Protective measurements: probing single quantum systems

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Making measurements on single quantum systems is considered difficult, almost impossible if the state is a priori unknown. Protective measurements suggest a possiblity to measure single quantum systems and gain some new information in the process. Protective measurement is described here, both in the original and generalized form. The degree to which the system and the apparatus remain entangled in a protective measurement, is assessed. Possible exterimental tests of protective measurements are also discussed.

Keywords: Expectation value, protective measurements, quantum systems, wavefunction collapse.

Introduction

QUANTUM mechanics has been a tremendously successful theory for describing microscropics systems. Till date there has not been a single experiment which can demonstrate a violation of quantum theory. The success of quantum mechanics is so widespread and robust that people have come to believe that it is the ultimate theory for describing microscopic systems, and that classical mechanics should be an approximation of quantum mechanics in an appropriate limit. However, this transition from quantum to classical has been a sticky issue since the time of the very inception of quantum theory.

While the state of a classical particle is adequately described by the knowledge of its position and momentum, quantum mechanics does not even allow simultaneous well-defined values of these two quantities. The state of a quantum particle is strangely described by a complex entity called the wavefunction. Although the dynamics of state, or the wavefunction, is exactly described by the Schrödinger equation, the meaning of measurable quantities takes an altogether different meaning. An observable described by an operator **A** is believed to have a well-defined value only if the state of the system is an eigenstate of this operator, namely

$$\mathbf{A}|a_n\rangle = a_n|a_n\rangle, \tag{1}$$

where a_n is called the eigenvalue of the observable. Eigenvalue is also the outcome in a measurement of the observable **A**. Measurements of **A** on identical copies of same system, in the state (say) $|a_k\rangle$, will all lead to the same result a_k. Thus, an eigenvalue is a well-defined value of **A** if the system is in its eigenstate.

However, if the system is in a state (say) $|\psi\rangle$ which is not an eigenstate of **A**, the value which one should assign to the observable described by **A** is ambiguous. All one can do is to define an *expectation value* of **A** as

$$\langle A\rangle = \langle\psi|\mathbf{A}|\psi\rangle. \tag{2}$$

A measurement of **A** in the state $|\psi\rangle$ would still yield an eigenvalue, one from the set $\{a_i\}$. However, the important difference is that measurements of **A** on identical copies of same system, in the state $|\psi\rangle$, will all lead to different eigenvalues. The state $|\psi\rangle$ can be expanded in terms of the eigenstates of **A** as

$$|\psi\rangle = \sum_n c_n|a_n\rangle, \tag{3}$$

where c_n are some complex numbers. The expectation value of **A** can now be written as

$$\langle\mathbf{A}\rangle = \sum_n |c_n|^2\langle a_n|\mathbf{A}|a_n\rangle = \sum_n |c_n|^2 a_n. \tag{4}$$

The above expression is conventionally interpreted as an average of the measurement results on a large ensemble of identically prepared systems in the state $|\psi\rangle$. A fraction $|c_k|^2$ of the total systems yields the eigenvalue a_k.

The point to note in the above example is that while the value of **A** is well-defined for a single system in an eigenstate $|a_k\rangle$, the expectation value $\langle\mathbf{A}\rangle$ in a state $|\psi\rangle$ cannot be defined for a single system. It appears to have a meaning only for a large number of measurements on an ensemble of identical systems.

Issues like the one described above still plague quantum theory, although they are mainly interpretational issues. What sense one should make of the formalism of quantum theory is not clear.

Quantum measurement process

While evolution of quantum systems is well understood, what happens in a measurement process is not clear. This

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is simply because the apparatus we use is classical, and how information from a quantum system is carried over to the classical apparatus is not part of the quantum formalism. Quantum theory merely postulates that in a measurement process, the value obtained is an eigenvalue of the observable being measured; it results in the reduction of the original state to the corresponding eigenstate of the observable. How this process comes about is not understood, and remains an open problem.

John von Neumann[1] was the first to attempt putting a quantum measurement process on a mathematical footing. According to him, a quantum measurement can be broken up into two processes.

- Process 1 is a unitary process which establishes correlations between the state of the system and state of the apparatus. It basically correlates the various amplitudes of the state of the system to various possible outcomes of the apparatus. The apparatus too has to be treated as a quantum system. For example, if the initial state $|\psi\rangle$ is given by $|\psi\rangle = \sum_{i=1}^{n} c_i|a_i\rangle$ and the initial state of the apparatus is given by $|d_0\rangle$, then process 1 is a unitary operation

$$|d_0\rangle\sum_{i=1}^{n} c_i|a_i\rangle \xrightarrow[\text{Process 1}]{} \sum_{i=1}^{n} c_i|d_i\rangle|a_i\rangle. \tag{5}$$

  What process 1 has done is to correlate the eigenstates of **A** with distinct states of the apparatus. The states $|d_i\rangle$ could, for example, correspond to some discrete positions of a pointer needle.
- Process 2 is a non-unitary process which picks out a single outcome from the superposition described by eq. (5)

$$\sum_{i=1}^{n} c_i|d_i\rangle|a_i\rangle \xrightarrow[\text{Process 2}]{} |d_k\rangle|a_k\rangle. \tag{6}$$

  It is obvious that process 2 cannot be realized through the Schrödinger evolution. The process constitutes the heart of the so-called *measurement problem*.

The mechanism behind process 2 has confounded scientists since the beginning of quantum theory. It is no surprise that researchers have come up with suggested resolutions which can be considered radical to fantastic like the Everett many worlds interpretation[2] or the GRW proposal[3].

Strong and (almost) impulsive measurement

Let us first put the von Neumann process 1 on a rigorous mathematical footing. Process 1 can be constructed by a suitable interaction between the system and the apparatus and a time evolution. Conventional quantum measurements may be considered as the result of a strong interaction between the system and the apparatus, active for a short duration of time. The Hamiltonian of the system and the apparatus may be written as

$$\mathbf{H}(t) = \mathbf{H_S} + \mathbf{H_A} + g(t)\mathbf{Q_S}\mathbf{Q_A} \approx g(t)\mathbf{Q_S}\mathbf{Q_A}, \tag{7}$$

where $\mathbf{H_S}$, $\mathbf{H_A}$ represent the free Hamiltonians of the system and the apparatus respectively, and $\mathbf{Q_S}$, $\mathbf{Q_A}$ the operators of the system and the apparatus respectively, through which they interact. We introduce another observable $\mathbf{R_A}$ conjugate to $\mathbf{R_A}$, such that $[\mathbf{R_A}, \mathbf{Q_A}] = i\hbar$ and $\mathbf{R_A}|r\rangle = r|r\rangle$.

The apparatus is prepared in an initial state $|\phi(r_0)\rangle$ which is a packet of $|r\rangle$ states, centred at $r = r_0$. The initial state of the system $|\psi_s\rangle$ can be expanded in terms of the eigenstates of $\mathbf{Q_S}$, $|\psi_s\rangle = \sum_i c_i|s_i\rangle$, where $\mathbf{Q_S}|s_i\rangle = s_i\,|s_i\rangle$. Let us assume that the measurement interaction is switched on at $t = 0$ and continues till $t = T$, with the proviso $\int_0^T g(t)\mathrm{d}t = 1$. The state, after the measurement interaction, is given by

$$\begin{aligned}
|\Psi(T)\rangle &= \mathrm{e}^{(-\mathrm{i}/\hbar)\mathbf{Q_S}\mathbf{Q_A}}\,|\psi_s\rangle|\phi(r_0)\rangle \\
|\Psi(T)\rangle &= \sum_i \mathrm{e}^{(-\mathrm{i}/\hbar)\mathbf{Q_S}\mathbf{Q_A}}\, c_i|s_i\rangle|\phi(r_0)\rangle \\
|\Psi(T)\rangle &= \sum_i c_i|s_i\rangle|\phi(r_0 + s_i)\rangle \\
&= c_1|s_1\rangle|\phi(r_0 + s_1)\rangle + c_2|s_2\rangle|\phi(r_0 + s_2)\rangle \\
&\quad + c_3|s_3\rangle|\phi(r_0 + s_3)\rangle + \ldots .
\end{aligned} \tag{8}$$

The measurement interaction results in various eigenstates of $\mathbf{Q_S}$ becoming entangled with packets of $|r\rangle$ states of the apparatus, localized at different values of r. A narrow packet localized at $r_0 + s_k$, for example, would imply a measured eigenvalue s_k of the system. However, at this stage there is not one outcome, but a superposition of various outcomes with different probabilities.

Quantum measurement of single systems

The preceding discussion of quantum measurement has an interesting consequence for single systems, i.e. systems for which an ensemble of identical copies is not available. If the state is a priori unknown and an ensemble of identical copies is available, one can perform many measurements on different copies, and from the resulting probabilities of various outcomes $|c_i|^2$, try to infer the values of C_i and reconstruct the original state using $|\psi_s\rangle = \sum_i c_i|s_i\rangle$.

However, if there is only a single system available, one can choose to make one measurement, which will give one a single eigenvalue (say) s_k, and the state would have collapsed to $|s_k\rangle$. The measurements give absolutely no information regarding the original state $|\psi_s\rangle$. This means

that if the state of a single system is unknown, it will always remain unknown. This is something profound and implies that an unknown reality cannot be unvieled, even in principle.

The expectation value of any observable always has a well-defined value in any state. But the question that arises is whether the expectation value has any meaning for a single quantum system. Since the only interpretation of the expectation value traditionally understood is in terms of repeated measurements on an ensemble, the answer seems to be that the expectation value has no meaning for a single quantum system.

However, if one could somehow *measure* the expectation value of an observable in a single measurement, one could argue that it has a meaning. If one could measure the expectation value of an unknown state, it would imply that the expectation value has an objective reality, and would probably lend credence to the objective reality of the quantum state itself.

Protective measurements

About 22 years ago, Aharonov, Anandan and Vaidman (AAV) proposed a quantum measurement scheme involving very weak and adiabatic measurements, which they called 'protective' measurements[4–10], where they claimed the possibility of actually measuring the expectation value of any observable in a restricted class of states. The proposal initially raised surprise and scepticism among many[11–18].

While conventional quantum measurements are considered strong and impulsive, protective measurements make use of the opposite limit where coupling between the system and the apparatus is *weak* and *adiabatic*. For protective measurements to work, the system should be in a non-degenerate eigenstate of its Hamiltonian. The interaction should be so weak and adiabatic that one cannot neglect the free Hamiltonians. Let the Hamiltonian of the combined system be

$$\mathbf{H}(t) = \mathbf{H_A} + \mathbf{H_S} + g(t)\mathbf{Q_A}\mathbf{Q_S}, \tag{9}$$

where various entities have the same meaning as in the preceding section. The coupling $g(t)$ acts for a long time T and is switched on and off smoothly. The interaction is normalized as $\int_0^T \mathrm{d}t\, g(t) = 1$, and is assumed to be small and almost constant for the most part, justifying the approximation, $g(t) \approx 1/T$. If $|\Psi(0)\rangle$ is the state vector of the combined apparatus–system just before the measurement process begins, the state vector after T is given by

$$|\Psi(T)\rangle = \mathcal{T} \exp\left(-\frac{\mathrm{i}}{\hbar} \int_0^T H(\tau)\mathrm{d}\tau \right) |\Psi(0)\rangle, \tag{10}$$

where $\mathcal{T}$ is the time ordering operator. Since the time dependence of the Hamiltonian is trivial, we may divide the interval $[0, T]$ into N equal intervals ΔT, so that $\Delta T = T/N$. Since the full Hamiltonian commutes with itself at different times during $[0, T]$, one can write eq. (10) as

$$|\Psi(T)\rangle = \left[\exp\left[-\frac{\mathrm{i}\Delta T}{\hbar} \left(\mathbf{H_A} + \mathbf{H_S} + \frac{1}{T}\mathbf{Q_A}\mathbf{Q_S} \right) \right] \right]^N |\Psi(0)\rangle. \tag{11}$$

In order to solve the dynamics, one has to worry about whether different operators sitting in the exponential commute with each other or not. Since designing the apparatus is in the hands of the experimenter, we consider the case when $\mathbf{Q_A}$ commutes with the free Hamiltonian of the apparatus, i.e., $[\mathbf{Q_A}, \mathbf{H_A}] = 0$, so that we can have eigenstates $|a_i\rangle$ such that $\mathbf{Q_A}|a_i\rangle = a_i\,|\,a_i\rangle$ and $\mathbf{H_A}|a_i\rangle = E_i^a\,|\,a_i\rangle$. The operators of the system, $\mathbf{Q_S}$, $\mathbf{H_S}$, may or may not commute with each other; the energy eigenstates of the system are given by

$$H_S|\mu\rangle = \mu|\mu\rangle. \tag{12}$$

The states $|a_i\rangle$ are also the exact eigenstates of the instantaneous Hamiltonian $H(t)$, in the apparatus subspace. So, the exact instantaneous eigenstates can be written in a factorized form $|a_i\rangle|\overline{\mu}\rangle$, where $|\overline{\mu}\rangle$ are defined by

$$\left(\mathbf{H_S} + \frac{1}{T} a_i \mathbf{Q_S} \right) |\overline{\mu}\rangle = \overline{\mu}|\overline{\mu}\rangle. \tag{13}$$

The system states $|\overline{\mu}\rangle$ depend on the eigenvalue of $\mathbf{Q_A}$. Let us assume the initial state to be a direct product of a non-degenerate eigenstate of $\mathbf{H_S}$, $|\nu\rangle$, and $|\phi(r_0)\rangle$:

$$|\Psi(0)\rangle = |\nu\rangle\,|\,\phi(r_0)\rangle. \tag{14}$$

Introducing complete set of exact eigenstates in the above equation, the wave function at a time T can now be written as

$$|\Psi(T)\rangle = \sum_{i,\mu} \mathrm{e}^{(\mathrm{i}/\hbar)E(a_i,\mu)N\Delta T}\, |a_i\rangle\overline{|\mu\rangle\langle\mu|}\,|\,\nu\rangle\langle a_i||\phi(r_0)\rangle, \tag{15}$$

where the exact instantaneous eigenvalues $E(a_i, \mu)$ can be written as

$$E(a_i, \mu) = E_i^a + \frac{1}{T}\overline{\langle\mu|}\,\mathbf{Q_S}\,\overline{|\mu\rangle} a_i + \overline{\langle\mu|}\,\mathbf{H_S}\,\overline{|\mu\rangle}. \tag{16}$$

Till this point we have not made any approximations, except for ignoring the switching on and switching off times. Now if the measurement interaction is very weak and highly adiabatic, $1/T$ is very small, so that

$$\overline{|\mu\rangle} = |\mu\rangle + \mathcal{O}(1/T) + \ldots. \tag{17}$$

In the large T limit, one can assume the states to be unperturbed, i.e., $|\mu\rangle \approx |\mu\rangle$. The energy eigenvalues now assume the form

$$E(a_i, \mu) \approx E_i^a + \frac{1}{T}\langle\mu|Q_S|\mu\rangle a_i + \langle\mu|H_S|\mu\rangle. \quad (18)$$

Assuming the states to be unperturbed and the energy to be first order in $1/T$ amounts to doing a first-order perturbation theory. In this approximation $\langle\bar{\mu}||\nu\rangle \approx 0$, and the sum over μ disappears and only the term where $\mu = \nu$ survives. This allows us to write the apparatus part of the exponent again in the operator form

$$|\Psi(T)\rangle \approx \mathrm{e}^{-(i/\hbar)\mathbf{H}_\mathbf{A}T-(i/\hbar)Q_\mathrm{A}\langle Q_S\rangle_\nu-(i/\hbar)\langle\mathbf{H}_\mathbf{S}\rangle_\nu T}|\nu\rangle\,|\phi(r_0)\rangle. \quad (19)$$

Since $\mathbf{Q_A}$ is an operator conjugate to $\mathbf{R_A}$, it will act as a generator of translation for $|r\rangle$ states. The second term in the exponent will shift the centre of the packet $|\phi(r_0)\rangle$ by an amount $\langle\nu|\mathbf{Q_S}|\nu\rangle$

$$|\psi(T)\rangle = \mathrm{e}^{-(i/\hbar)\mathbf{H}_\mathbf{A}T-(i/\hbar)\nu T}|\nu\rangle|\phi(r_0 + \langle\mathbf{Q_S}\rangle_\nu)\rangle.$$

Thus, at the end of the measurement interaction, the centre of the apparatus packet $|\phi(r_0)\rangle$ shifts by $\langle\nu|\mathbf{Q_S}|\nu\rangle$. The apparatus thus records, not the eigenvalue of $\mathbf{Q_S}$ as in eq. (8), but its expectation value in the initial *unknown* state $|\nu\rangle$. Not only that, within this approximation the system and the apparatus are not entangled.

Some clarifications

Protective measurements were widely misunderstood and resulted in a lot of criticim[11–19]. Here we list some features of protective measurements which should clarify various issues that were raised.

- Protective measurements do not require that the state of the system be a priori fully known.
  Example: atom in a trap where one may not know the exact potential, but does know that the atom will be in the ground state.
  One may have made an energy measurement on a system to know that it is in a particular energy eigenstate, but without the knowledge of the Hamiltonian one cannot know what the eigenstate is and hence finding the expectation value of an observable is not possible.
- The shift in the pointer state is proportional to the expectation value of the observable being measured.
- The expectation value is obtained in a single measurement on a single system.
- As shown in the preceding analysis, an observable whose expectation value is measured, need not commute with the Hamiltonian of the system. An objection with a contradicting claim was raised by Uffink[19]. Gao[20] pointed out the flaw in the argument.
- The system is not entangled with the apparatus after the measurement.
- The state of the system does not change after the measurement (within the approximation used).
- Expectation value of another operator can be measured, after the measurement of one.

Generalized protective measurements

It has also been demonstrated[8] that protective measurements can, in principle, be performed even in the most general case where $[\mathbf{Q_A}, \mathbf{H_A}] \neq 0$ and $[\mathbf{Q_S}, \mathbf{H_S}] \neq 0$. However in actual practice, finding the right observables for the apparatus, and satisfying all the constraints may be a formidable challenge. This is so because in this case, the initial apparatus is not supposed to be a packet of eigenstates of the operator conjugate to $\mathbf{Q_A}$. Rather it is supposed to be a packet of eigenstates of the operator conjugate to an operator $\mathbf{Y}$ defined as

$$\mathbf{Y} = \sum_j \langle\mathbf{Q_A}\rangle_{a_j}|a_j\rangle\langle a_j|, \quad (20)$$

where various entities have the same meaning as in the preceding analysis. Whether such an operator can always be found in practice, is an open question.

In this analysis we have not considered the dynamical effect of the 'free' Hamiltonian of the apparatus. This Hamiltonian will cause the spreading of the packet of the initial apparatus state. In normal course of action, one would have ignored this effect. However, since the protective measurements are supposed to be adiabatic, the effect of the Hamltonian of the apparatus, though small, will be cumulative. In other words, the pointer packet may spread considerably during the course of protective measurement interaction. Finding the centre of a large packet, to read out the measured expectation value, would be a difficult task and one may have to apply some special techniques to do so[8].

Does it really work for a single system?

The success of protective measurements crucially depends on the assumption that the entanglement between the system and the apparatus can be made negligibly small, but the system will still shift the pointer state by a finite amount.

Let us now quantify the effect of the terms we have neglected till now. If we consider the state of the system and apparatus to be perturbed to first order in $1/T$, as opposed to being unperturbed in the preceding analysis, the final state, to next higher order in $1/T$, would look like

$$|\Psi(T)\rangle \approx e^{-(i/\hbar)H_A T-(i/\hbar)^{\nu T}}|\nu\rangle|\phi(r_0+\langle \mathbf{Q_S}\rangle_\nu)\rangle$$

$$+\frac{1}{T}\sum_{\mu(\neq\nu)}\alpha_{\mu\nu}e^{-(i/\hbar)H_A T-(i/\hbar)^{\mu T}}|\mu\rangle|\phi(r_0+\langle \mathbf{Q_S}\rangle_\mu)\rangle, \quad (21)$$

where $\alpha_{\mu\nu}$ involve matrix elements of $\mathbf{Q_S Q_A}$ among various unperturbed states, and unperturbed energies. The above is an entangled state and in a real measurement there is a probability that the original state of the system, which was $|\nu\rangle$ to begin with, gets changed to (say) $|\mu\rangle$ and the apparatus state gets shifted by $\langle \mathbf{Q_S}\rangle_\mu$. The probability of this happening is proportional to $1/T^2$.

One can see that by increasing T and weakening the interaction, the probability of the protective measurement failing can be made smaller, but can never be made zero. In general, it has been rigorously shown that the *state disturbance* in protective measurements scales as $1/T^2$ (ref. 21). This indicates that although a practical implementation of protective measurement is possible, for a single unknown state, one can never be sure that the protective measurement has yielded the expectation value in the original state. Hence it cannot be used to argue for a strict objective reality of the wavefunction.

Experimental realization

The idea of protective measurements was proposed more than two decades ago, but an experimental demonstration of the same is still lacking. The reason for this is that there are several constraints which the system and the apparatus should satisfy before one can carry out a successful protective measurement. The adiabatic nature of interaction may also present some difficulty.

Cold atoms for testing protective measurements

A proposal was made for testing protective measurements using cold atoms in a Stern–Gerlach-like set-up[22]. We briefly describe the same in the following (see Figure 1). Low velocity of cold atoms may be exploited for achieving adiabaticity to some degree. The Hamiltonian of the atom with mass m and magnetic moment μ can be written as

$$\mathbf{H} = \frac{\mathbf{P}^2}{2M} - \mu B_0 \vec{\boldsymbol{\sigma}}\cdot\vec{n}_0 - \mu g(t) B_i \mathbf{x}\,\vec{\boldsymbol{\sigma}}\cdot\vec{n}. \quad (22)$$

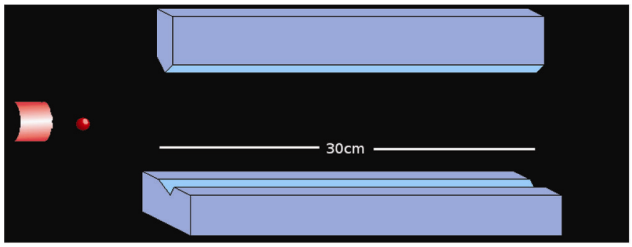

Figure 1. A schematic diagram of a Stern–Gerlach-type experiment with cold atoms.

Initial system–apparatus state

$$|\Psi(0)\rangle = |+\rangle|\phi_p(0,\varepsilon)\rangle, \quad \vec{\sigma}\cdot\vec{n}_0|\pm\rangle = \pm|\pm\rangle, \quad (23)$$

where $|\phi_p(0,\varepsilon)\rangle$ is a Gaussian wavepacket in the momentum space, with zero average momentum and a width ε in momentum space. The state after a time T is given by

$$|\Psi(T)\rangle = e^{-iHT}|+\rangle|\phi_p(0,\varepsilon)\rangle. \quad (24)$$

The position operators $\mathbf{x}$ will act as a generator of translation in momentum space, and the system-apparatus state, at the end of the measurement interaction, is given by

$$|\Psi(T)\rangle \approx e^{-iP_x^2 T/2M} e^{-i\mu B_0 T} e^{\mu B_i \vec{n}_0\cdot\vec{n}x}|+\rangle|\phi_p(0,\varepsilon)\rangle$$

$$= e^{-i\mu B_0 T}|+\rangle|\phi_p(\langle \mu B_i\vec{\sigma}\cdot\vec{n}\rangle_+, \varepsilon(T))\rangle. \quad (25)$$

The state of the spin and the state of the atom are disentangled and $|\phi_p(\langle \mu B_i\vec{\sigma}\cdot\vec{n}\rangle_+, \varepsilon(T))\rangle$ is a Gaussian with a momentum $\langle \mu B_i\vec{\sigma}\cdot\vec{n}\rangle_+$, and a width $\sqrt{\varepsilon^2 + T^2/M^2\varepsilon^2}$.

If one uses the following experimental parameters: $\varepsilon = 1$ mm, $L = 30$ cm, $B_0 = 1$ Gauss and atom velocity: $v \sim 1$ cm/s, then momentum shift $\gg$ momentum spread.

As the atom travels after coming out of the interaction region, its position will shift, because of the non-zero average momentum. Position shift after 30 sec evolution after interaction will about 2 cm.

Conclusion

We have described protective measurements which are a promising tool for probing single systems, i.e. systems for which an ensemble of identical quantum states is not available. Protective measurements lend a new experimental meaning to the quantum expectation value which, traditionally, has meaning only in the context of many eigenvalue measurements over an ensemble. Although protective measurements can be practically used to measure the expectation value in a single measurement, the non-zero error which is always present, rules out using the same to assign any object reality to the wavefunction. An experimental test of protective measurements should be possible and one proposal for the same has been described here.

1. von Neumann, J., *Mathematische Grundlagen der Quantenmechanik*, Springer, 1932; English translation in *Quantum Theory and Measurements* (eds Wheeler, J. A. and Zurek, W. H.), Princeton University Press, 1983.
2. Everett III, H., 'Relative state' formulation of quantum mechanics. *Rev. Mod. Phys.*, 1957, **29**(3), 454–462.
3. Ghirardi, G. C., Rimini, A. and Weber, T., Unified dynamics for microscopic and macroscopic systems. *Phys. Rev. D*, 1986, **34**(2), 470–491.

4. Aharonov, Y. and Vaidman, L., Measuring the Schrödinger wave of a single particle. *Phys. Lett. A*, 1993, **178**(1–2), 38–42.
5. Aharonov, Y., Anandan, J. and Vaidman, L., Meaning of the wave function. *Phys. Rev. A*, 1993, **47**(6), 4616–4626.
6. Aharonov, Y., Anandan, J. and Vaidman, L., The meaning of protective measurements. *Found. Phys.*, 1996, **26**(1), 117–126.
7. Aharonov, Y., Masser, S., Popescu, S., Tollaksen, J. and Vaidman, L., Adiabatic measurements on metastable systems. *Phys. Rev. Lett.*, 1996, **77**(6), 983–987.
8. Hari Dass, N. D. and Qureshi, T., Critique of protective measurements. *Phys. Rev. A*, 1999, **59**(4), 2590–2601.
9. Vaidman, L., Protective measurements. In *Compendium of Quantum Physics: Concepts, Experiments, History and Philosophy* (eds Greenberger, D., Hentschel, K. and Weinert, F.), Springer, 2009, pp. 505–508.
10. Gao, S. (ed.), *Protective Measurement and Quantum Reality*, Cambridge University Press, 2015.
11. Schwinger, J., Quantum mechanics: not mysterious. *Science*, 1993, **262**(5135), 826–827.
12. Unruh, W. G., Reality and the measurement of the wave function. *Phys. Rev. A*, 1994, **50**(1), 882–887.
13. Rovelli, C., Comment on 'Meaning of the wave function'. *Phys. Rev. A*, 1994, **50**(3), 2788–2792.
14. Ghose, P. and Home, D., An analysis of the Aharonov–Anandan-Vaidman model. *Found. Phys*., 1995, **25**(7), 1105–1109.
15. Choudhury, S. R., Dasgupta, P. and Datta, K., Can the wave function of a single system be measured? University of Delhi preprint, 1995.
16. Alter, O. and Yamamoto, Y., Protective measurement of the wave function of a single squeezed harmonic-oscillator state. *Phys. Rev. A*, 1996, **53**(5), R2911–R2914.
17. Alter, O. and Yamamoto, Y., Reply to 'Comment on 'Protective measurement of the wave function of a single squeezed harmonic-oscillator state'. *Phys. Rev. A*, 1997, **56**(1), 1057–1059.
18. Samuel, J. and Nityananda, R., Comment on 'Meaning of the wave function'. arXiv:gr-qc/9404051.
19. Uffink, J., How to protect the interpretation of the wave function against protective measurements. *Phys. Rev. A*, 1999, **60**(5), 3474–3481.
20. Gao, S., On Uffink's criticism of protective measurements. *Stud. Hist. Philos. Mod. Phys.*, 2013, **44**(4), 513–518.
21. Schlosshauer, M., State disturbance and pointer shift in protective quantum measurements. *Phys. Rev. A*, 2014, **90**(5), 052106-1 to 052106-12.
22. Hari Dass, N. D., Cold atoms for testing quantum mechanics and parity violation in gravitation. arXiv:quant-ph/9908085.

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