A SEMIPHENOMENOLOGICAL THEORY OF SUPERFLUID LIQUID ^{3}He

R. E. AMRITKAR AND N. KUMAR
Department of Physics, Indian Institute of Science, Bangalore-560012

ABSTRACT

It is shown that the recently observed two phase transitions, namely, A (Second order) and B (First order) in liquid ^{3}He below 3 mdeg K can be understood semi-quantitatively in terms of an effective spin Hamiltonian, incorporating an intra-pair and an inter-pair spin dependent interactions. We assume BCS type pairing with 1-odd ($S = 1$) in phase-A and 1-even ($S = 0$) in phase-B. It is shown that the experimental observations are in fair agreement with the theory.

RECENT experimental observations[1-3] show that liquid ^{3}He undergoes a second order phase transition at 2·65 mdeg K (Phase-A) and a first order transition at 2·0 mdeg K (Phase-B). Possibility of the new phases being of superfluid nature, with pair condensation in higher angular momentum states ($l > 0$) has been suggested[4,5]. However, no explicit model Hamiltonian describing the two phase transitions and the physical properties of the two phases, has so far been suggested.

In this paper, we propose a simple model Hamiltonian which seems to describe these phases fairly well, in terms of an intra-pair and an inter-pair spin-dependent interaction. We take cue from the fact that the transitions appear to involve the dynamics of the nuclear spins in an essential fashion, in the reciprocal space.

We write the effective spin Hamiltonian phenomenologically, as

$$H = \Delta \sum_{\vec{k}} \vec{s}_{\vec{k}} \cdot \vec{s}_{-\vec{k}} - \frac{D}{N} \sum_{\vec{k} \neq \vec{k}'} S^z_{\vec{k}} S^z_{\vec{k}'} - H_0 \sum_{\vec{k}} S^z_{\vec{k}} \qquad (1)$$

with the total spin $S_{\vec{k}} = s_{\vec{k}} + s_{-\vec{k}}$ of spins of the nuclei paired in the time reversed states $\vec{k}, -\vec{k}$. The summation is restricted to a phase space shell (of width δ) around the Fermi surface. Δ is the intra-pair excitation energy required to go from a spin singlet ($S_{\vec{k}} = 0$) to a spin triplet state ($S_{\vec{k}} = 1$), along with the *concomitant* orbital excitation to preserve overall antisymmetry. Note that the Hamiltonian contains the intra-pair spin transition and also the cooperative feature of the spin pairs [first and second terms of eq. (1) respectively].

The partition function for the Hamiltonian in eq. (1) can be evaluated exactly. We have,

$$Z = \mathrm{Tr}\{\exp(-\beta H)\} = 2\sqrt{N}\, g_1 \int_{-\infty}^{\infty} dx\, e^{Nf(x)} \qquad (2)$$

with,

$$f(x) = -\pi x^2 + \ln\left\{\frac{1}{2}\left(\frac{1}{g} e^{\beta\Delta} + 1\right) + \cosh(\beta H_0 + \sqrt{4\pi\beta D}\, x)\right\},$$

where $g = g_1/g_0$ is the ratio of the orbital degeneracies in the triplet (S=1, l odd) and the singlet (S=0, l even) states. In deriving eq. (2) we have used the Stratonovich identity,

$$e^{a^2} = \int_{-\infty}^{\infty} dx\, e^{\{-\pi x^2 + 2\sqrt{\pi}\, ax\}} \qquad (3)$$

to linearize the term $D\,(\sum_{\vec{k}} S^z_{\vec{k}})^2$ occurring in the exponent. The evaluation of the integral in eq. (2) is done by saddle point method. The analysis of the free energy in the limit of zero magnetic field and for $\eta \equiv \Delta/D > 1$ and $g > 1{\cdot}45$ shows two phase transitions; a second order phase transition at $T = T_{CA}$ and another phase transition at a lower temperature $T = T_{CB}$ which may be first or second order depending on the value of η. The two critical temperatures T_{CA} and T_{CB} are given by the roots of the equation,

$$\frac{1}{2}\left(\frac{1}{g} e^{\beta\Delta} + 3\right) = 2\beta D. \qquad (4)$$

For temperatures $T_{CB} < T < T_{CA}$, the system shows spontaneous magnetization $M_s(T)$ per pair, (in units of twice the nuclear Bohr magneton) given by the transcendental equation,

$$M_s(T) = \frac{\sinh(2\beta D M_s)}{\frac{1}{2}\left(\frac{1}{g} e^{\beta\Delta} + 1\right) + \cosh(2\beta D M_s)}. \qquad (5)$$

Following qualitative features can be easily noted. Phase-A is an ordered phase with pairs in the triplet state (S=1, l odd), while in phase-B the pairs are in singlet state (S=0, l even). For the choice of the parameters $g=3$, $\eta = 1{\cdot}45$ and $T_{CA}/T_{CB} \simeq 1{\cdot}3$, it is found that the spontaneous magnetization in phase-A increases monotonically with decreasing temperature and falls abruptly to zero at T_{CB}, the fall being independent of the field and of the same order of magnitude as has been observed[3]. Clearly A transition is second order and B is first order. Also the fall in the

magnetization decreases as we decrease the ratio T_{cA}/T_{cB} and near the triple point in the P-T phase diagram the B transition becomes of second order, again being consistent with the experimental findings[6]. The NMR frequency shift $\nu^2_{liq}(T, H) - \nu^2_{sol}(T, H)$ was found[1] to be proportional to $(1 - T/T_{cA})$ and also independent of H in the phase-A. For zero field case this would imply $\nu^2_{liq}(T, H = 0) \propto (1 - T/T_{cA})$ in the phase-A. The shift vanishes abruptly at B transition. It is found that the solution of eq. (5), for the above-mentioned values of the parameters. gives a linear behaviour of $M_s^2(T)$ in the phase-A (H=0) and an abrupt vanishing at B, confirming the experimental facts. The other experimental observations like the specific heat jumps at A transition and the constancy of the transverse magnetic susceptibility $[\chi_\perp(T)]$ in phase-A and then an abrupt fall in $\chi_\perp(T)$ at B[1] also follow from our theory. The specific heat jump is found to be proportional to D. Adding a transverse field term to the Hamiltonian and ignoring commutators involving the total spins, we get $\chi_\perp(T) \simeq 2/D$, a constant in the phase-A, and at the B transition a discontinuous drop given by,

$$\frac{\chi_\perp(T \to T^-_{cB})}{\chi_\perp(T \to T^+_{cB})} \simeq \frac{D}{k_B T_{cB}\left[\frac{1}{g}\exp(\Delta/k_B T_{cB}) + 3\right]} \simeq \cdot 25. \quad (6)$$

In conclusion, we have been able to explain the anomalous nature of the two phases in liquid ^{3}He below 3 mdeg K on the basis of an effective spin Hamiltonian with semiphenomenological parameters. Detailed calculations will be published elsewhere.

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CONCENTRATION OF JAPANESE ENCEPHALITIS (JBE) VIRAL ANTIGENS PREPARED FROM VERO CELL CULTURE BY SUCROSE DIALYSIS

C. N. VENKATESHAN, S. D. KELKAR, C. N. DANDAWATE AND S. N. GHOSH
Virus Research Centre, Poona-411001, India

ABSTRACT

Concentration by sucrose dialysis of HA and CF antigens of JBE virus prepared from Vero cell culture is described. This method is rapid, reliable, economical and does not require any special equipment or material. Antigens thus concentrated can be employed in serological tests.

INTRODUCTION

THE production of high titred arboviral antigens from the infected suckling mouse brains is a procedure routinely employed in most of the laboratories working on arboviruses. Clarke and Casal's[1] sucrose acetone (SA) extraction method is most suitable for the preparation of antigens for haemagglutination-inhibition (HI) as well as complement fixation (CF) tests. However, the procedure is rather laborious, potentially hazardous and requires voluminous acetone. The tissue culture system for the production of arboviral antigens is more convenient and less hazardous, but usually the titres of antigens are comparatively poor[2-6]. Several procedures are employed for concentrating viral antigens[4,6-20]. Concentration by sucrose dialysis of HI and CF antigens prepared from Vero cell culture is described in this communication. The Japanese encephalitis virus, a Togavirus, (Andrewes)[21] is employed in the study.

MATERIAL AND METHODS

Sucrose is commonly used for preparing density gradient. It is hygroscopic and highly soluble in water and has little effect on viruses. Infected tissue culture fluids (ITCF) harvested on different days were dialysed against solid sucrose or saturated sucrose solution and the concentrated antigens were tested for haemagglutination (HA), HI and CF tests to find out the potentialities of antigens for use in the routine diagnostic serological tests.

Growth and maintenance of Vero culture, methods for inoculation of virus JBE P3 (P 20778) and harvesting ITCF are described separately (Rai, J. *et al.*, in preparation).

Concentration of Antigens by Dialysis against Sucrose.—The antigen to be concentrated was placed in a dialysis bag. Both ends of the bag were tied and the bag was kept in a beaker or a measuring cylinder. Solid sucrose covered the dialysis bag all around and the container was kept at room