i(s)).$$

Travel-time estimation by time-slice method was explained in Figure 4.

Yoshimura and Suga compared two sets of travel time estimated by the instantaneous method and the time slice method using Automatic Vehicle Identification (AVI) data as true values. They found that the instantaneous method gave rise to large errors at both increased and decreased hours of traffic congestion and that the time slice method could follow actual travel time fluctuation without delay. Thus the time slice method is more suitable for offline application rather than on-line application when speed varies over time and also provides better results over the instantaneous method. This path travel time is considered as a dependent variable for travel time variation analysis.

**Travel-time variations across different sections:** Travel time statistical parameters such as mean, median and standard deviation for all the sections were estimated. Travel-time distribution was also plotted for the three sections and presented in Figure 5. The probability and cumulative distribution is a visual tool representation of travel-time variability over the period. Mean travel time of section 1 was 605 s lower than sections 2 and 3, which had values of 718 s and 760 s respectively. The standard deviation of travel time for the section 1 was 310 s higher than that of the section 2 was 346 s. The 95th percentile of travel time in the case of section 1 was 1296, this value was lower than section 3.

**Regression analysis for travel-time variation**

MLR analysis was carried out to understand the influence of all the incidents on travel-time variation. The estimated MLR model coefficients for the entire year’s data...
of all the routes are presented in Table 1. The sign of estimated coefficients of all the variables was positive. This indicates that all the incidents showed a positive contribution to travel-time variation, which is logical since travel time increases with increase in the occurrence of incidents. Based on *t*-statistics values it can be concluded that except road work all incidents contributed significantly to travel-time variation in section 1. Whereas in section 2 other incidents and sections road works *t*-statistics values were less than 1.95. This indicates that these parameters are insignificant with respect to travel-time variation. The estimated $R^2$ values were very low for all the three routes, which indicates that the uncertainty explained (11%) by these values is very low.

Further, in this study travel-time variation was studied by considering one variable each from the demand side, supply side and external effects. The volume of traffic entering from the beginning of each section for a period of 1 h was considered as a variable from the demand-side factor. The number of accidents occurring in that hour within the section was considered as a supply-side factor and the amount of rainfall measured within the hour for the corresponding section was considered as the external factor. The influence of the number of traffic accidents on travel time was considered as a supply-side factor. Due to traffic accidents some portion of the road section had been blocked; this had an impact on the capacity of the section and further influenced the travel-time variation. Therefore, in this study we have assumed the number of accidents as a supply-side factor in the travel-time variation model.

**SURE analysis for travel-time variation**

The travel-time variation model was implemented for working-day data. On working days, in 3243 out of a total of 5976 observations, travel time mainly varied due to traffic volume, traffic accident and rainfall. MLR analysis was carried out for each section individually and the residual error obtained by this model was used for estimating the error covariance matrix (Table 2). From Table 2, it can be observed that off-diagonal elements of the error covariance matrix are non-zero; this emphasizes that the error residuals for all the three sections are dependent on one another. Therefore, there is the possibility of gain (positive or negative) in model coefficients obtained by the MLR model. Using this error covariance
matrix, SURE model coefficients were estimated. Table 3 presents the model coefficients, standard error (SE) and t-statistics value of both MLR and SURE models for all three sections of the study area.

The SE obtained by the SURE model for the three sections was lower than the MLR model. Particularly, SE was significantly lower than the constant parameter, traffic accident and rainfall parameter. In the case of the traffic volume parameter, SE was nominal. Figure 6 is a graphical representation of SE for the traffic accidents and rainfall parameters obtained by using the MLR and SURE models. Furthermore, the reduction of SE percentage as the ratio of the difference between SE obtained by both MLR and SURE models to that obtained by MLR model was estimated. The reduction of SE percentage was more significant for the traffic accident parameter is greater by 16% and 10% lesser. In case of rainfall parameter, it was greater by 9% and lesser by 6%. From SE consideration, it can be concluded that the model coefficients obtained by the SURE model are more appropriate than the MLR model.

The model coefficients obtained by the SURE model were lower than those obtained with the MLR model. Particularly, significant difference was observed in traffic accident and rainfall parameters (Figure 7). The SURE model coefficients obtained for the rainfall parameter were 42% lower than the MLR model coefficients. This is possibly due to the fact that the rainfall variables are highly correlated with the unobserved characteristics in the individual sections of the study area. Thus the results indicate that independent models overestimate the travel-time under the influence of error correlation among various sections.

The SE obtained by the SURE model for constant term was lower than that obtained with the MLR model for all sections. Particularly more significant was observed in section 3. The corresponding t-statistics values were also found to improve in all the three sections. This indicates that the MLR model underestimates the travel time during free-flow situations when there is no uncertainty from the supply side and external effect. The $R^2$ values were considered as goodness-of-fit measures for SURE and MLR models. The $R^2$ values will account for the variation in the travel-time variable as explained by the independent variable considered in the model. The $R^2$ value was estimated in using eq. (7).

$$R^2 = \left(1 - \frac{\text{SEE}}{\text{SST}}\right),$$

where \(\text{SEE}\) is the sum squared error \(= \sum_{i=1}^{n}(y_{it} - \hat{y}_{it})^2\), \(\text{SST}\) is the total sum squared error \(= \sum_{i=1}^{n}(y_{it} - \bar{y}_{it})^2\), where \(y_{it}\) is the observed travel time for section \(i\) of time interval \(t\), \(\hat{y}_{it}\) is the estimated travel time for section \(i\) of time interval \(t\) and \(\bar{y}_{it}\) is the average travel time for section \(i\) of time interval \(t\).

The $R^2$ values of the MLR and SURE models were almost the same for all three sections. The $R^2$ value was around 0.12 for sections 1 and 2; for section 3, the MLR and SURE model $R^2$ value was around 0.22. Travel-time variation analysis under various uncertainties is a complex phenomenon; therefore, both models have a small percentage of variation. This may be improved by increasing the uncertainty parameters from the supply-side and demand-side of the system.

**Conclusion**

In this study travel-time variation under the influence of traffic volume (uncertainty element from demand side),
traffic accidents (uncertainty element from supply side) and rain-fall (uncertainty element from external factor) was analysed. The archived supersonic vehicular detectors data of the Kobe route (route no. 3) of the Hanshin Expressway was considered for the analysis. Initially, section-level travel time was estimated. Then path-level travel time was estimated adopting the time slice method. This travel time was studied under the influence of uncertainties from traffic volume, traffic accidents and intensity of rainfall. Contemporaneous error correlation among various sections was analysed adopting the SURE model. The result of the SURE model was compared with the MLR model. The significant findings from this study are summarized as follows.

- If there is error correlation among various sections due to elements of uncertainty, the SURE model is more efficient than the MLR model.
- SE obtained using the SURE model is less than that with the MLR model for all the parameters in all the sections. The reduction of SE percentage was more significant in the case of the traffic accident parameter, which was greater by about 16% and lesser by 10%. This emphasizes that model coefficients obtained by this method are more appropriate than those with the MLR model.
- The coefficients estimated by the MLR model underestimate the travel time compared to the SURE model.
- Except for free-flow situations, the results observed by the independent models have overestimated the travel time under the influence of error correlation among various sections due to traffic volume as the demand-side factor, traffic accidents as the supply-side factor and rainfall as an external factor on the transport system.

### Table 3. Model coefficients estimated by MLR and SURE models

<table>
<thead>
<tr>
<th>Variable</th>
<th>MLR model</th>
<th>SURE model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimated coefficient</td>
<td>Standard error</td>
</tr>
<tr>
<td><strong>Section 1: Tsukiyama–Ikutagawa</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>503.384</td>
<td>9.945</td>
</tr>
<tr>
<td>Traffic volume (no. vehicles/hr)</td>
<td>0.066</td>
<td>0.006</td>
</tr>
<tr>
<td>Rainfall (mm/h)</td>
<td>15.694</td>
<td>3.646</td>
</tr>
<tr>
<td><strong>Section 2: Ikutagawa–Nishinomiya</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Traffic volume (no. vehicles/hr)</td>
<td>0.036</td>
<td>0.003</td>
</tr>
<tr>
<td>Rainfall (mm/h)</td>
<td>14.091</td>
<td>2.412</td>
</tr>
<tr>
<td><strong>Section 3: Nishinomiya IC–Awaza</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Traffic volume (no. vehicles/hr)</td>
<td>0.283</td>
<td>0.010</td>
</tr>
<tr>
<td>Traffic accidents (no.)</td>
<td>254.820</td>
<td>27.962</td>
</tr>
<tr>
<td>Rainfall (mm/h)</td>
<td>23.198</td>
<td>5.784</td>
</tr>
</tbody>
</table>


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